Distributed Energy Efficient Fair User Association in Massive MIMO Enabled HetNets

Dantong Liu, Lifeng Wang, Yue Chen, Tiankui Zhang, Kok Keong Chai, and Maged Elkashlan

***Abstract*—Massive multiple-input and multiple-output (MIMO) and heterogeneous networks (HetNets) have been recognized as key enabling technologies for future fifth generation (5G) mobile networks. However, the circuit power consumption of massive MIMO scales with the tremendous number of antennas. As a result, the problem of energy efficient user association in massive MIMO enabled HetNets is of vital importance. We investigate the energy efficient user association problem in massive MIMO enabled HetNets, and formulate the network logarithmic utility maximization problem. Based on the Lagrangian dual analysis, a low complexity distributed user association algorithm is developed for energy efficient fair user association while considering quality of service (QoS) provision for users. Simulation results demonstrate the effectiveness of the proposed algorithm in improving the energy efficiency and user fairness, compared to other user association algorithms.**

***Index Terms*—HetNets, massive MIMO, energy efficient, user association.**

I. Introduction

Heterogeneous Networks (HetNets) escalates the spectrum efficiency by using a mix of macrocells and small cells. On the other hand, massive multiple-input and multiple-output (MIMO) technology promises enormous enhancement in spectrum efficiency by transmitting independent data streams via a tremendous number of antennas simultaneously to multiple users sharing the same transmission resource. These benefits have put massive MIMO and HetNets in the spotlight of preliminary fifth generation (5G) discussions [1].

Although massive MIMO promises unprecedented increase in spectrum efficiency, the circuit power consumption also scales with the tremendous number of antennas. As such, the energy efficient radio resource allocation (RRA) in massive MIMO systems has prompted significant research [2].

User association, as an indispensable functionality of RRA, has great impact on the performance of the wireless network. Most existing research on user association in HetNets has focused on single antenna HetNets (see [3], [4] and the citations therein). The research on user association in massive MIMO enabled HetNets is limited and still in its infancy. In [5], an optimal user association algorithm is proposed for massive MIMO networks to maximize the sum logarithmic user data rate, and an interesting observation is given that despite allowing association to multiple base stations (BSs) in the convex formulation, the single BS association performs close to the globally optimal solution.

While the aforementioned laid a good foundation in understanding user association in massive MIMO enabled HetNets, the energy efficiency of user association in such a scenario is less well understood. In [6], a beamforming vector optimization is achieved by minimizing power consumption while considering quality of service (QoS) constraint. Different from [6], in order to investigate both the spectrum efficiency and power consumption, we adopt energy efficiency (bits/Joule) as a fundamental performance metric. We propose an energy efficient fair user association in massive MIMO enabled HetNets, and formulate the sum logarithmic user energy efficiency maximization problem. Based on the Lagrangian dual analysis, a low complexity distributed user association algorithm is proposed to ensure energy efficient fair user association, under QoS constraint, which has been proven to converge to the global optimum of the dual problem. From the simulation results, we conclude some insights into the effect of transmit power and antenna number of massive MIMO on the energy efficient user association.

II. System Model

We focus on the 2-tier downlink HetNets where tier 1 is modeled as macrocell and tier 2 as picocell. Macro BSs (MBSs) provide basic coverage, whereas Pico BSs (PBSs) are deployed in the coverage area of each MBS to enhance capacity. We assume large-scale antenna array is implemented at MBS, while PBSs are single antenna BSs. Let $i \in \mathcal{B}$ index i-th BS, where $i = 1$ indicates the MBS, and others are PBSs, with all the BSs assumed to share the same frequency band. We have a set of single antenna users $\mathcal{U}$ distributed in one macrocell geographical area, with $j \in \mathcal{U}$ to index users. We assume each user can only associate with one BS at any time, since associating one user with multiple BSs incurs more implementation difficulties than that with single BS. Then the proposed user association algorithm is applied to decide which BS i will serve which user j.

