Data-Oriented View for Convolutional Coding With Adaptive Irregular Constellations

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***Abstract*— Current wireless systems offer various use-cases where conventional channel capacity focused performance criteria might not apply. Following the similar perspective, convolutional encoding can find more room due to its low complexity and low decoding delay. Besides, it has been also shown that error performance of a convolutional encoder can be improved further by using adaptive irregular constellations. A new performance measure, data-oriented approach, was recently proposed for the transmission of small data packets, i.e. mission-critical IoT applications, over fading channels. In this letter, delay performance gain resulting from convolution coding optimized irregular constellations is investigated. Then, we derive a new performance criterion based on delay and finite block length constraints. Based on this criterion, we design irregular constellations together with convolutional coding for short packet transmission.**

***Index Terms*— Small data transmission, delay outage rate, constellation design, error correction coding.**

I. Introduction

TRADITIONAL broadband wireless systems have been designed for large packet sizes without imposing delay constraints [1]. More refined analysis of those metrics can be found in [2]. Without delay constraints, channel coding techniques utilizing the iterative decoding process; such as turbo and low-density parity check (LDPC) codes, gained considerable popularity for decades and there was a tense competition in channel code design for reaching Shannon channel capacity. Then, polar coding was introduced in [3]; it is the first channel coding technique achieving the channel capacity over symmetric binary input discrete memoryless channels [3].

However, it might not be possible to deploy the powerful coding techniques in every single scenario due to budget constraints, delay requirements and implementation complexity in current wireless systems. The low-throughput devices are mainly low-powered low-rate simple sensors/actuators [4] so the superiority of capacity-approaching/achieving codes might disappear in those cases. For the low latency communication, iterative decoding structure can cause an outage in communication after exceeding predefined delay constraint when the number of iterations is high [5]. For instance, the superiority of LDPC codes might disappear in ultra-reliable low latency communication (URLLC) use cases where the frame length is expected to be less than four hundred bits [6].

Nevertheless, convolutional encoders are still used in current wireless systems. For instance, tail-biting convolutional encoders are supported in the downlink of narrow band Internet of Things (NB-IoT) at the user equipment [7]. In addition, terminated and tail-biting convolutional encoders might be promising for short packet transmission over sensor networks. In fact, the former one is more preferable due to having less data rate loss [8]. More interestingly, it was shown that convolutional encoders can outperform polar codes for medium-sized packets in low latency scenarios [9]. Similar to those, the advantages of convolutional encoders over capacity-approaching codes were presented in [10] with strict delay constraints.

Recently, the availability of optimization techniques results in existing growth of using unequally spaced, *irregular*, constellations and the superiority of those constellations over conventional symmetric constellations were presented in [11]. A comprehensive overview about deploying irregular constellation designs and its practicability issues was given in [12]. For convolutionally coded transmission, [13] presents the error performance analysis when irregular constellations are deployed and the optimized irregular constellations obtained from this analysis were presented in [14]. However, these studies were based on long term average channel characteristics, so called channel-oriented approach, and lacked of reflecting any specific parameter related with individual data transmission.

In the channel-oriented approach, the error analysis was mainly based on fading statistics and a constellation to be used. However, future mission-critical IoT applications might require different parameters, amount of information bits to be transmitted and available bandwidth, to maintain certain quality-of-service (QoS) constraints for particular short-time packet transmission [15], [16]. When short packet transmission is considered, Shannon capacity is not achievable and the

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capacity formula assuming cannot be adopted since it lacks of characterizing the reliability and latency. In those cases, [17] proposed a more refined analysis of the maximum achievable rate as a function of the block length and target decoding error probability. Then, this non-asympotic maximal achievable rate analysis was extended to various practical systems [18]–[20].

In this letter, the use of irregular constellation first investigates through decoding delay. It is shown that irregular constellations which were optimized based on the generalized error probability can provide lower decoding delay in addition to providing higher reliability. Then, data-oriented approach in constellation design is introduced for convolutional coded finite block length transmission. The simulation results show that the irregular constellations based on individual data transmission yields considerable gain for a given block length and target error probability.

