# Symbol-Level Precoding With Constellation Rotation in the Finite Block Length Regime

Steven Kisseleff, *Member, IEEE*, Wallace A. Martins, *Senior Member, IEEE*, Symeon Chatzinotas, *Senior Member, IEEE*, and Björn Ottersten, *Fellow, IEEE*

***Abstract*— This letter tackles the problem of optimizing the parameters of a symbol-level precoder for downlink multi-antenna multi-user systems in the finite block length regime. Symbol-level precoding (SLP) is a non-linear technique for multi-user wireless networks, which exploits constructive interference among co-channel links. Current SLP designs, however, implicitly assume asymptotically infinite blocks, since they do not take into account that the design rules for finite and especially short blocks might significantly differ. This letter fills this gap by introducing a novel SLP design based on discrete constellation rotations. The rotations are the added degree of freedom that can be optimized for every block to be transmitted, for instance, to save transmit power. Numerical evaluations of the proposed method indicate substantial power savings, which might be over 99% compared to the traditional SLP, at the expense of a single additional pilot symbol per block for constellation de-rotation.**

***Index Terms*— Symbol-level precoding (SLP), finite block length, short packets, spatial diversity.**

## I. Introduction

THE capacity gains stemming from full frequency-reuse downlink transmissions is significantly impacted by the underlying multi-user interference (MUI). Multi-antenna transmitters with channel-state information (CSI) can tackle MUI via precoding. Classical linear precoding usually targets interference mitigation [1] or even cancellation [2], which can be obtained via symbol-independent linear precoders designed solely based on the available CSI [3], [4]. A more efficient strategy is to constructively overlap the signals propagating via different paths using symbol-level precoding (SLP) [5].

SLP encompasses a set of techniques that can benefit from the otherwise harmful MUI effects by shaping the transmitted waveforms so as to induce constructive interference at each user [6], [7]. In other words, SLP does not target interference mitigation, but rather it exploits MUI. SLP is a non-linear technique that uses CSI along with users' data symbols to form the precoder. Several SLP schemes exploiting different properties of the communication setup have been proposed [5]–[11]. In [5], the authors have shown how to split the interference into constructive and destructive parts, and how to eliminate the destructive part via a selective channel inversion. In [6], the authors have shown how to rotate the otherwise destructive interference components so as to transform them into useful power. In [7], [8], the authors have shown different optimization strategies, including sum power minimization and max-min fairness problem for phase shift keying (PSK) modulations. Those results have been extended to multi-level modulations in [9]. Other aspects related to, for instance, non-linear channels [10] or faster-than-Nyquist redundant signaling [11] have also been addressed. We refer the reader to the comprehensive survey in [12] for further details.

Recently, the authors in [13] have discovered a new degree of freedom for the design of SLP-based multi-user systems, namely: the rotation of the symbol constellation of each user as part of the overall precoding procedure. They have assumed a continuous-phase rotation for each user and formulated a non-convex optimization problem to determine the optimal transmit vectors in each symbol interval along with the optimal constellation rotation for all symbol intervals within the coherence time. In this context, performance gains of 3-6 dB have been reported. However, the work in [13] does not allow for general packet lengths, and is not suitable for packets substantially smaller than the channel coherence time, since the solution to the underlying optimization problem needs to be obtained before each packet transmission. Besides, the continuous-phase approach has some practical challenges when it comes to informing the receivers about the optimal constellation de-rotation.[1] In practice, the transmitter could notify the receivers about the respective phase offsets using one of the following two strategies: (i) transmit the encoded phase as part of the data packet; or (ii) transmit a sequence of known pilot symbols. The problem with the first method is the overhead, which scales with the accuracy of phase quantization and with the number of users. The problem with the second method is the necessity of phase estimation, which may only be sufficiently accurate in case of a large number of pilots. Moreover, phase errors can substantially increase the symbol-error rate (SER). Hence, both strategies lead to a tradeoff between overhead and SER.

