A Relaying Scheme Based on Diagonalization for Multi-Relay Symmetric MIMO Communication Networks

Yongzhi Yu and Weikun Zhang

***Abstract*— This letter investigates an effective relaying scheme for the existing multi-relay symmetric multiple-input–multiple-output (MIMO) communication networks, with a source–destination pair that communicates through amplify-and-forward (AF) multi-relay nodes. The maximum mutual information (MI) is taken as the design criterion with per relay power constraint. Considering the objective function can be maximized by diagonalizing the matrix of the determinant, a low-complexity and decentralized design is developed to solve this problem. Based on mathematics theory and variable substitution, the objective function is constituted with the sum of multiple diagonal matrices by eigenvalue decomposition (ED) and singular value decomposition (SVD), the original optimization problem is transformed into a standard scalar convex optimization problem. Finally, the Karush–Kuhn–Tucker (K.K.T) conditions are employed to obtain the optimal values of the diagonal elements for all power allocation matrices. Numerical results show that the proposed scheme can significantly improve the system performance.**

***Index Terms*— Multi-relay, relaying scheme, diagonalization, K.K.T conditions, system capacity.**

I. Introduction

RELAYING schemes have become more popular because of their implementation simplicity and low-complexity, an effective relaying scheme can improve the communication quality between the source and the destination.

For a practical MIMO relay network, it is an inevitable tendency to apply multi-relay, so research on multi-relay MIMO networks has significant sense. Recently, MIMO multi-relay networks have been examined in [1]–[4]. A conventional simple amplify-and-forward (SAF) relaying scheme was introduced in [1]. In [2] a relaying scheme for matrix triangularization was proposed, the effective channel matrix was transformed into a triangular form by the phase control matrix. Aiming to overcome a performance loss in terms of the network capacity in [2], a scheme of QR decomposition (QRD) at the destination was proposed in [3]. Fu *et al.* [4] investigated a diagonal relaying scheme under a total relay power constraint to maximize the information rate, the scheme had a much better spectral efficiency than some of the existing methods when the relay power was moderated to high.

However, in AF multi-relay symmetric MIMO networks, conventional relaying schemes need high computational complexity as well as the optimization is done in a centralized manner; they focus on channel diagonalization or giving the precoding matrix directly. There are no investigations that the objective function could be maximized by diagonalizing the matrix expression. Motivated by this fact, the main contributions of this letter are as follows. First, considering the specialization of the MI expression, the objective function could be maximized by diagonalizing the matrix of the determinant based on mathematical theory. Second, exploiting the low-complexity and decentralized relaying scheme, each relay precoding matrix is solved separately and the objective function is transformed into the sum of multiple diagonal matrices by ED and SVD, based on which the form of the each relay precoder is derived. Finally, compared with the conventional relaying scheme, our proposed scheme shows significant gains, in particular, it is more obvious when the number of relays increases.

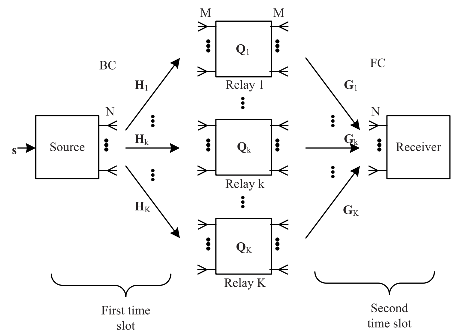

Fig. 1. System model of two-hop AF multi-relay symmetric MIMO relay network.

Some notations are used in this letter. We adopt bold face letters to denote vectors and matrices, in which lower case represents vectors and upper case represents matrices. $\mathbf{E}[\cdot]$ denotes the expectation operation. $(\cdot)^H$ stands for the Hermitian transpose, diag$(\cdot)$ is defined as the diagonal matrix. tr$(\cdot)$ represents the matrix trace. $|\cdot|$ denotes the determinant operation of a square matrix and $(\cdot)^{-1}$ represents matrix inversion. The operation $(x)_+$ returns $\max(x, 0)$.

II. System Model

In this section, the two-hop AF multi-relay symmetric MIMO relay network model is considered, which comprises a source–destination pair both equipped with N antennas, source and destination are communicating through K relay nodes equipped with M antennas, as illustrated in Fig. 1.