In order to formulate the user association problem, we define the association matrix $\mathbf{X}$ as

$$x_{ij} = \begin{cases} 1, & \text{if user } j \text{ is associated with BS } i \\ 0, & \text{otherwise} \end{cases}. \tag{1}$$

A. Modeling the Massive MIMO MBS User Rate

We denote M as the number of MBS antennas, and S as the maximum number of downlink data stream that MBS can transmit simultaneously on any given resource block (RB) with equal power allocation. We assume the time-division duplexing (TDD) operation with reciprocity-based channel state estimation and perfect channel state information (CSI).[1] We refer to

[1] Since user association is carried out in a large time scale compared to the change of channel, the fast fading is averaged out, and perfect CSI is considered [3], [7].

Manuscript received April 13, 2015; revised June 24, 2015; accepted July 6, 2015. Date of publication July 9, 2015; date of current version October 8, 2015. The associate editor coordinating the review of this paper and approving it for publication was M. Matthaiou.

D. Liu, Y. Chen, K. K. Chai, and M. Elkashlan are with the School of Electronic Engineering and Computer Science, Queen Mary University of London, London E1 4NS, U.K. (e-mail: d.liu@qmul.ac.uk; yue.chen@qmul.ac.uk; michael.chai@qmul.ac.uk; maged.elkashlan@qmul.ac.uk).

L. Wang is with the Department of Electronic & Electrical Engineering, University College London, London WC 1E7JE, U.K. (e-mail: lifeng.wang@ucl.ac.uk).

T. Zhang is with the School of Information and Communication Engineering, Beijing University of Posts and Telecommunications, Beijing 100876, China (e-mail: zhangtiankui@bupt.edu.cn).

Digital Object Identifier 10.1109/LCOMM.2015.2454504

the massive MIMO regime as the case where $1 \ll S \ll M$. In this regime, user data rate tends to concentrate on deterministic limits when the number of antennas grows large, i.e., the effect of small-scale fading disappears [5].[2] Here we borrow the data rate formula in [5] for the linear zero-forcing beamforming (LZFBF) precoding. In practice, the total number of users associated with MBS is always larger than S. We assume the users associated with MBS are served by resource sharing. Thus when user j is associated with MBS equipped with large antenna array, the achievable downlink data rate r_{1j} can be expressed as

$$r_{ij} = \left(\sum\nolimits_{j\in\mathcal{U}} x_{ij}\right)^{-1} S\log(1+\gamma_{ij}),\ i=1, \tag{2}$$

where

$$\gamma_{ij} = (M-S+1)\,\frac{\frac{P_i^t}{S}g_{ij}}{\sigma^2+\sum_{i'\neq i}P_{i'}^t g_{i'j}},\ i=1. \tag{3}$$

Here, we adopt a Round Robin scheduler as in [4]. Equal resource sharing among users associated with MBS is assumed. P_i^t denotes the transmit power of BS i, σ^2 is the noise power level, and g_{ij} is the large-scale fading channel power gain between BS i and user j. Note that the SINR γ_{1j} is scaled by $(M-S+1)/S$ due to the array gain and the equal power assignment from massive MIMO over the Rayleigh channel fading [8]. The data rate expression in (2) is simple, accurate and well adopted in system optimization.

B. Modeling the Single Antenna PBS User Rate

For single antenna PBS, we assume that all PBSs allocate the time-frequency resources among the associated users equally. The long-term downlink data rate of user j when associated with PBS $i, i>1$ is

$$r_{ij} = \left(\sum\nolimits_{j\in\mathcal{U}} x_{ij}\right)^{-1} \log(1+\gamma_{ij}),\ i>1, i\in\mathcal{B}, \tag{4}$$

where

$$\gamma_{ij} = \frac{P_i^t g_{ij}}{\sigma^2+\sum_{i'\neq i}P_{i'}^t g_{i'j}},\ i>1, i\in\mathcal{B}. \tag{5}$$