The contributions of this letter are as follows:

- We motivate the analysis of SLP in the finite block length regime from the statistical perspective;

Manuscript received February 19, 2021; revised March 12, 2021; accepted April 4, 2021. Date of publication April 8, 2021; date of current version July 10, 2021. This work was supported by Luxembourg National Research Fund (FNR) under the CORE project RISOTTI C20/IS/14773976. The associate editor coordinating the review of this letter and approving it for publication was P. Yang. *(Corresponding author: Steven Kisseleff.)*

The authors are with the Interdisciplinary Centre for Security, Reliability and Trust (SnT), University of Luxembourg, 1855 Luxembourg City, Luxembourg (e-mail: steven.kisseleff@uni.lu; wallace.alvesmartins@uni.lu; symeon.chatzinotas@uni.lu; bjorn.ottersten@uni.lu).

Digital Object Identifier 10.1109/LCOMM.2021.3071838

[1] Note that these challenges can be avoided if phase synchronization is applied in each block. However, the overhead required for the synchronization is only justified when the block duration is similar to the coherence time. In our work, we assume that the coherence time is much longer than the average block duration.

This work is licensed under a Creative Commons Attribution 4.0 License. For more information, see https://creativecommons.org/licenses/by/4.0/

- A small discrete set of possible constellation rotations is selected, which is very promising from the perspective of optimization complexity as well as practicality and robustness of the solution;
- An implementation strategy based on a computationally intensive offline calculation and a very quick online calculation is proposed;
- A complexity analysis is provided and the impact of the power consumption during online calculation on the overall performance is investigated.

This letter is organized as follows. The system model for the signal transmission using SLP and ZF precoding is described in Section II. In Section III, a novel method for SLP in finite block length regime is proposed. The performance of the proposed method is evaluated in Section IV and compared with the performance of benchmark schemes. Section V concludes the letter.

## II. System Model

We consider a classical multi-user multiple-input single-output (MU-MISO) downlink system with $T$ transmit antennas and $N \leq T$ single-antenna users.[2] The base station (BS) simultaneously transmits the symbols $s_n[k]$ intended for each user $n$ in each symbol interval $k$. These symbols are stacked in the vector $\mathbf{s}[k] = [\, s_1[k]\; s_2[k]\; \ldots\; s_N[k]\,]^\mathsf{T}$. If ZF precoding is applied using matrix $\mathbf{W}_{\text{ZF}} \in \mathbb{C}^{T\times N}$, we obtain the transmit vector

$$\mathbf{x}[k] = \mathbf{W}_{\text{ZF}}\mathbf{s}[k]\,. \tag{1}$$

If SLP is applied, the transmit vector $\mathbf{x}[k]$ is optimized in such a way to guarantee that the scaled version of the received signal at the $n$th user pertains to the extended constellation region $\mathcal{S}_n[k] \subset \mathbb{C}$ associated with symbol $s_n[k]$. In a quadrature PSK (QPSK) constellation, for instance, if $s_n[k] = (1+\mathrm{j})/\sqrt{2}$, with $\mathrm{j}^2 = -1$, then the corresponding extended constellation region would be $\mathcal{S}_n[k] = \{a + \mathrm{j}b \,|\, a \geq 1/\sqrt{2}, b \geq 1/\sqrt{2}\}$. As there are four constellation points, there are four different extended constellation regions.[3] For further details on the definition of $\mathcal{S}_n[k]$ for different constellations, please refer to [12] and references therein.

We assume that perfect CSI is available at the BS and that the channel coherence time is much longer than the symbol interval. These assumptions allow us to consider that the channel coefficients of the $n$th user, stacked in vector $\mathbf{h}_n \in \mathbb{C}^{T\times 1}$, are fixed during the transmission of several symbols. In this context, the receive signal is

$$y_n[k] = \mathbf{h}_n^\mathsf{T}\mathbf{x}[k] + \omega_n[k]\,, \tag{2}$$

where the complex-valued noise signal $\omega_n[k]$ has variance $\sigma_n^2$.