In the first time slot, the source broadcasts the signal vector $\mathbf{s} \in \mathbb{C}^{N\times 1}$ to all the relay nodes through the backward channels (BCs). Let $\mathbf{H}_k \in \mathbb{C}^{M\times N}$ $(k = 1, 2, \cdots, K)$ stand

Manuscript received March 22, 2017; accepted April 2, 2017. Date of publication April 7, 2017; date of current version August 10, 2017. This work was supported by the Fundamental Research Funds for the Universities KY10800150020. The associate editor coordinating the review of this letter and approving it for publication was M. Di Renzo. *(Corresponding author: Yongzhi Yu.)*

The authors are with the College of Information and Communication Engineering, Harbin Engineering University, Harbin 150001, China (e-mail: yuyongzhi@hrbeu.edu.cn; xinweikunxin@126.com).

Digital Object Identifier 10.1109/LCOMM.2017.2693188

for the channel matrix from the source to the kth relay node. The received signal $\mathbf{y}_{r,k} \in \mathbb{C}^{M\times 1}$ at the kth relay node is

$$\mathbf{y}_{r,k} = \mathbf{H}_k\mathbf{s} + \mathbf{n}_{r,k} \quad (1)$$

Where $\mathbf{n}_{r,k}$ is complex additive white Gaussian noise(AWGN) at the kth relay node.

In the second time slot, the kth relay precoding matrix $\mathbf{Q}_k \in \mathbb{C}^{M\times M}$ processes the received signal $\mathbf{y}_{r,k}$, and then all relay nodes simultaneously forward their linear processed signal to the destination through the forward channels (FCs). The transmitted single $\mathbf{x}_{r,k}$ at the kth relay node and the kth relay power constraint are given as

$$\mathbf{x}_{r,k} = \mathbf{Q}_k(\mathbf{H}_k\mathbf{s} + \mathbf{n}_{r,k}) \quad (2)$$

$$\mathrm{tr}\{\mathbf{Q}_k((P_s/N)\mathbf{H}_k\mathbf{H}_k^H + \sigma_r^2\mathbf{I}_M)\mathbf{Q}_k^H\} \le P_r \quad (3)$$

Where P_s and P_r are defined as the maximum transmission power at the source and all the relay nodes, respectively. σ_r^2 is noise power at each relay node. Let $\mathbf{G}_k \in \mathbb{C}^{N\times M}$ $(k = 1, 2, \cdots, K)$ stand for the channel matrix from the kth relay node to the destination, the received signal $\mathbf{r} \in \mathbb{C}^{N\times 1}$ at the destination is given by

$$\mathbf{r} = \sum_{k=1}^{K}\mathbf{G}_k(\mathbf{Q}_k\mathbf{H}_k\mathbf{s} + \mathbf{Q}_k\mathbf{n}_{r,k}) + \mathbf{n}_d = \overline{\mathbf{H}}\mathbf{s} + \mathbf{n} \quad (4)$$

Where $\mathbf{n}_d \in \mathbb{C}^{N\times 1}$ is the complex AWGN at the destination. $\overline{\mathbf{H}} = \sum_{k=1}^{K}\mathbf{G}_k\mathbf{Q}_k\mathbf{H}_k$ is the equivalent channel matrix from the source to the destination, $\mathbf{n} = \sum_{k=1}^{K}\mathbf{G}_k\mathbf{Q}_k\mathbf{n}_{r,k} + \mathbf{n}_d$ is the equivalent noise between the source and the destination. For simplicity, we assume that all the BCs and FCs are independently and identically distributed and experience the same Rayleigh flat fading.

III. Relay Precoding Scheme Design

In this section, the objective function is maximized by diagonalizing the matrix of the determinant. We propose a low-complexity and decentralized relaying scheme that has an evident performance improvement in the communication network. According to the problem model and the letter [5], the optimization problem of the MI maximization is given by

$$\max_{\mathbf{Q}_k, k=1,\cdots,K} C = (1/2)\log_2|\mathbf{I}_N + \overline{\mathbf{H}}\mathbf{R_s}\overline{\mathbf{H}}^H\mathbf{R}_\mathbf{n}^{-1}| \quad (5)$$

$$s.t.\ \mathrm{tr}\{\mathbf{Q}_k(\mathbf{I}_M + \rho_s\mathbf{H}_k\mathbf{H}_k^H)\mathbf{Q}_k^H\} \le \rho_r M, \quad k = 1, 2, \cdots, K \quad (6)$$

