Energy Harvesting Communications Under Energy Underflow Constraints

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***Abstract*— We focus on energy management in a transmission system that harvests energy from the environment and stores the harvested energy in a battery. Because of the constraint in certain batteries that we cannot drain the stored energy completely, we study energy underflow, which is a result of the case the energy in the battery falls below a certain level. We formulate the available space to store more energy in the battery. Then, using the stochastic steady-state relation between the energy arrival and demand processes, we provide a closed-form expression for the energy underflow probability in the battery. Finally, we employ different energy demand processes from the battery, and then compare the average data service rates in the wireless channel under different energy underflow constraints.**

***Index Terms*— Energy harvesting, energy storage management, energy underflow probability, large deviation principle.**

I. Introduction

ONE driving motivation behind the search for sustainable energy sources and management is to find methods and techniques for self-sustainable communication devices. Here, research challenges arise generally due to the differences in energy storage mechanisms and the stochastic aspects of natural energy sources, e.g., wind and solar. Therefore, the deliberate use of energy has become another performance objective in wireless communications along with spectral efficiency and quality-of-service (QoS) guarantees. The concern lies in estimating the periodicity and magnitude of the exploited energy source, deciding which parameters to tune, and simultaneously avoiding premature energy depletion before next recharge cycle, while providing certain communication performance levels [1]. In order to understand energy storage dynamics, researchers have implemented several mathematical models. Among many of these models, queueing theory-based energy quantization models can reflect energy storing characteristics and generally provide less-complicated analysis when invoked in wireless communication studies [2].

The new perspective towards this concept of including sources of harvested energy and energy conversion mechanisms requires us to understand energy storage technologies and their performance levels. Some researchers have focused on the information-theoretic aspects and addressed how much information we can transfer under energy harvesting constraints, e.g., [3], [4]. Another aspect has become the data-link performance. In particular, we refer to a transmitter with a data buffer and a battery, where the objective is to manage the transmission rates while monitoring both the traffic load and the stored energy [5]. For instance, in delay-constrained transmissions, using Markov processes to model the battery, and hence the overall system, the authors obtained the effective capacity of the channel under energy harvesting constraints [6]. We finally refer interested readers to the research on power management policies in batteries [7], [8] and the stochastic modeling of these batteries [9], [10] as well.

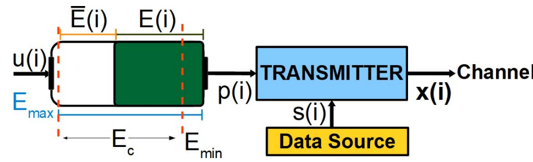

Fig. 1. System model.

In energy harvesting wireless communications, the efficient use of the harvested energy is of paramount importance. Moderate energy usage leads to energy overflows, and hence energy losses due to the limited battery size, whereas excessive energy usage results in energy outages that cause transmission interruptions. One other practical concern with certain batteries is that we cannot drain all the energy from a battery but we should use a certain amount of the available energy in order to extend the battery life [11]. In this case, rather than energy outages, controlling energy underflows (i.e., the energy level falls below a certain threshold in a battery) serves the purpose of energy management both to extend battery life and to prevent data transmission interruptions. In this context, the authors of [12] regarded the energy underflow probability as the battery constraint and attained the optimal constant power allocation policy that maximizes the effective capacity.

We focus on energy management in an energy harvesting wireless communication system, where the system employs a battery to store the harvested energy. We characterize the energy underflows in the battery and provide the energy underflow probability. Regarding a discrete battery model and basing our analysis on a queueing-theoretic approach, we provide an analytical framework that is simple, yet comprehensive, and can encapsulate the characteristics of a generic energy harvesting communication system, where transmission outages are to be avoided and the battery life should be prolonged.

II. System Model

A. Energy Harvesting and Storage

As seen in Figure 1, the system stores the harvested energy in a battery. The total storage capacity of the battery is $E_{\max}$ units of energy.[1] If the battery is full during charging, the systems dissipates the excess energy, e.g., in the form

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[1]Depending on the transmission protocol and the energy harvester type, energy demands and arrivals vary considerably. For instance, transceivers require output power levels in the order of $2 - 100$ mW to communicate within a range of 30 meters while solar cells produce 15 mW/cm^2 [13].

of heat. Moreover, the system avoids battery depletion as much as possible, which follows extreme discharging. Specifically, it does not permit the amount of energy in the battery to be below a certain level, i.e., $E_{\min}$. Therefore, we refer to the feasible battery capacity, E_c, as the difference between $E_{\max}$ and $E_{\min}$, i.e., $E_c = E_{\max} - E_{\min}$. Furthermore, $E(i)$ and $\bar{E}(i)$ denote the level of the available energy and the available space to store more energy, respectively, at the end of the i^{th} time frame. Hence, we have $E_{\max} = E(i) + \bar{E}(i)$.