The optimization of $\mathbf{x}[k]$ is typically done with respect to the transmit power under some quality-of-service (QoS) constraints. An example of such an optimization problem is similar to [12]

$$\underset{\mathbf{x}[k]\in\mathbb{C}^{N\times 1}}{\text{minimize}} \; \|\mathbf{x}[k]\|_2^2\,,$$
$$\text{subject to: } \mathbf{h}_n^\mathsf{T}\mathbf{x}[k] \in \sigma_n\sqrt{\gamma_n}\cdot\mathcal{S}_n[k], \quad \forall n, \tag{3}$$

where $\gamma_n$ denotes the signal-to-interference-plus-noise ratio (SINR) of the $n$th user. The expression $\sigma_n\sqrt{\gamma_n}\cdot\mathcal{S}_n[k]$ denotes the set containing all elements of $\mathcal{S}_n[k]$ scaled by $\sigma_n\sqrt{\gamma_n}$. Using the geometry of the extended SLP regions, the problem can be formulated as a non-negative least-squares (NNLS) problem [14], which can be efficiently solved [15], [16]. Usually, it is possible to substantially reduce the transmit power due to the added degrees of freedom in the SLP formulation, as compared to the ZF precoder. Besides, a distinct advantage of SLP over ZF precoding is a slight improvement of the symbol error rate, since the point $\mathbf{h}_n^\mathsf{T}\mathbf{x}[k]$ is often further apart from the boundary of the detection regions, as illustrated in Fig. 1, thus reducing the probability of error.

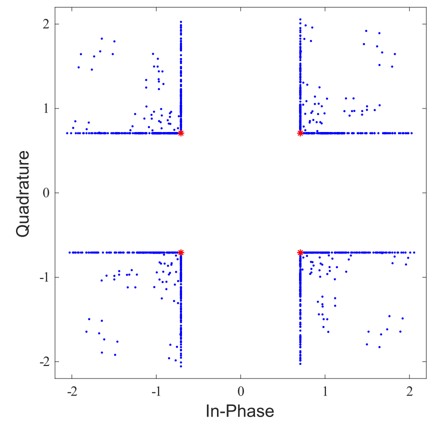


Fig. 1. Scatter plot of the noiseless received signal for one of six jointly precoded users using QPSK modulation, with $\sigma_n\sqrt{\gamma_n} = 1$. The majority of signals lie on the boundary of the four extended constellation regions.

In general, the solution to the underlying NNLS problem will be on the boundary of the non-negative $2N$-dimensional *orthant*,[4] i.e. on the boundary of regions $\sigma_n\sqrt{\gamma_n}\cdot\mathcal{S}_n[k]$ for some of the users $n$. Fig. 1 indeed shows most of the optimal points $\mathbf{h}_n^\mathsf{T}\mathbf{x}[k]$ lying on the boundary of regions $\sigma_n\sqrt{\gamma_n}\cdot\mathcal{S}_n[k]$ — a behavior observed not only in our own extensive simulations but also in other works from the literature [13]. This indicates that it might be possible to obtain power savings if the symbols $s_n[k]$ that generated the noiseless received signals on the boundary of $\sigma_n\sqrt{\gamma_n}\cdot\mathcal{S}_n[k]$ were precoded along with a different set of symbols from the other users. In other words, some combinations of symbols comprising vector $\mathbf{s}[k]$ favor power savings when compared to other combinations. This fact can be exploited in a finite block length regime by optimizing the combinations for each block, as we shall discuss in the following.

## III. SLP in Finite Block Length Regime

In this section we propose a method to obtain power savings based on the broken symmetry of the symbol constellations in the finite block length regime.

[2]Typically, in order to maximize capacity while still allowing for the channel-matrix invertibility, the number of users to be served is equal to the number of transmit antennas at the base station, i.e. $T = N$.

[3]For higher-order multi-level constellations, an extended constellation region might actually be comprised of a single point [9].

[4]There are $2N$ real dimensions to be optimized for $N$ users.

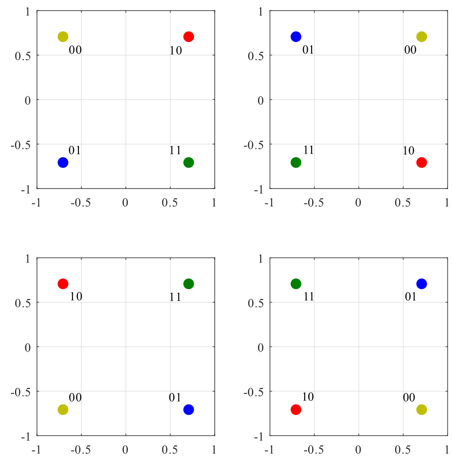


Fig. 2. Possible rotations of a QPSK constellation.