Where $\rho_s = P_s/N\sigma^2$ and $\rho_r = P_r/M\sigma^2$ are defined as the signal-to-noise ratio (SNR) coefficient of the BC and the FC, respectively. The factor 1/2 in (5) comes from the fact that the efficiency of the signal drops by one-half in two time instances. In the following discussion, we will ignore this factor. $\mathbf{R_n}$ is the covariance matrix of the equivalent noise $\mathbf{n}$ between the source and destination, whose expression is the following:

$$\mathbf{R_n} = \mathbf{E}[\mathbf{n}\mathbf{n}^H] = \sum_{k=1}^{K}\mathbf{G}_k\mathbf{Q}_k\mathbf{R}_{\mathbf{n}_{r,k}}\mathbf{Q}_k^H\mathbf{G}_k^H + \mathbf{R}_{\mathbf{n}_d} \quad (7)$$

By substituting (7) back into the objective function of (5), we can obtain the expression of MI as the following:

$$C = \log_2|\mathbf{I}_N + \rho_s(\sum_{k=1}^{K}\mathbf{G}_k\mathbf{Q}_k\mathbf{H}_k\mathbf{H}_k^H\mathbf{Q}_k^H\mathbf{G}_k^H(\mathbf{I}_N + \mathbf{G}_k\mathbf{Q}_k\mathbf{Q}_k^H\mathbf{G}_k^H)^{-1})| \quad (8)$$

Where we assume that equation (9) is set up aiming for the optimization to be done in a decentralized manner and equation (10) can be proved by the property $(\mathbf{I}+\mathbf{AB})^{-1} = \mathbf{I} - \mathbf{A}(\mathbf{I}+\mathbf{BA})^{-1}\mathbf{B}$ and $(\mathbf{A}+\mathbf{BCD})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}(\mathbf{C}^{-1} + \mathbf{DA}^{-1}\mathbf{B})^{-1}\mathbf{DA}^{-1}$.

$$\sum_{k=1}^{K}\mathbf{G}_k\mathbf{Q}_k\mathbf{H}_k(\mathbf{H}_{\overline{k}})^H(\mathbf{Q}_{\overline{k}})^H(\mathbf{G}_{\overline{k}})^H = \mathbf{0} \quad (9)$$

$$\mathbf{G}_k\mathbf{Q}_k\mathbf{H}_k\mathbf{H}_k^H\mathbf{Q}_k^H\mathbf{G}_k^H(\mathbf{I}_N + \sum_{k=1}^{K}\mathbf{G}_k\mathbf{Q}_k\mathbf{Q}_k^H\mathbf{G}_k^H)^{-1} = \mathbf{G}_k\mathbf{Q}_k\mathbf{H}_k\mathbf{H}_k^H\mathbf{Q}_k^H\mathbf{G}_k^H(\mathbf{I}_N + \mathbf{G}_k\mathbf{Q}_k\mathbf{Q}_k^H\mathbf{G}_k^H)^{-1} \quad (10)$$

Where the subscript $\overline{k}$ is the complementary number of k. The original optimization problem (5)-(6) can be rewritten as

$$\max_{\mathbf{Q}_k, k=1,\cdots,K}\log_2|\mathbf{I}_N + \rho_s(\sum_{k=1}^{K}\mathbf{G}_k\mathbf{Q}_k\mathbf{H}_k(\mathbf{I}_N + \mathbf{G}_k\mathbf{Q}_k\mathbf{Q}_k^H\mathbf{G}_k^H)^{-1}\mathbf{H}_k^H\mathbf{Q}_k^H\mathbf{G}_k^H)| \quad (11)$$

$$s.t.\ \mathrm{tr}\{\mathbf{Q}_k(\mathbf{I}_M + \rho_s\mathbf{H}_k\mathbf{H}_k^H)\mathbf{Q}_k^H\} \le \rho_r M, \quad k = 1, 2, \cdots, K \quad (12)$$

From the Hadamard inequality [5], the determinant of a positive definite matrix is maximized only when the matrix is a diagonal matrix. To guarantee the maximization of the objective function in (11), each part matrix of the determinant must be diagonalized. The diagonalization requirement in (11) motivates us to employ matrix decomposition.