II. System Model and Generalized Error Analysis

In a rate-r convolutional coded transmission, the information bits in lth frame $\mathbf{b}_l = [b_{l,1} \cdots b_{l,N_c}]$ are encoded (one frame has N_c information bits) and the encoded bits, $\mathbf{c}_l = [c_{l,1} \cdots c_{l,N_s}]$ where $r = N_c/N_s$, are fed to the bit-to-symbol mapper in which transmitting symbols with a length of $L = N_s/\log 2(M)$, $\mathbf{s}_l = [s_{l,1}, \cdots, s_{l,N_s/\log 2(M)}]$, are assigned from a choice of constellation alphabet χ of size M. Then, the received signal for the i^{th} symbol in the l^{th} frame can be written as

$$r_{l,i} = h_l s_{l,i} + n_{l,i}, \tag{1}$$

where $n_{l,i}$ is the additive white Gaussian noise (AWGN) sample with zero-mean and $N_0/2$ noise variance per dimension and $h_l s_{l,i}$ is the fading channel coefficient which is random but remain constant over the one block length. In the receiver side, soft-decision Viterbi decoding is used by assuming that the perfect channel state information (CSI) and corresponding constellation, χ, are known throughout the decoding process and instantaneous received SNR can be explicitly defined as $\gamma = |h_l|^2 |s_{l,i}|^2 / |n_{l,i}|^2$.

The error performance calculation of convolutional encoder along with any type of constellation, P_b, comes with the cost of increased complexity and its details can be found in [21], [22]. Basically, the analysis therein was based on constructing product-state matrix, $\mathbf{S}(I)$ and each entry of $\mathbf{S}(I)$ is formulated as

$$[\mathbf{S}(I)]_{(u,v),(\bar{u},\bar{v})} = \Pr(u \to \bar{u}|u) \sum_n p_n I^{a(u\to\bar{u}) \oplus a(v\to\bar{v})} D_{(u,v),(\bar{u},\bar{v})}, \tag{2}$$

where the summation in (2) is over all possible n parallel transitions depending on a given encoder, p_n denotes the probability of the nth parallel transition between $(u \to \bar{u})$ if it exists, otherwise $p_n = 1$. $\Pr(u \to \bar{u}|u)$ is the conditional probability of a transition from state u to state $\bar{u}$, given state u and $\mathrm{W}(a(i \to j))$ denotes the Hamming weight of the information sequence for the information bits $a(i \to j)$ belonged by a transition from i to j where $i \in \{u, v\}$ and $j \in \{\bar{u}, \bar{v}\}$ [22].

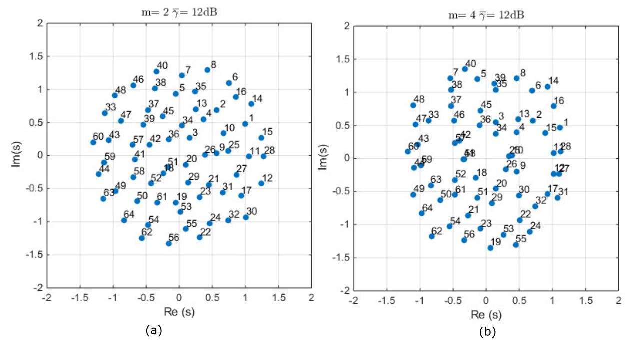

Fig. 1. (a) Optimized irregular constellation for $[5,7]_8$ coded case at $\overline{\gamma} = 12$ dB (m = 2) (b) Optimized irregular constellation for $[5,7]_8$ coded case at $\overline{\gamma} = 12$ dB (m = 4).

III. Optimized Irregular Constellations for Infinite Block Length

In this subsection, the performance of irregular constellations [14] designed for rate-1/2 convolutional encoder $[5,7]_8$ are investigated over Nakagami-*m* fading channels in terms of decoding delay over long block length cases where m denotes Nakagami-*m* fading parameter. Specifically, the decoding delay can be defined as the time interval from the moment at which the information is sent to the moment at which the decoding process is completed [23]. Then, the decoding delay can be calculated by subtracting the sum of the encoding time and the channel delay from the overall delay. As the quantity of the decoding delay, the window-length of back-search limit in Viterbi decoder, τ, is simply used as in [10].