### A. General Approach

For the sake of motivation, let us first consider the infinite block length regime. As mentioned earlier, the combination of symbols comprising vector $\mathbf{s}[k]$ impacts the SLP performance. We note that most of the symbol constellations used in practice are circularly symmetric, e.g. PSK and rectangular quadrature amplitude modulation (QAM). Consider, for instance, an $M_n$-PSK constellation for the $n$th user. The symmetry of the $M_n$-PSK constellation can be illustrated by rotating all its points by multiples of $\frac{2\pi}{M_n}$, after which the constellation points overlap with the original ones, as depicted in Fig. 2 for $M_n = 4$. Hence, we note that there are $M_n$ different rotations of the constellation, which can be achieved via multiplication of symbols from the original constellation by $\mathrm{e}^{\mathrm{j}\phi_n}$, with $\phi_n \in \{0, \frac{2\pi}{M_n}, 2\frac{2\pi}{M_n}, \ldots, (M_n-1)\frac{2\pi}{M_n}\}$. The resulting constellations are equivalent to the original one. The number of combinations of constellation points of all users is given by $Q = \prod_n M_n$. As an example, with $N = 8$ and $M_n = 4, \forall n$, we then obtain $Q = 4^8 = 65536$ different possibilities for vector $\mathbf{s}[k]$. Each point of the symbol constellation of each user may occur with equal probability. Correspondingly, the rotation of the constellation by a multiple of $\frac{2\pi}{M_n}$ does not lead to any performance gain in the infinite block length regime.

Now, consider transmissions with a finite block length $L$ comprised of the vectors $\mathbf{s}[k], \mathbf{s}[k-1], \ldots, \mathbf{s}[k-L+1]$. Even in the case when all vectors in a block are different, only a small fraction $L/Q$ of the total number of symbol combinations is used; for instance, when $L = 128$ and $M_n = 4, \forall n$, then $\frac{L}{Q} = \frac{128}{65536} \approx 0.2\%$.[5] For block lengths $L$ that are currently used in practical systems, the majority of possible symbol combinations actually do not appear in a block. *This fact breaks the symmetry of the symbol constellation, since not all symbols are equally probable for the precoder in L transmissions.* Correspondingly, the rotation of each constellation in the finite block length regime may lead to different performance, i.e. different transmit power. By selecting in each block the optimal constellation rotation $\phi_n$ for each user $n$, significant performance gains can be obtained. A similar observation has been made in [13], where the block length has been restricted by the coherence time.[6] In order to obtain the optimal transmit vectors in each block of length $L$, the problem (3) needs to be modified:

$$\underset{\substack{\mathbf{x}[k-\ell],\phi_n \\ \ell\in\{0,\ldots,L-1\} \\ n\in\{1,\ldots,N\}}}{\text{minimize}} \ \frac{1}{L}\sum_{\ell=0}^{L-1} \|\mathbf{x}[k-\ell]\|_2^2 \ ,$$

$$\text{subject to: } \mathbf{h}_n^{\mathsf{T}}\mathbf{x}[k-\ell] \in \sigma_n\sqrt{\gamma_n}\,\mathrm{e}^{\mathrm{j}\,\phi_n}\cdot\mathcal{S}_n[k-\ell], \forall n, \ell,$$
$$\phi_n \in \left\{0, \frac{2\pi}{M_n}, \ldots, (M_n-1)\frac{2\pi}{M_n}\right\}, \quad \forall n. \quad (4)$$

Similarly to [13], this problem is non-convex and cannot be solved directly using the methods of convex optimization. Below, we propose a practical solution, which substantially reduces the optimization complexity.

### B. Practical Implementation Aspects

We assume a long coherence time, i.e. corresponding to the combined length of a large number of blocks. Hence, possible variations of the CSI are small and rare, which allows us to decouple the offline optimization from the online calculation.