In the following, we analyze the structures of the optimal relay precoding matrices for all relay nodes. First, for the first relay node, let $\mathbf{A}_1 = \mathbf{I}_N + \rho_s\mathbf{G}_1\mathbf{Q}_1\mathbf{H}_1(\mathbf{I}_N + \mathbf{G}_1\mathbf{Q}_1\mathbf{Q}_1^H\mathbf{G}_1^H)^{-1}\mathbf{H}_1^H\mathbf{Q}_1^H\mathbf{G}_1^H$, according to the lemma [6]: if $\mathbf{X} \in \mathbb{C}^{N\times N}$ and $\mathbf{Y} \in \mathbb{C}^{N\times N}$ are two positive definite matrices, then

$$|\mathbf{X}+\mathbf{Y}| = |\mathbf{X}||\mathbf{I} + \mathbf{X}^{-1/2}\mathbf{Y}\mathbf{X}^{-1/2}| \quad (13)$$

The objective function in (11) can be rewritten as

$$\log_2|\mathbf{A}_1| + \log_2|\mathbf{I}_N + \mathbf{A}_1^{-1/2}\rho_s(\sum_{i=2}^{K}\mathbf{G}_i\mathbf{Q}_i\mathbf{H}_i(\mathbf{I}_N + \mathbf{G}_i\mathbf{Q}_i\mathbf{Q}_i^H\mathbf{G}_i^H)^{-1}\mathbf{H}_i^H\mathbf{Q}_i^H\mathbf{G}_i^H)\mathbf{A}_1^{-1/2}| \quad (14)$$

For the first relay node, we consider the ED of $\mathbf{H}_1\mathbf{H}_1^H$ and $\mathbf{G}_1^H\mathbf{G}_1$

$$\mathbf{H}_1\mathbf{H}_1^H = \mathbf{U}_1\mathbf{\Lambda}_1\mathbf{U}_1^H \quad \mathbf{G}_1^H\mathbf{G}_1 = \mathbf{V}_1\mathbf{D}_1\mathbf{V}_1^H \quad (15)$$

Where $\mathbf{\Lambda}_1 = \mathrm{diag}(\alpha_{1,1}, \cdots, \alpha_{1,M})$ with $\alpha_{1,m} \ge 0$, and $\mathbf{D}_1 = \mathrm{diag}(\beta_{1,1}, \cdots, \beta_{1,M})$ with $\beta_{1,m} \ge 0$, $m = 1, 2, \cdots, M$.

$\mathbf{U}_1$ and $\mathbf{V}_1$ are unitary matrices. Then the optimal form of the first relay precoding matrix is written as

$$\mathbf{Q}_1 = \mathbf{V}_1\mathbf{F}_1\mathbf{U}_1^H \tag{16}$$

Where $\mathbf{F}_1 = \text{diag}(f_{1,1}, \cdots, f_{1,M})$ with $f_{1,m} \geq 0$ is denoted as the first relay power allocation matrix.

By substituting (15)-(16) back into the expression for $\mathbf{A}_1$, and according to the matrix operation rules of the expression (19) in [7] and [8], the first term of the objective function in (14) is rewritten as

$$\log_2|\mathbf{A}_1| = \log_2|\mathbf{I}_N + \rho_s(\mathbf{\Lambda}_1\mathbf{F}_1^2\mathbf{D}_1(\mathbf{I}_N + \mathbf{D}_1\mathbf{F}_1^2)^{-1})| \tag{17}$$

Obviously, the computation result of $\mathbf{A}_1$ in (17) is a diagonal matrix, for the second term of the objective function in (14) can be given as follows

$$\log_2|\mathbf{I}_N + \rho_s\mathbf{A}_1^{-1/2}\mathbf{G}_2\mathbf{Q}_2\mathbf{H}_2(\mathbf{I}_N + \mathbf{G}_2\mathbf{Q}_2\mathbf{Q}_2^H\mathbf{G}_2^H)^{-1} \mathbf{H}_2^H\mathbf{Q}_2^H\mathbf{G}_2^H\mathbf{A}_1^{-1/2} + \mathbf{A}_1^{-1/2}\rho_s(\sum_{i=3}^{K}\mathbf{G}_i\mathbf{Q}_i\mathbf{H}_i(\mathbf{I}_N + \mathbf{G}_i\mathbf{Q}_i\mathbf{Q}_i^H\mathbf{G}_i^H)^{-1}\mathbf{H}_i^H\mathbf{Q}_i^H\mathbf{G}_i^H)\mathbf{A}_1^{-1/2}| \tag{18}$$