Now, let $u(i)$ and $p(i)$ be the energy arrival rate at the battery and the energy demand rate from the battery in the i^{th} time frame, respectively. Then, we express the energy level in the battery at the end of the i^{th} time frame as

$$E(i) = \min\left\{\left[E(i-1) + u(i) - p(i)\right]^{+}, E_{\max}\right\}, \tag{1}$$

where $[\cdot]^{+} = \max\{\cdot, 0\}$. Subsequently, the available space in the battery at the end of the same time frame becomes

$$\begin{aligned}\bar{E}(i) &= E_{\max} - E(i)\\ &= E_{\max} - \min\left\{\left[E(i-1) + u(i) - p(i)\right]^{+}, E_{\max}\right\}\\ &= \max\left\{E_{\max} - \left[E(i-1) + u(i) - p(i)\right]^{+}, 0\right\}\\ &= \left[E_{\max} - \left[E(i-1) + u(i) - p(i)\right]^{+}\right]^{+}\\ &= \left[\min\left\{E_{\max} - E(i-1) - u(i) + p(i), E_{\max}\right\}\right]^{+}\\ &= \left[\min\left\{\bar{E}(i-1) + p(i) - u(i), E_{\max}\right\}\right]^{+}\\ &= \min\left\{\left[\bar{E}(i-1) + p(i) - u(i)\right]^{+}, E_{\max}\right\}. \end{aligned} \tag{2}$$

Notice the analogy between the actual energy level in (1) and the available space to store more energy in (2). In (1), when $E(i-1) + u(i) - p(i) \geq E_{\max}$, the system dissipates the excess energy, i.e., energy waste. In (2), when $\bar{E}(i-1) + p(i) - u(i) \geq E_{\max}$, an energy outage follows and the data transmission stops. While the control of energy waste is important from the perspective of using energy resources as efficient as possible, the control of energy outages is necessary to prevent transmission interruptions. We further know that while the entire battery capacity can be used for energy storage, the battery life is extended if only a fraction of the capacity is used [11]. We can consider the feasible battery capacity, E_c, as the fraction of the storage capacity that is used for charging and discharging. When the actual available space in the battery becomes more than E_c, i.e., $\bar{E}(i) > E_c$, we assume that an energy underflow occurs. Thus, the energy demand process that avoids energy underflows as much as possible so that the battery life is extended and the number of transmission interruptions is minimized is a research question. Hence, $\Pr\{\bar{E}(i) \geq E_c\}$ becomes a battery performance metric.

B. Channel Input-Output Model

The transmitter takes $s(i)$ bits from the source and sends a vector of N symbols, i.e., $\mathbf{x}(i)$, following the encoding and modulation of $s(i)$ bits. Hence, the input-output relation in the channel in the i^{th} time frame is

$$\mathbf{y}(i) = \mathbf{x}(i)h(i) + \mathbf{w}(i), \tag{3}$$

where $\mathbf{y}(i)$ and $\mathbf{x}(i)$ are the channel output and input vectors, respectively, and $\mathbf{w}(i)$ is the additive complex Gaussian noise vector with independent and identically distributed zero-mean samples. The covariance matrix of $\mathbf{w}(i)$ is $\mathbb{E}_{\mathbf{w}}\left\{\mathbf{w}(i)\mathbf{w}(i)^{\dagger}\right\} = \sigma_w^2 \mathbf{I}_{N\times N}$, where σ_w^2 is the average noise power and $\mathbf{I}_{N\times N}$ is the identity matrix. The input vector is constrained as follows: $\mathbf{x}(i)^{\dagger}\mathbf{x}(i) \leq p(i)$. Above $\mathbf{w}(i)^{\dagger}$ and $\mathbf{x}(i)^{\dagger}$ are the conjugate transposes of $\mathbf{w}(i)$ and $\mathbf{x}(i)$, respectively. Moreover, we consider a block-fading channel and assume that the channel fading coefficient, $h(i)$, stays constant during one time frame and changes independently from one frame to another. Herein, assuming that N is relatively large, and hence the decoding error probability is negligible, we set the transmission rate in the channel in one time frame to the instantaneous mutual information between the channel input and output. Specifically, the number of bits that are retrieved from the data source in the i^{th} frame is set to the instantaneous mutual information, i.e., $s(i) = I(\mathbf{x}(i); \mathbf{y}(i))$. We also note that $s(i)$ refers to the data service rate in the channel in the i^{th} time frame.