At first, consider a straightforward implementation approach, which consists of the following steps:

- For each combination of rotations, calculate the optimal transmit vectors $\mathbf{x}[k-\ell]$ for all symbols of the block.
- Select the combination with the lowest total power consumption via $\arg\min_{\phi_n,\forall n} \sum_{\ell=0}^{L-1} \|\mathbf{x}[k-\ell]\|_2^2$.

This implementation requires the optimization to be performed online. The total number of possible combinations of rotations is[7] $S = \prod_n M_n$. The resulting complexity corresponds to $SL$ executions of the NNLS algorithm and $SL$ additions (to determine the average consumed power per block). According to [15], the complexity of each iteration of the fast NNLS algorithm equals $\mathcal{O}(TN^2)$. The algorithm is run until convergence, which may require up to $N$ iterations. Hence, in the worst case, the total complexity of the online execution of this implementation is equal to $\mathcal{O}(SLTN^3)$.

Alternatively, the optimization can be partially done offline by exploiting the symmetry of the symbol constellations. In fact, the rotated constellations are identical to the respective original one, if they are rotated by multiples of $\frac{2\pi}{M_n}$. Hence, we propose to calculate the transmit vectors for all symbol combinations of all users offline. Specifically, the optimization in (3) is solved for each symbol combination using the NNLS method. From the transmit vectors, the corresponding power consumption is obtained and stored in a look-up table (LUT).

The online calculation comprises the identification of the symbols pertaining to the rotated symbols of the block and the summation of the corresponding transmit power obtained

[5]In practice, the ratio of used combinations to the total number of combinations may be even lower due to possible repetitions of symbol combinations.

[6]The authors assume that the block length does not affect the system performance too much, such that long coherence time and correspondingly large block length are favorable.

[7]Note that for PSK modulation, the number of combinations of rotations $S$ is equal to the number of symbol combinations $Q$. This is not true for QAM modulation with $M_n > 4$, where $Q \gg S$.

**Algorithm 1** Proposed Implementation

**Input:** all combinations of symbols $\mathbf{s}[q], 1 \leq q \leq Q$;
**Output:** transmit vectors $\mathbf{x}[k-\ell], \forall \ell, \phi_n, \forall n$;

1: **for** $q = 1$ **to** $Q$ **do** {Offline calculation}
2: Solve (3) for $\mathbf{s}[q]$ using $\mathcal{S}_n[q], \forall n \Rightarrow \mathbf{x}[q] \rightarrow$ LUT;
3: **end for**
4: Set $P[m] = 0,\ 1 \leq m \leq S$; {Online calculation}
5: **for** $m = 1$ **to** $S$ **do**
6: Obtain $\phi_n, \forall n$ pertaining to $m$th comb. of rotations;
7: **for** $\ell = 0$ **to** $L-1$ **do**
8: $\mathbf{s}[q^*] = [s_1[k-\ell]\, \mathrm{e}^{\mathrm{j}\,\phi_1}, \ldots, s_N[k-\ell]\, \mathrm{e}^{\mathrm{j}\,\phi_N}]^\intercal \Rightarrow q^*$;
9: LUT $\rightarrow \mathbf{x}[q^*]$, $\mathbf{x}[k-\ell] = \mathbf{x}[q^*]$;
10: $P[m] \leftarrow P[m] + \|\mathbf{x}[q^*]\|_2^2$;
11: **end for**
12: **end for**
13: $m^* = \arg\min_m P[m]$;
14: Execute lines 6-11 using $m^*$ to get $\mathbf{x}[k-\ell]$.

from the LUT. Then, the combination with the lowest power is selected via full search. This overall complexity of the online calculation is $\mathcal{O}(SLN)$, which is lower than the complexity of a straightforward implementation discussed above by a factor of $TN^2$. The pseudocode of the proposed method is shown in Algorithm 1.

### C. Remarks

As mentioned earlier, the main challenge of the continuous-phase optimization approach is the need to notify the receiver. This drawback is still present with the proposed discrete-phase optimization method. However, the overhead or the required accuracy of pilot phase estimation is much lower due to a very limited set of possible rotations with only $M_n$ points from the entire range $[0, 2\pi]$ for $M_n$-PSK constellations. Specifically, a single pilot symbol for each block of length $L$ is enough to indicate the orientation of the constellation.