Let $\mathbf{A}_2 = \mathbf{I}_N + \rho_s\mathbf{A}_1^{-1/2}\mathbf{G}_2\mathbf{Q}_2\mathbf{H}_2(\mathbf{I}_N + \mathbf{G}_2\mathbf{Q}_2\mathbf{Q}_2^H\mathbf{G}_2^H)^{-1} \mathbf{H}_2^H\mathbf{Q}_2^H\mathbf{G}_2^H\mathbf{A}_1^{-1/2}$, the expression in (18) can be rewritten as the following:

$$\log_2|\mathbf{A}_2| + \log_2|\mathbf{I}_N + \mathbf{A}_2^{-1/2}\mathbf{A}_1^{-1/2}\rho_s(\sum_{i=3}^{K}\mathbf{G}_i\mathbf{Q}_i\mathbf{H}_i(\mathbf{I}_N + \mathbf{G}_i\mathbf{Q}_i\mathbf{Q}_i^H\mathbf{G}_i^H)^{-1}\mathbf{H}_i^H\mathbf{Q}_i^H\mathbf{G}_i^H)\mathbf{A}_1^{-1/2}\mathbf{A}_2^{-1/2}| \tag{19}$$

The diagonalization requirement in (19) motivates us to apply SVD as follows

$$\mathbf{H}_2\mathbf{A}_1^{-1/2} = \mathbf{U}_2(\mathbf{\Lambda}_2^{1/2})\mathbf{\Gamma}_2^H \quad \mathbf{G}_2^H\mathbf{G}_2 = \mathbf{V}_2\mathbf{D}_2\mathbf{V}_2^H \tag{20}$$

Where $\mathbf{\Lambda}_2 = \text{diag}(\alpha_{2,1}, \cdots, \alpha_{2,M})$ with $\alpha_{2,m} \geq 0$, and $\mathbf{D}_2 = \text{diag}(\beta_{2,1}, \cdots, \beta_{2,M})$ with $\beta_{2,m} \geq 0$, $m = 1, 2, \cdots, M$. $\mathbf{U}_2$, $\mathbf{V}_2$ and $\mathbf{\Gamma}_2$ are unitary matrices. The optimal form of the second relay precoding matrix is

$$\mathbf{Q}_2 = \mathbf{V}_2\mathbf{F}_2\mathbf{U}_2^H \tag{21}$$

Where the second relay power allocation matrix is defined as $\mathbf{F}_2 = \text{diag}(f_{2,1}, \cdots, f_{2,M})$ with $f_{2,m} \geq 0$. The first term of the objective function in (19) can be expressed as (22). The computation result of $\mathbf{A}_2$ in (22) is also a diagonal matrix.

$$\log_2|\mathbf{A}_2| = \log_2|\mathbf{I}_N + \rho_s(\mathbf{\Lambda}_2\mathbf{F}_2^2\mathbf{D}_2(\mathbf{I}_N + \mathbf{D}_2\mathbf{F}_2^2)^{-1})| \tag{22}$$

To simplify the analysis, we directly discuss the kth relay node, the cost function is given by

$$\log_2|\mathbf{A}_k| + \log_2|\mathbf{I}_N + \mathbf{A}_k^{-1/2}\cdots\mathbf{A}_1^{-1/2}\rho_s(\sum_{i=k+1}^{K}\mathbf{G}_i\mathbf{Q}_i\mathbf{H}_i (\mathbf{I}_N + \mathbf{G}_i\mathbf{Q}_i\mathbf{Q}_i^H\mathbf{G}_i^H)^{-1}\mathbf{H}_i^H\mathbf{Q}_i^H\mathbf{G}_i^H)\mathbf{A}_1^{-1/2}\cdots\mathbf{A}_k^{-1/2}| \tag{23}$$