III. Energy Underflow Characterization

We invoke the large deviation principle and queuing theory [14] and regard the similarity between a single server queueing system and the aforementioned energy storage model in (1), hence the available space model in (2). We know that the steady-state queue length distribution in single server queues, assuming it exists, has a characteristic decay rate [15]. Following the same analysis in [15], let us assume that the maximum capacity of the battery is infinite, i.e., $E_{\max} = \infty$, and denote the decay rate of the tail distribution of the available space to store more energy in the battery in the steady-state by μ. Then, we provide the following definition:

Definition 1: Given a stationary and ergodic energy arrival process, $u(i)$, and a stationary and ergodic energy demand process, $p(i)$, under the stability condition[2] $\mathbb{E}\{p(i)\} < \mathbb{E}\{u(i)\}$, the decay rate of the tail distribution of the available space in the battery becomes

$$\mu \triangleq -\lim_{E_{\text{th}}\to\infty} \frac{\log \Pr\{\bar{E}(\infty) \geq E_{\text{th}}\}}{E_{\text{th}}}, \tag{4}$$

where E_{th} is the threshold for the steady-state available space in the battery. We call μ the available space decay rate.

The expression (4) states that when there is a large threshold, we can approximate the probability of the available space being greater than a given threshold with an exponential function of μ and E_{th}. Particularly, let us assume that the total storage capacity of the battery, $E_{\max}$, is very large. Then, the energy underflow probability is approximated with an exponential function of μ and the feasible battery capacity, E_c, i.e., $\Pr\{\bar{E}(i) \geq E_c\} \approx e^{-\mu E_c}$. Hence, given an energy arrival process, smaller μ refers to a high energy underflow probability and an energy demand process that consumes the energy in the battery very quickly, whereas larger μ refers

[2]Because we want to minimize the number of transmission interruptions and extend the battery life, we need to stabilize $\bar{E}(i)$. Hence, we have $\mathbb{E}\{p(i)\} < \mathbb{E}\{u(i)\}$ as the stability condition.

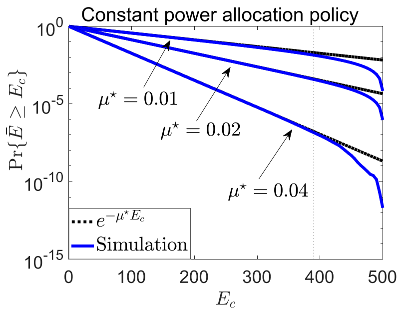

Fig. 2. The energy underflow probability vs. the feasible battery capacity.

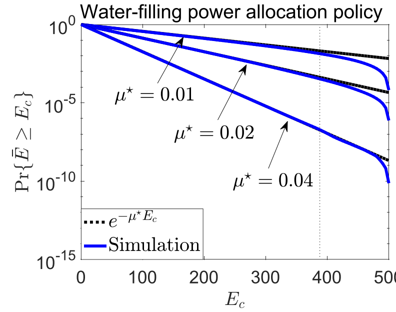

Fig. 3. The energy underflow probability vs. the feasible battery capacity.

to a low energy underflow probability and an energy demand process that utilizes the available energy in a moderate way.

Now, assuming $E_{\max} = \infty$, we redefine the instantaneous available space in the battery given in (2) as $\bar{E}(i) = \left[\bar{E}(i-1) + p(i) - u(i)\right]^+$. Then, by considering a work-conserving[3] energy demand process and that $u(i)$ and $p(i)$ are independent of each other, we have a unique $\mu^\star > 0$ such that [14, Remark 9.1.2]

$$\Lambda_u(-\mu^\star) + \Lambda_p(\mu^\star) = 0. \quad (5)$$

Above, $\Lambda_u(\mu) = \lim_{t\to\infty} \frac{1}{t} \log \mathbb{E}_u\Big\{e^{\mu \sum_{i=1}^t u(i)}\Big\}$ and $\Lambda_p(\mu) = \lim_{t\to\infty} \frac{1}{t} \log \mathbb{E}_p \left\{e^{\mu \sum_{i=1}^t p(i)}\right\}$ are the log-moment generating functions of the energy arrival and demand processes, respectively. The expression (5) states that given an energy arrival process and an available space decay rate for a feasible battery capacity, we can adjust the energy demand parameters under the stability condition $\mathbb{E}\{p(i)\} < \mathbb{E}\{u(i)\}$. For instance, we can adjust the average energy demand rate. Specifically, considering the battery characteristics, μ is a design parameter that tells about the transmission parameters.