The de-rotation of the constellation is another problem of the continuous-phase approach. Interestingly, this problem does not occur with the proposed discrete phase, since the detection can be done using the same constellation in all data packets independently of their actual rotation. The reason is the aforementioned circular symmetry, such that the constellations before and after rotation are identical. Hence, the receiver can simply detect the symbols using the original constellation and assign the bits of the rotated constellation later.

Finally, the complexity of the online calculation is much lower with the proposed method. Specifically, after an offline SLP optimization for all combinations of symbols, the selection of the optimal rotation can be done very quickly for any packet/block length.

## IV. Numerical Results

We consider a set of $N$ randomly scattered users in the distance between 10 m and 100 m from the base station. Furthermore, we assume $T = N$. For the signal transmission, we assume carrier frequency of 2.4 GHz and path loss exponent of 3.5. The channel phase is selected randomly from

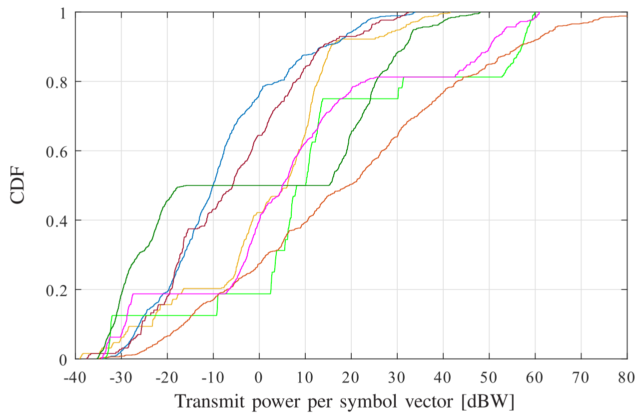


Fig. 3. Examples of transmit power CDFs for $N = 6$ users. Each curve corresponds to a different channel realization.

$[0, 2\pi]$. Due to the complexity of the proposed precoder, which scales exponentially with the number of constellation points, we focus on QPSK transmissions only. We select the transmit power of each precoding scheme such that the scaling of the symbol constellation at the receiver is $\sigma_n\sqrt{\gamma_n} = 10^{-7}$. If a different signal quality is desirable, the transmit power obtained in our simulations needs to be scaled accordingly. This, however, does not change the performance gains, which will be demonstrated in this section. For each set of parameters, we average the consumed power over given block length in 1500 channel realizations and 200 blocks per realization, which results in averaging over 300000 blocks.

We start with the motivation for the selection of the optimal rotation in the finite block length regime. Fig. 3 shows several examples of the cumulative distribution function (CDF) of the transmit power for $N = 6$ users. Different curves in this figure correspond to different channel realizations. While the CDF shape varies from realization to realization, we consistently observe a very large discrepancy between the maximum and the minimum transmit power values of each respective CDF, sometimes leading to a difference in multiple orders of magnitude. With infinite block length, all symbol combinations are equally probable and the average transmit power is likely to be close to the middle or upper part of the CDF. As mentioned earlier, the rotation optimization can be interpreted as selection of the constellation points pertaining to the lowest power consumption. Through this, it is possible for each block to select the symbol combinations in the lower part of the CDF, which leads to a much lower average power consumption and large performance gains.

In Fig. 4, we show the average required transmit power with $N = 3$ users and different block lengths. We can observe that the power consumption with constellation rotation (rotated SLP) is substantially lower than with traditional SLP and ZF solution. Furthermore, we observe that the performance gain compared to the traditional SLP reduces with increasing block length, since the probability of occurrence of each symbol combination becomes more and more balanced. However, since the increase of the consumed power with increasing block length is rather slow, we still can appreciate a gain of 2.7 dB using the proposed method for a block length of 96 symbols. On the other hand, it is clearly favorable to utilize