Where we also consider variable substitution $\mathbf{A}_k = \mathbf{I}_N + \rho_s\mathbf{A}_{k-1}^{-1/2}\cdots\mathbf{A}_1^{-1/2}\mathbf{G}_k\mathbf{Q}_k\mathbf{H}_k(\mathbf{I}_N + \mathbf{G}_k\mathbf{Q}_k\mathbf{Q}_k^H\mathbf{G}_k^H)^{-1}\mathbf{H}_k^H\mathbf{Q}_k^H \mathbf{G}_k^H\mathbf{A}_1^{-1/2}\cdots\mathbf{A}_{k-1}^{-1/2}$, we employ the SVD to obtain the optimal structure of the kth relay precoding matrix.

$$\mathbf{H}_k\mathbf{A}_1^{-1/2}\cdots\mathbf{A}_{k-1}^{-1/2} = \mathbf{U}_k(\mathbf{\Lambda}_k^{1/2})\mathbf{\Gamma}_k^H \quad \mathbf{G}_k^H\mathbf{G}_k = \mathbf{V}_k\mathbf{D}_k\mathbf{V}_k^H \tag{24}$$

$$\mathbf{Q}_k = \mathbf{V}_k\mathbf{F}_k\mathbf{U}_k^H \tag{25}$$

Where $k = 3, 4, \cdots, K$, the discussion of $\mathbf{\Lambda}_k$, $\mathbf{D}_k$, $\mathbf{F}_k$ and $\mathbf{U}_k$, $\mathbf{V}_k$, $\mathbf{\Gamma}_k$ are same as $\mathbf{\Lambda}_2$, $\mathbf{D}_2$, $\mathbf{F}_2$ and $\mathbf{U}_2$, $\mathbf{V}_2$, $\mathbf{\Gamma}_2$. The first term of the cost function in (23) can be given by

$$\log_2|\mathbf{A}_k| = \log_2|\mathbf{I}_N + \rho_s(\mathbf{\Lambda}_k\mathbf{F}_k^2\mathbf{D}_k(\mathbf{I}_N + \mathbf{D}_k\mathbf{F}_k^2)^{-1})| \tag{26}$$

The computation result of $\mathbf{A}_k$ in (26) is also a diagonal matrix as $\mathbf{A}_i, i = 1, 2, \cdots, k-1$. Finally, the original cost function in (11) can be transformed into the sum of multiple diagonal matrices

$$C = \sum_{k=1}^{K}\log_2|\mathbf{A}_k| = \sum_{k=1}^{K}\log_2|\mathbf{I}_N + \rho_s(\mathbf{\Lambda}_k\mathbf{F}_k^2\mathbf{D}_k(\mathbf{I}_N + \mathbf{D}_k\mathbf{F}_k^2)^{-1})|, \quad k = 1, 2, \cdots, K \tag{27}$$

Moreover, according to the property $\text{tr}(\mathbf{YXY}^{-1}) = \text{tr}(\mathbf{X})$ and $\text{tr}(\mathbf{XY}) = \text{tr}(\mathbf{YX})$, the transmission power at the kth relay node can be given as

$$\text{tr}\{(\mathbf{I}_N + \rho_s\mathbf{\Lambda}_k)\mathbf{F}_k^2\} \leq \rho_r M, \quad k = 1, 2, \cdots, K \tag{28}$$

Finally, based on the mathematical derivation and matrix decomposition, we transform the original optimization problem into a scalar optimization problem, which is easier to solve.

$$\max_{|f_{k,m}|^2, k=1,\cdots,K} \sum_{k=1}^{K}(\sum_{m=1}^{M}\log_2(1 + \rho_s\frac{\alpha_{k,m}|f_{k,m}|^2\beta_{k,m}}{1 + \beta_{k,m}|f_{k,m}|^2})) \tag{29}$$

$$s.t.\ |f_{k,m}|^2 \geq 0, \quad m = 1, 2, \cdots, M \tag{30}$$

$$\sum_{m=1}^{M}(1 + \rho_s\alpha_{k,m})|f_{k,m}|^2 \leq \rho_r M \tag{31}$$

The problem in (29)-(31) is a standard convex optimization problem with respect to $|f_{k,m}|^2$. K.K.T conditions are applied to obtain the optimal values of the diagonal elements for all relay power allocation matrices. This optimization problem is similar to a problem solved in [9].

$$|f_{k,m}|^2 = \frac{1}{2\beta_{k,m}(1 + \rho_s\alpha_{k,m})} [\sqrt{\rho_s^2\alpha_{k,m}^2 + 4\rho_s\alpha_{k,m}\beta_{k,m}\mu_k^*} - \rho_s\alpha_{k,m} - 2]_+ \tag{32}$$

Where $\mu_k^*, k = 1, 2, \cdots, K$ are Lagrangian multipliers, which must be satisfied with the power constraint at the kth relay node: $\sum_{m=1}^{M}(1 + \rho_s\alpha_{k,m})|f_{k,m}|^2 = \rho_r M$. In this letter, the bisection method is used to solve μ_k^*.