Decoding delay is proportional to τ in convolutionally coded scenarios and to the iteration number Q in LDPC coded scenarios. For this purpose, the required average SNR values to obtain a predefined BER threshold, $P_{b,th}$, are calculated via Monte Carlo simulations for optimized irregular constellations and M-QAM cases. In order to give a basic comparison, a convolutionally coded SISO system is considered, where rate-1/2 convolutional encoder $[5,7]_8$ is employed and in order to increase the coding rate from $1/2$ to $3/4$ for 64-ary signalling cases, $[1\ 1\ 0\ 1; 0\ 1\ 1\ 1]$ is used as puncturing pattern [24]. The snapshots of optimized irregular constellations corresponding to $\overline{\gamma} = 12$ dB are given in Fig. 1 for different m values and the required SNR values for $m = \{2, 4\}$, are plotted with respect to different τ values where $P_{b,th} = 10^{-4}$ in Fig. 2. It can be seen from Fig. 2, 1-2 dB SNR gain can be obtained by using SNR-adaptive optimized irregular constellations for the same decoding delay.

Now, the same convolutionally coded SNR-adaptive system is compared with rate-1/2 LDPC code which was used in DVB-S2 standard [25] for the block length of 19200 information bits. When 16-ary constellations are used along with $Q = \{1, 2, 3, 4, 8\}$, it can be observed from Fig. 3, the required SNR values to reach 10^{-4} of P_b in case of convolutionally coded case can be less than the ones for LDPC coded cases for lower iteration number and the decoding delay of LDPC coded scenarios is higher than convolutional coded cases for those values. Without limits on decoding delay and the number of iterations, LDPC code reaches the target error performance

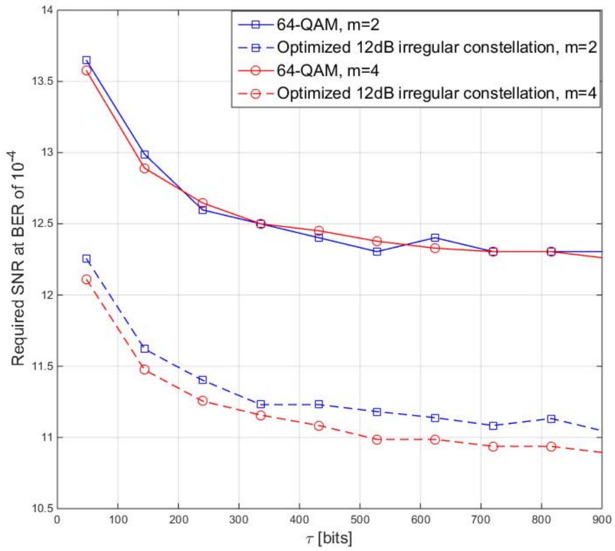

Fig. 2. The required SNR values of SNR-adaptive optimized 64-ary signalling model and SNR-independent 64-QAM model where rate-1/2 $\left([5,7]_8\right)$ convolutional encoder is used for all cases.

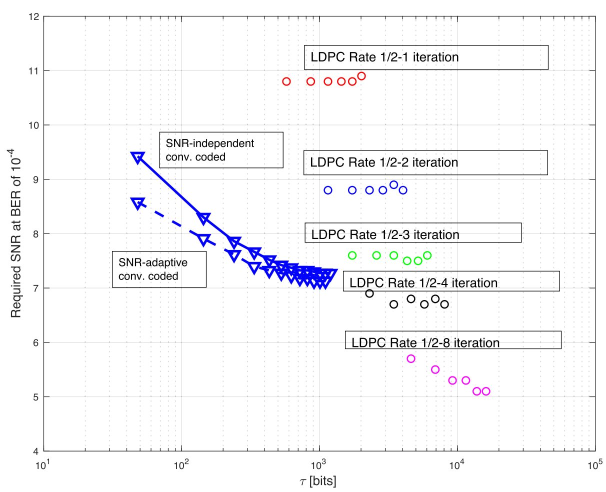

Fig. 3. The required SNR values of SNR-adaptive optimized 16-ary signalling model with $[5,7]_8$ and LDPC encoder with 16-QAM.

value with smaller SNR values as expected. Therefore when transmission is restricted by the decoding delay (less than a few hundred bits), convolutional decoding is preferred even from channel-oriented aspect.