C. Power Consumption Model for Massive MIMO MBS

The power consumption of conventional BS is proportional to the radiated transmit power [9]. However, such an assumption in massive MIMO systems is misleading since an infinite energy efficiency can be achieved as $M\to\infty$. As such, we adopt the model in [10] which clearly specifies how the power consumption P_i scales with the antenna number M.

$$P_i = P_i^t/\eta_i + \sum\nolimits_{m=0}^{3} C_{m,0}S^m + \sum\nolimits_{m=0}^{2} C_{m,1}S^m M,\ i=1, \tag{6}$$

where η_i is the efficiency of power amplifier of BS i, $C_{m,0}$ and $C_{m,1}$ are coefficients. The total power consumption includes the power consumption of transceiver chains, coding and decoding, channel estimation and precoding, and architectural costs, as detailed in [10].

[2]Notice that the length of the coherence interval has negligible effect on user association.

D. Power Consumption Model for Single Antenna PBS

For single antenna PBS, we adopt the conventional linear power consumption model in [9] which is composed of the static and adaptive power consumptions. The adaptive power consumption is linear to the radiated transmit power of PBSs, and the static power consumption is related to the power consumption of transceiver chains. Based on [9], the power consumption of BS i P_i is given by

$$P_i = P_i^t/\eta_i + P_i^s,\ i\neq 1, i\in\mathcal{B}, \tag{7}$$

where P_i^s is the static power consumption of BS i.

III. Problem Formulation

The energy efficiency (bits/Joules) of user j when associated with BS i is defined as r_{ij}/P_i. As shown in [11], maximizing the sum energy efficiency within a system will result in extremely unfair throughput allocation. To preserve some degree of fairness and avoid user starvation, we adopt the energy efficiency proportional fairness criterion as in [11], where the utility of user j when associated with BS i is defined as $\mu_{ij} = \log(r_{ij}/P_i)$. The logarithm function is widely used to construct utility function, as it is concave and hence has diminishing benefits, which encourages user fairness and load balancing.

Our problem is to find optimal user association matrix $\mathbf{x}$ that maximizes the overall network utility which is given by

$$\max_{\mathbf{x}} \quad \sum\nolimits_{j\in\mathcal{U}}\sum\nolimits_{i\in\mathcal{B}} x_{ij}\mu_{ij} \tag{8}$$

$$\sum\nolimits_{i\in\mathcal{B}} x_{ij} = 1,\ \forall j \tag{9}$$

$$\sum\nolimits_{j\in\mathcal{U}} x_{ij} = K_i,\ \forall i \tag{10}$$

$$\sum\nolimits_{i\in\mathcal{B}} x_{ij}\gamma_{ij} \geq \gamma_j^{\min},\ \forall j \tag{11}$$

$$x_{ij}\in\{0,1\},\ \forall j, \forall i, \tag{12}$$

where $\gamma_j^{\min}$ is the minimum downlink SINR required by user j. The constraint (9) and (12) imply that one user can only be associated with one BS at any time. The constraint (11) indicates that the user's received downlink SINR cannot be smaller than the minimum SINR requirement in order to fulfil QoS provision for users.

A. Lagrangian Dual Analysis

For the optimal solution of the optimization problem, the Lagrangian function can be obtained as below

$$L(\mathbf{x},\mathbf{K},\mathbf{a},\mathbf{b}) = \sum_{j\in\mathcal{U}}\sum_{i\in\mathcal{B}} x_{ij}\log(c_{ij}) - \sum_{i\in\mathcal{B}} K_i\log(K_iP_i)$$
$$-\sum_{i\in\mathcal{B}} a_i\left(\sum_{j\in\mathcal{U}} x_{ij} - K_i\right) - \sum_{j\in\mathcal{U}} b_j\left(\gamma_j^{\min} - \sum_{i\in\mathcal{B}} x_{ij}\gamma_{ij}\right), \tag{13}$$

where $c_{ij} = S\log(1+\gamma_{ij})$, if $i=1$, and otherwise, $c_{ij} = \log(1+\gamma_{ij})$. In (13), a_i and b_j are nonnegative Lagrange multipliers.