IV. Performance Analysis

Given a stochastic energy arrival process, we assume that a constant-level energy demand process (a constant power allocation policy) exists, i.e., $p(i) = p$. Then, we have $\Lambda_p(\mu) = \mu p$. Invoking the result in (5), we show

$$p = \lim_{t\to\infty} -\frac{1}{t\mu^\star} \log \mathbb{E}_u \left\{e^{-\mu^\star \sum_{i=1}^t u(i)}\right\} \text{ for } \mu^\star > 0. \quad (6)$$

Particularly, as long as the the constant-level energy demand is smaller than or equal to p, the desired energy underflow probability for a given feasible battery capacity, i.e., $Pr\{\bar{E}(i) \geq E_c\} \approx e^{-\mu^\star E_c}$, is sustained. In (6), if the energy arrivals are independent and identically distributed, the constant-level energy demand becomes $p = -\frac{1}{\mu^\star} \log \mathbb{E}_u \left\{e^{-\mu^\star u(i)}\right\}$. Subsequently, having the probability density function of the energy arrivals as $f_u(u) = \lambda e^{-\lambda u}$ and obtaining the constant-level energy demands for given available space decay rate values, as $p = -\frac{1}{\mu^\star} \log \left(\frac{\lambda}{\lambda+\mu^\star}\right)$, we plot the energy underflow probability results of the finite-size battery simulations in Fig. 2 along with the exponential underflow probability results. We set the maximum battery capacity to $E_{\max} = 500$ energy units and the average energy arrival to $\lambda^{-1} = 1$, and consider the following available space decay rate values: $\mu = 0.01$, 0.02 and 0.04. As seen in Fig. 2, by employing the exponential function approximation, we can determine the energy underflow probability for feasible battery capacity values that are smaller than 400 units. Moreover, we consider the water-filling power allocation policy[4] and set the energy demand to

$$p(i) = N\sigma_w^2 \left[\frac{1}{z_0} - \frac{1}{z(i)}\right]^+, \quad (7)$$

where $z(i) = |h(i)|^2$ is the channel fading gain power, z_0 is a cutoff value such that $\int_{z_0}^\infty p(i) f_z(z) dz < \mathbb{E}\{u(i)\}$ and (5) are satisfied, and $f_z(z)$ is the probability density function of z. Keeping the same settings in Fig. 2, we employ the energy demand process in (7) and simulate the finite-size battery. As seen in Fig. 3, we can again approximate the energy underflow probability with an exponential function very closely for E_c values smaller than 400 energy units. Notice also that the energy demand rate decreases with increasing μ in both Fig. 2 and Fig. 3 because the system requires strict control over energy underflow violations.

As for the throughput levels in the channel, we assume that the channel fading information is available at both the transmitter and the receiver, and consider the average data service rate[5] in the channel, $\mathbb{E}_s\{s(i)\}$. We set the signal-to-noise ratio (SNR) to -10 dB, i.e., $10\log_{10}\left(\lambda^{-1}/(N\sigma_w^2)\right) = -10$ dB. Particularly, we define the SNR as the ratio between the average energy arrival rate to the average total noise power in the channel in one time frame. Then, the average data service rate in the channel becomes $\mathbb{E}_s\{s(i)\} = \mathbb{E}_z\left\{N \log_2\left(1 + p_t(i) z(i)/\sigma_w^2\right)\right\}$, where the input is complex Gaussian distributed[6] and the transmission power for one data

[3]We consider that the transmitter has always data to transmit. Therefore, it consumes certain amount of energy as long as there is energy in the battery. Therefore, we regard the energy demand process as work-conserving.

[4]The water-filling power allocation policy is optimal when the elements of the input vector are Gaussian distributed under an average power constraint and when the energy source is available at any time [16, Sec. 4.2.4]. However, we cannot say that this is the optimal policy when there are other constraints such as a finite-size battery and a stochastic energy arrival process.