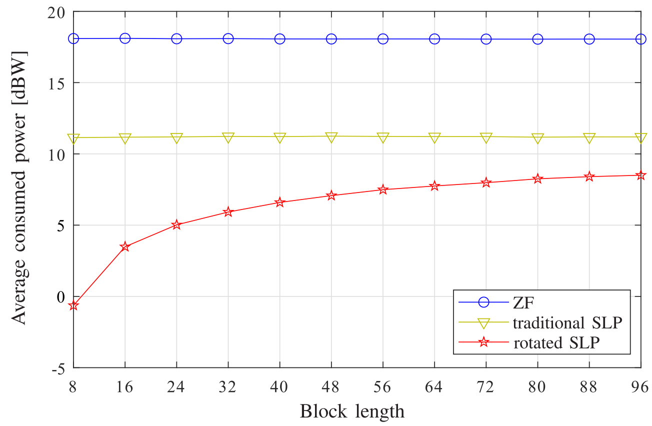


Fig. 4. Average power consumption with $N = 3$ users vs. block length.

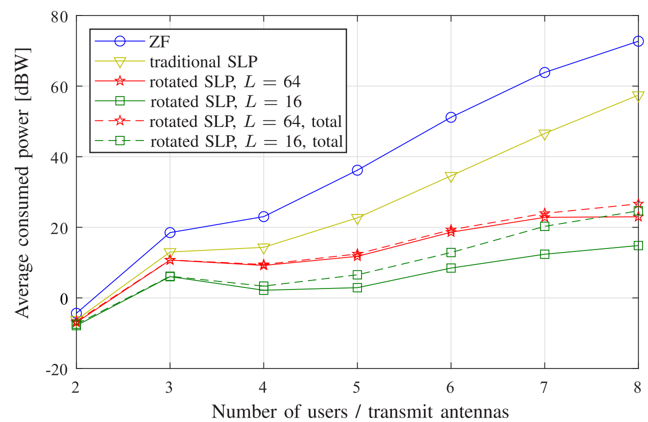


Fig. 5. Average power consumption with $L \in \{16, 64\}$ vs. number of users.

shorter block lengths, such as $L = 16$ symbols per block, which leads to a gain of 7.7 dB.

Fig. 5 shows the average consumed power with two block lengths and different numbers of users. We observe that the power consumption increases with increasing number of users. Furthermore, we observe that the gap between the proposed method and the benchmark schemes (ZF and traditional SLP) increases with increasing number of users and with decreasing block length. Specifically, with a large number of users, e.g. $N = 8$, and $L = 64$ symbols per block, we obtain gains of 49.7 dB and 34.4 dB with respect to ZF and traditional SLP, respectively. With $L = 16$ symbols per block, we obtain gains of 57.9 dB and 42.6 dB with respect to the two benchmark schemes. These gains demonstrate the importance of the short block lengths. Specifically, the gain of 34.4 dB corresponding to 99.96% power savings compared to the traditional SLP is very promising and justifies the loss in spectral efficiency associated with the need to transmit an additional pilot symbol per block. Moreover, we show the results obtained by taking into account the additional power consumption associated with the online calculation part of the proposed implementation. For this, we assume a signal bandwidth of 1 MHz and that each operation (addition, multiplication) requires at most 0.5 nJ, cf. [17]. We observe that the total power consumption still substantially outperforms both baseline schemes. Nevertheless, with $L = 64$ and $N = 8$, we obtain a degradation of 3.6 dB due to the computational complexity, which motivates the development of more advanced search methods in future.

## V. Conclusion

In this work, a novel method of SLP for finite block lengths has been proposed. This method is based on the offline calculation of precoded vectors and the online full search for the optimal rotations of individual symbol constellations. Numerical results indicate a very high performance of the proposed solution in terms of power consumption compared to the existing benchmark schemes, such as traditional SLP and ZF. Since most of the practical communication systems rely on finite block lengths, this method is very promising and relevant for the design of future wireless networks.

The proposed method can be further enhanced via constellation shaping, e.g. by specifically designing the applied forward error correction code in order to favor a certain group of symbols pertaining to a low total transmit power. However, this approach is beyond the scope of this work.