IV. Simulation Results

In this section, the capacity performance of the proposed relaying scheme and conventional relaying schemes are numerically illustrated. For verifying the performance superiority of the proposed relaying scheme, there are several conventional relaying schemes to be compared as follows:

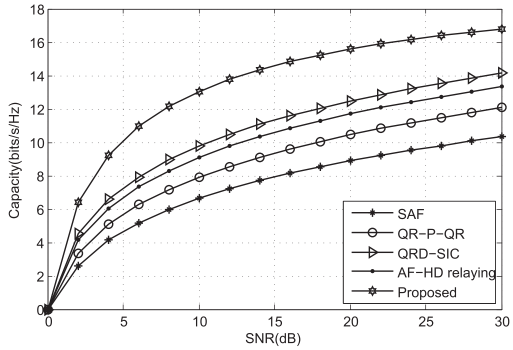

Fig. 2. The capacity performance versus SNR for conventional and proposed relaying schemes with $M = N = 4$ and $K = 2$.

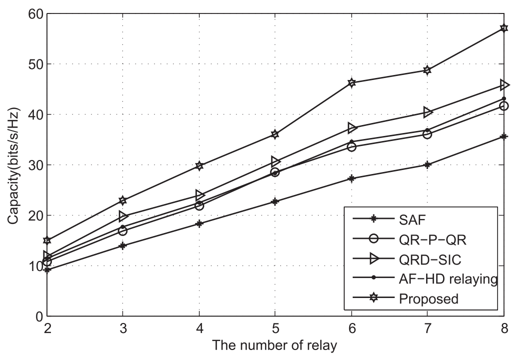

Fig. 3. The capacity performance versus the number of relays for conventional and proposed relaying schemes.

1) SAF relaying scheme in [1]: $\mathbf{Q}_k = \sqrt{P_r/\text{tr}(\mathbf{H}_k\mathbf{R}_s\mathbf{H}_k^H + \mathbf{R}_{\mathbf{n}_{r,k}})}\mathbf{I}_M$;
2) QR-P-QR relaying scheme in [2]: $\mathbf{Q}_k = \mathbf{Q}2_k\mathbf{O}_k\mathbf{Q}1_k^H$;
3) QRD-SIC relaying scheme in [3]: $\mathbf{Q}_k = \mathbf{G}_k^H\mathbf{H}_k^H$;
4) AF-HD relaying scheme in [10]:$\mathbf{Q}_k = \mathbf{V}_{rd,k}\sqrt{(\mathbf{\Lambda}_{sr,k}^2 - \mathbf{\Lambda}_{sr,k}\mathbf{\Lambda}_{rd,k})/(\mathbf{\Lambda}_{sr,k}^2 - \mathbf{\Lambda}_{rd,k}^2)}(\mathbf{U}_{sr,k})^H$.

Fig. 2 displays the comparisons of capacity performance for conventional and proposed relaying schemes in the $M = N = 4, K = 2$ configuration. For simplicity, SNR is employed to denote all signal-to-noise ratio in the following as SNR=SNR$_{sr}$=SNR$_{rd}$. For the capacity performance versus SNR curves, the capacity performance of the proposed relaying scheme is more than that of the conventional relaying schemes in all SNR ranges. In particular, the capacity performance gain achievable by the proposed relaying scheme is at most 3 bits/s/Hz higher than the gain achieved by the QRD-SIC relaying scheme.

Now, we investigate the impact of the number of relays. Fig. 3 shows the capacity performance versus the number of relays for conventional and proposed relaying schemes. Without loss of generality, we observe that the capacity performance of all relaying schemes increases when the number of relays increases. For all considered number of relays, the capacity performance of the conventional relaying schemes is less than that of the proposed relaying scheme.