[5]We can also consider the effective capacity as the performance measure, which defines the maximum constant data arrival rate at the transmitter buffer that the data service process can support under QoS constraints, i.e., $C_E(\theta) = \lim_{t\to\infty} -\frac{1}{t\theta} \log \mathbb{E}_s \left\{e^{-\theta \sum_{i=1}^t s(i)}\right\}$, where θ is the QoS exponent [17].

[6]We assume that a sufficient averaging of the additive noise in one frame is possible and that we can obtain a necessary power allocation among the samples of $\mathbf{x}(i) = [x_1(i)^\dagger, \cdots, x_N(i)^\dagger]^\dagger$ so that we have $\mathbb{E}_x\{\mathbf{x}(i)^\dagger \mathbf{x}(i)\} = N p_t(i)$ and $x_j(i)$ $\mathcal{CN}(0, p_t(i))$ for $j \in \{1, \cdots, N\}$. However, one can perform our analysis with arbitrarily distributed input modulations as well.

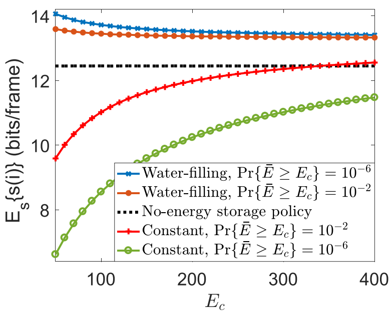

Fig. 4. The average service rate vs. the feasible battery capacity.

symbol is $p_t(i) = \min\{p(i), E(i-1)+u(i)\}/N$. Specifically, if the demanded energy is not met, the transmission power is set to $E(i-1)+u(i)$. Then, we plot the average service rate as a function[7] of the feasible battery capacity in Fig. 4 when the energy underflow probability is 10^{-6} and 10^{-2}, respectively. Recall that the maximum battery capacity is 500 energy units. We compare the average service rates obtained when the transmitter employs the constant and water-filling power allocation policies with the average service rate that is obtained when the transmitter employs the no-energy storage policy, i.e., the transmitter utilizes the energy as soon as it is harvested, where we have $p(i) = u(i)$. We can see that the water-filling power allocation policy is better than the other policies, and the constant power allocation policy performs the worst. The reason is that in order to guarantee the energy underflow probability constraint, the constant power level is set to a value that is much lower than the average energy arrival rate, which leads to excessive energy waste. On the other hand, the water-filling power allocation policy consumes more harvested energy when the channel fading is good, and drains more energy from the battery. This decreases the probability of energy waste. Notice also that the constant power allocation policy performance increases with the increasing energy underflow probability. Even, it performs better than the no-energy storage policy when E_c is larger. This is because the constant-level energy demand increases with the increasing energy underflow probability, which leads to less energy waste. However, when the transmitter employs the water-filling power allocation policy, the increasing energy underflow probability causes many energy outages. As a result, the transmitter may lose the chance of spending more energy in the channel when the channel attenuation is low, which causes a decrease in the average service rate. Finally, the performance of the water-filling power allocation policy decreases with increasing E_c but the performance of the constant power allocation policy increases. This is because increasing E_c causes more energy outages for the water-filling power allocation policy and less energy waste for the constant power allocation policy. The water-filling power allocation policy runs better when there are less energy outages and the constant power allocation policy runs better when there is less energy waste.

[7]Having $\Pr\{\bar{E}(i) \geq E_c\} \approx e^{-\mu E_c}$, we know that μ and E_c are inverse of each other, i.e., $\mu = \frac{1}{E_c}\log\left(\frac{1}{\Pr\{\bar{E}(i)\geq E_c\}}\right)$. Therefore, plotting the average service rate in the channel as a function of E_c serves the same purpose with plotting the average service rate as a function of μ.

V. Conclusion

We have proposed a framework for energy management in a transmission system that harvests energy from the environment for data transmission and stores the harvested energy in a battery. We have initially characterized the energy underflows at a battery. Then, formulating the available space decay rate in the battery, we have provided an approximation for the energy underflow probability as a function of the available space decay rate and the feasible battery capacity. We have further simulated a finite-size battery and compared the simulation results with the analytical results. We have evaluated the constant and water-filling power allocation policies and compared them with the no-energy storage policy. We have shown that the water-filling power allocation policy runs better when there are less energy outages and the constant power allocation policy runs better when there is less energy waste.

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