## References


[1] T. K. Y. Lo, "Maximum ratio transmission," *IEEE Trans. Commun.*, vol. 47, no. 10, pp. 1458–1461, Oct. 1999.

[2] M. Joham, W. Utschick, and J. A. Nossek, "Linear transmit processing in MIMO communications systems," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 2700–2712, Aug. 2005.

[3] M. Alodeh *et al.*, "Symbol-level and multicast precoding for multiuser multiantenna downlink: A state-of-the-art, classification, and challenges," *IEEE Commun. Surveys Tuts.*, vol. 20, no. 3, pp. 1733–1757, 3rd Quart., 2018.

[4] E. Björnson, M. Bengtsson, and B. Ottersten, "Optimal multiuser transmit beamforming: A difficult problem with a simple solution structure," *IEEE Signal Process. Mag.*, vol. 31, no. 4, pp. 142–148, Jul. 2014.

[5] C. Masouros and E. Alsusa, "Dynamic linear precoding for the exploitation of known interference in MIMO broadcast systems," *IEEE Trans. Wireless Commun.*, vol. 8, no. 3, pp. 1396–1404, Mar. 2009.

[6] C. Masouros, "Correlation rotation linear precoding for MIMO broadcast communications," *IEEE Trans. Signal Process.*, vol. 59, no. 1, pp. 252–262, Jan. 2011.

[7] M. Alodeh, S. Chatzinotas, and B. Ottersten, "Constructive multiuser interference in symbol level precoding for the MISO downlink channel," *IEEE Trans. Signal Process.*, vol. 63, no. 9, pp. 2239–2252, May 2015.

[8] A. Li and C. Masouros, "Interference exploitation precoding made practical: Optimal closed-form solutions for PSK modulations," *IEEE Trans. Wireless Commun.*, vol. 17, no. 11, pp. 7661–7676, Sep. 2018.

[9] M. Alodeh, S. Chatzinotas, and B. Ottersten, "Symbol-level multiuser MISO precoding for multi-level adaptive modulation," *IEEE Trans. Signal Process.*, vol. 16, no. 8, pp. 5511–5524, Aug. 2017.

[10] D. Spano, M. Alodeh, S. Chatzinotas, and B. Ottersten, "Symbol-level precoding for the nonlinear multiuser MISO downlink channel," *IEEE Trans. Signal Process.*, vol. 66, no. 5, pp. 1331–1345, Mar. 2018.

[11] W. A. Martins, D. Spano, S. Chatzinotas, and B. Ottersten, "Faster-than-Nyquist signaling via spatiotemporal symbol-level precoding for multi-user MISO redundant transmissions," in *Proc. IEEE Int. Conf. Acoust., Speech Signal Process. (ICASSP)*, May 2020, pp. 5090–5094.

[12] A. Li *et al.*, "A tutorial on interference exploitation via symbol-level precoding: Overview, state-of-the-art and future directions," *IEEE Commun. Surveys Tuts.*, vol. 22, no. 2, pp. 796–839, 2nd Quart., 2020.

[13] M. Alodeh and B. Ottersten, "Joint constellation rotation and symbol-level precoding optimization in the downlink of multi-user MISO channels," 2020, *arXiv:2011.03935*. [Online]. Available: https://arxiv.org/abs/2011.03935

[14] J. Krivochiza, J. C. Merlano-Duncan, S. Andrenacci, S. Chatzinotas, and B. Ottersten, "Closed-form solution for computationally efficient symbol-level precoding," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Dec. 2018, pp. 1–6.

[15] R. Bro and S. De Jong, "A fast non-negativity-constrained least squares algorithm," *J. Chemometrics*, vol. 11, no. 5, pp. 393–401, Sep. 1997.

[16] A. Haqiqatnejad, F. Kayhan, and B. Ottersten, "An approximate solution for symbol-level multiuser precoding using support recovery," in *Proc. IEEE 20th Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, Jul. 2019, pp. 1–5.

[17] D. Molka, D. Hackenberg, R. Schöne, and M. S. Müller, "Characterizing the energy consumption of data transfers and arithmetic operations on ×86−64 processors," in *Proc. IEEE Int. Conf. Green Comput.*, Aug. 2010, pp. 123–133.