Table I shows the computation-complexity analysis of all relaying schemes. In this letter, the flop number [11] is adopted to measure the complexity of the schemes.

TABLE I
THE COMPUTATION COMPLEXITY ANALYSIS OF ALL RELAYING SCHEMES

Scheme	Each Relay node	Destination Node
SAF	$16(2N^3 + N^2)$	0
QR-P-QR	$8(3N^3 + 2N^2)$	$8(N^3 + N^2)$
QRD-SIC	$16(N^3 + N^2)$	$8((K+1)N^3 + (K+2)N^2)$
Proposed	$8(3N^3 + 2N^2)$	0
AF-HD relaying	$\mathcal{O}((N^3 \times K)) + \mathcal{O}((N \times K \times t_{tol} \times l_{iter}))$ [10]	

For simplicity, only the flop numbers of matrix multiplication are analysed, it is assumed that $\mathbf{H}$ is a $m \times n$ matrix and $\mathbf{G}$ is a $n \times p$ matrix, then the flop number of the matrix multiplication $\mathbf{HG}$ are $8mnp$. As shown in Table I, there is a low-complexity operation of the proposed relaying scheme, and it provides a good tradeoff between the complexity and the capacity performance compared with the other relaying schemes.

V. CONCLUSION

For the AF multi-relay symmetric MIMO communication networks, a relaying scheme of the objective function diagonalization based on the ED and SVD has been developed aiming to optimize the relay precoding matrix under the MI maximization with the per relay power constraint. For each relay node, we established the optimal structure of the relay precoding matrix separately and the optimal values of the diagonal elements for the power allocation matrices are solved by the K.K.T conditions. Simulation results show that, in all SNR ranges, the system capacity performance of the proposed relaying scheme is more effective than the conventional relaying schemes.

REFERENCES

[1] Y. M. Cho, J. Yang, and J. Seo, "Sum MSE minimization for downlink multi-relay multi-user MIMO network," *KSII Trans. Internet Inf. Syst.*, vol. 8, no. 8, pp. 2722–2742, 2014.

[2] H. Shi, T. Abe, T. Asai, and H. Yoshino, "Relaying schemes using matrix triangularization for MIMO wireless networks," *IEEE Trans. Commun.*, vol. 55, no. 9, pp. 1683–1688, Sep. 2007.

[3] Y. Zhang, H. W. Luo, and W. Chen, "Efficient relay beamforming design with SIC detection for dual-hop MIMO relay networks," *IEEE Trans. Veh. Technol.*, vol. 59, no. 8, pp. 4192–4197, Oct. 2010.

[4] Y. Fu, L. Yang, and W.-P. Zhu, "A nearly optimal amplify-and-forward relaying scheme for two-hop MIMO multi-relay networks," *IEEE Commun. Lett.*, vol. 14, no. 3, pp. 229–231, Mar. 2010.

[5] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York, NY, USA: Wiley, 1991.

[6] D. S. Bemstein, *Matrix Mathematics*. Princeton, NJ, USA: Princeton Univ. Press, 2005.

[7] Z. Y. Zhang and L. Qiu, "Optimal progressive precoder design for ARQ packet retransmissions in nonregenerative MIMO relay systems," in *Proc. Veh. Technol. Conf.*, May 2012, pp. 1–5.

[8] Z. Zhang, L. Qiu, and J. Xu, "Progressive linear precoder optimization for ARQ packet retransmissions in nonregenerative MIMO relay systems," *IEEE Trans. Signal Process.*, vol. 61, no. 1, pp. 68–80, Jan. 2013.

[9] Y. Yu and Y. Hua, "Power allocation for a MIMO relay system with multiple-antenna users," *IEEE Trans. Signal Process.*, vol. 58, no. 5, pp. 2823–2835, May 2010.

[10] D. W. K. Ng, E. S. Lo, and R. Schober, "Dynamic resource allocation in MIMO-OFDMA systems with full-duplex and hybrid relaying," *IEEE Trans. Commun.*, vol. 60, no. 5, pp. 1291–1304, May 2012.

[11] Z. Shen, R. Chen, J. G. Andrews, R. W. Heath, and B. L. Evans, "Low complexity user selection algorithms for multiuser MIMO systems with block diagonalization," *IEEE Trans. Signal Process.*, vol. 54, no. 9, pp. 3658–3663, Sep. 2006.