IV. Data-Oriented Approach

Now, the framework of finding optimal constellation points for a given encoder and fading parameter is transformed into data-oriented constellation design framework where the amount of data and available bandwidth are placed in the center of the performance evaluation. Specifically, the data-oriented approach introduces delay outage ratio (DOR) as an alternative to decoding delay, defined as the probability that the required information delivery time exceeds the predefined transmission time threshold, (T_{th}) [16]. For the given H information bits, $\text{DOR}_{\text{T}_{\text{th}}}$ is expressed as,

$$\text{DOR}_{\text{T}_{\text{th}}} = \Pr\left[\text{T}_{\text{th}} < \frac{\text{H}}{\text{R}\left(\text{L},\epsilon\right)}\right] \tag{3}$$

where $R\left(L,\epsilon\right)$ is an actual rate fo the transmission depending on the block length, L, and achievable block error probability, ϵ. Then, the achievable rate can be expressed as [19]

$$R\left(L,\epsilon\right) = C\left(\gamma\right) - \Delta C\left(\gamma\right) - \frac{\gamma_o}{B}\sqrt{\frac{V\left(\gamma\right)}{n}}Q^{-1}\left(\epsilon\right) + \frac{1}{2}\frac{\log_2 L}{L}, \tag{4}$$

where

$$V\left(\gamma\right) = \frac{1}{M}\sum_{i=1}^{M}\text{Var}\left[\log_2\left(\sum_{j=1}^{M}\exp\left(|Z|^2 - |\tilde{s}_i + Z - \tilde{s}_j|^2\right)\right)\right]. \tag{5}$$

Herein, Z is a zero-mean complex Gaussian distribution with unit variance, $\tilde{s}_i$, $\tilde{s}_j$ are the normalized constellation points of a M-QAM constellation with average power of γ, $\Delta C\left(\gamma\right)$ corresponds to the difference between ergodic capacity $C\left(\gamma\right)$ and the achievable capacity of a given encoder when infinite block length is considered and $V\left(\gamma\right)$ is the channel dispersion measuring the stochastic variability of the channel with respect to a deterministic channel when M-QAM constellations are used and γ_o is correction term for actual dispersion resulting from any other discrete finite-sized constellations [19]. It was shown in [20] that a iterative solution of γ for (4) with acceptable error, $\hat{\Gamma}^{(t)}$, is possible with very fast convergence in terms of ϵ for $\epsilon \leq 0.5$. Specifically, $\hat{\Gamma}^{(t)}$ is formulated as [26]

$$\hat{\Gamma}^{(t)} = 2^{\frac{H}{BT_{th}} + \frac{\gamma_o}{B\sqrt{L}}M^{(t-1)}\log_2(e)Q^{-1}(\epsilon)} - 1, \tag{6}$$

along with $M^{(t)} = V\left(\hat{\Gamma}^{(t)}\right)$ where t is an iteration number. After utilizing [Eq. (19), [26]], (3) can be written in an equivalent form of

$$\text{DOR}_{\text{T}_{\text{th}}} = \text{F}_{\gamma}\left(\hat{\Gamma}^{(t)}\right). \tag{7}$$

Herein, $\text{F}_{\gamma}\left(\cdot\right)$ is the cumulative distribution function (CDF) of γ. Now, it can be seen from (7) that data-oriented approach restricts instantaneous SNR values to be greater than $\Gamma^{(t)}$ in order to prevent the delay outage whereas γ can take any value in interval $(0,\infty)$ for the channel-oriented cases. Then, $D^{\text{data-oriented}}_{(u,v),(\bar{u},\bar{v})}$ for $T_{th} > T$ can be expressed as

$$D^{\text{data-oriented}}_{(u,v),(\bar{u},\bar{v})} = \int_{\Gamma^{(t)}}^{\infty} e^{-\Delta\gamma} f_{\gamma}\left(\gamma\right) d\gamma. \tag{8}$$

In case of Nakagami-*m* fading cases, the squared envelope of Nakagami-faded channel coefficient follows the Gamma distribution [27], which is,

$$f_{\gamma}\left(\gamma\right) = \frac{\gamma^{m-1}e^{-\gamma\frac{m}{\Omega}}}{\overline{\gamma}^{m}m^{-m}\Gamma(m)}, \tag{9}$$

where $\overline{\gamma}$ is the average fading power, m is the shaping parameter $m \geq 0.5$, and $\Gamma\left(\cdot\right)$ is the Gamma function [28]. After combining (8)-(9) and using [28, eq. 3.35.2] one can obtain

$$D^{\text{data-oriented}}_{(u,v),(\bar{u},\bar{v})} = \frac{m^m\omega^m\Omega^{-m}E_{1-m}\left(\omega\left(\Delta + \frac{m}{\Omega}\right)\right)}{\Gamma(m)}, \tag{10}$$

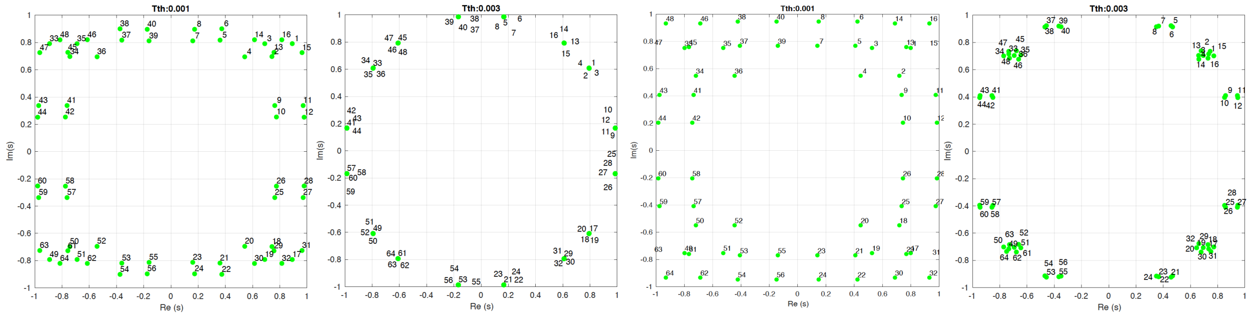

Fig. 4. The evolution of optimized 64-ary irregular constellations obtained from data-oriented approach for different T_{th} and m values (the first two constellations $m = 1$, the last two constellations $m = 2$) when $\overline{\gamma} = 20$ dB, $n = 80$ and $\epsilon = 10^{-3}$.

Algorithm 1 The PSO Algorithm With Swarm Size P for Constellation Search

Initialize P particles' positions, x_i^0, and the velocity of each particle, v_i^0.
Evaluate fitness value of each particle from $\mathrm{DOR_{T_{th}}}$ given in (11).
Set g_i^0 as the particle best value.
Calculate $g_p^0 = \min\{g_i^0\}$ as the swarm best value.
for $n = 1 : N_{iter}$ **do**
  for $i = 1 : P$ **do**
    $x_i^n = x_i^{n-1} + v_i^n$
    $v_i^n = v_i^{n-1} + r_1 c_1 \left(g_i^{n-1} - x_i^{n-1}\right) + r_2 c_2 \left(g_p^{n-1} - x_i^{n-1}\right)$
    if $\mathrm{DOR_{T_{th}}}$ for $x_i^n \leq \mathrm{DOR_{T_{th}}}$ for g_i^n **then**
      $g_i^n = x_i^n$
  Find g_p^n
return $x_i^{N_{iter}} \rightarrow \chi\left(m, \overline{\gamma}\right) = \{s_0, s_1, \cdots, s_{M-1}\}$

where $w = \hat{\Gamma}^{(t)}$ and $\Delta = |s_{l,i} - \hat{s}_{l,i}|^2$. Herein, $\hat{s}_{l,i}$ corresponds an erroneous symbol resulting from an erroneous decoder transition, and $E_v(\cdot)$ is the exponential integral function [28]. Then, the delay outage expression for convolutionally coded system can be formulated as

$$\mathrm{DOR_{T_{th}}} = 1 - \left(1 - \mathrm{P_{b|T<T_{th}}}\right)^{\mathrm{N_c}} \left(1 - \mathrm{F}_\gamma\left(\hat{\Gamma}^{(t)}\right)\right), \quad (11)$$

when the bit error event in each bit is assumed to be independent from each other herein.

V. Optimized Irregular Constellations for Finite Block Length

In order to determine optimal symbol points locations, a set of symbols minimizing (11), the optimizer uses a metaheuristic evolutionary algorithm which is based on swarm intelligence to find out optimized irregular constellations over different T_{th} values. Particle swarm optimization (PSO) technique is chosen in this letter because of its less computational load and fewer tuning parameters as compared to the other evolutionary algorithms. The PSO algorithm used in finding SNR-adaptive irregular constellations is summarized in Algorithm 1.

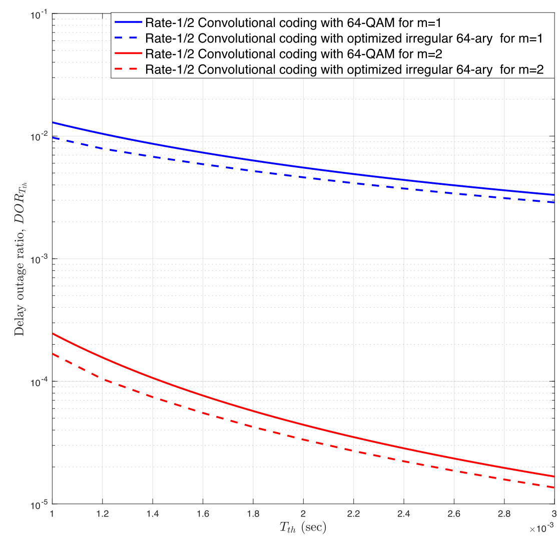

Fig. 5. DOR performance of convolutionally coded transmission when 64-QAM and 64-ary signaling cases for a given simulation parameter set.

In order to find optimized irregular constelllations which minimizes (11), after utilizing [29], γ_o can be calculated from

$$\gamma_o = \exp(-2L\delta^2/D_{max}^2)/\exp(-2L\delta^2/D_{M-QAM}^2), \quad (12)$$

where D_{max} corresponds to the largest power difference in any chosen pair of symbols in M-ary constellation such that $D_{max} = \max_{\tilde{s}_i, \tilde{s}_j \in \chi} |\tilde{s}_i - \tilde{s}_j|$, D_{M-QAM}^2 is given in [eq. (29), [29] and δ is chosen as 0.1 as in [29]. The parameters used in PSO optimizer in [30] are chosen in constellation search and in order to keep the average transmitted symbol energy not exceeding certain threshold and fair comparison with the reference constellations, the following constraint is also taken into account during the process.

After running the PSO optimizer for different T_{th} and m values, it has been observed that there are variations of symbol point locations based on those parameters. For example, Fig. 4 illustrates those variations in 64-ary signaling over Nakagami-m fading cases with $m = 1$ and $m = 2$ for different T_{th} values, respectively. Without of loss generality, in this case, H, B, and $\overline{\gamma}$ are chosen as 200, 2 MHz and 20 dB, respectively. As it can be seen from the figure, optimized irregular constellations vary with respect to delay constraint and fading characteristic.

Since uniform distribution among the constellation points are optimal when the symbol point locations are distributed on the circle under finite block length constraint [19], the irregular constellations have circular grid. For better channel conditions, it might be lower T_{th} leading to higher $\hat{\Gamma}^{(t)}$ or higher m values, the constellations turn into more irregular shapes. The performance enhancement resulting from irregular constellations obtained from data-oriented approach is shown in Fig. 5 where 64-QAM constellation is chosen as benchmark for the comparison. Fig. 5 illustrates that the gap between 64-QAM and 64-ary irregular cases is almost the same with different thresholds values and higher performance gain is obtained with higher m values. The exact coordinates of symbol point locations of optimized irregular 64-ary constellations used in Fig. 5 for the other T_{th} values can be found in [31].

VI. Conclusion

The use of irregular optimized constellations where optimal locations of the symbol points were found based on long term channel statistics was already proposed for many systems. In this letter, the achievable gain resulting from the use of irregular constellations in convolutionally coded transmission was presented by comparing them with powerful capacity approaching LDPC code in terms of decoding delay. Then, data-oriented approach was introduced to convolutionally coded system for finding optimized constellations over small data transmissions. The simulation results show that the irregular optimized constellations obtained from data-oriented approach provides enhanced transmission strategy regarding to finite block length and delay constraints.

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