It follows that the dual function $g(\cdot)$ can be written as

$$\min_{\mathbf{a},\mathbf{b}} g(\mathbf{a},\mathbf{b}) = \begin{cases} \max\limits_{\mathbf{x},\mathbf{k}} L(\mathbf{x},\mathbf{K},\mathbf{a},\mathbf{b}) \\ \text{s.t. } \sum_{i\in\mathcal{B}} x_{ij} = 1,\ \forall j \\ \quad x_{ij}\in\{0,1\},\ \forall j,\ \forall i \\ \quad \mathbf{a},\mathbf{b}\geq\mathbf{0} \end{cases}. \tag{14}$$

The maximization of Lagrangian has the following analytic solution

$$x_{ij}^* = \begin{cases} 1, & \text{if } i = i^* \\ 0, & \text{if } i \neq i^* \end{cases}, \tag{15}$$

where

$$i^* = \arg\max_i \left(\log(c_{ij}) - a_i + b_j\gamma_{ij}\right). \tag{16}$$

Setting $\partial L/\partial K_i = 0$, K_i^* can be derived as

$$K_i^* = e^{a_i-1}/P_i. \tag{17}$$

Since the dual function $g(\mathbf{a}, \mathbf{b})$ is not a differentiable function, we cannot directly obtain the closed-form expression for the optimal solution. To obtain the optimal solution of this dual problem, we adopt the subgradient method to update **a** and **b** as

$$a_i(t+1) = a_i(t) - \delta(t)\left(K_i(t) - \sum\nolimits_{j\in\mathcal{U}} x_{ij}(t)\right), \ \forall i, \tag{18}$$

$$b_j(t+1) = b_j(t) - \delta(t)\left(\sum\nolimits_{i\in\mathcal{B}} x_{ij}(t)\gamma_{ij} - \gamma_j^{\min}\right), \ \forall j, \tag{19}$$

where t represents the t-th iteration, and $\delta(t)$ is the step size. With the updated $a_i(t)$ and $b_j(t)$, $x_{ij}(t)$ and $K_i(t)$ can be updated accordingly via (15)–(17). Since the dual problem is always convex, the subgradient method is guaranteed to converge to the globally optimum of the dual problem (14).

There is an interesting interpretation of **a** and **b** as follows: We can regard **b** as the dissatisfactory factor of users. With $\sum_{i\in\mathcal{B}} x_{ij}\gamma_{ij}$ interpreted as the received SINR of user j, according to (19), when the received SINR is larger than the minimum required SINR, b_j will decrease, which will in turn reduce the weight of SINR value when choosing the serving BS via (16). We can also regard **a** as the inter-price between users and BSs. With $\sum_{j\in\mathcal{U}} x_{ij}$ interpreted as the amount of service demanded by users, and K_i as the amount of service BS i wants to supply, the inter-price **a** represents the *law of supply and demand*. As shown in (18), if the service demand $\sum_{j\in\mathcal{U}} x_{ij}$ is larger than the service supply K_i of BS i, the price a_i will increase, otherwise, a_i will decrease. In this sense, in line with (15) and (16), if BS i is overloaded, the inter-price a_i will go up, and fewer users will be associated with it.

B. Convergence Analysis

Lemma 1: The proposed subgradient decent method will converge to the optimum of dual problem $g(\cdot)$

Proof: The derivatives of the dual function $g(\cdot)$ are

$$\partial g(\mathbf{a}, \mathbf{b})/\partial a_i = K_i(a_i) - \sum\nolimits_{j\in\mathcal{U}} x_{ij}(a_i, b_j), \tag{20}$$

$$\partial g(\mathbf{a}, \mathbf{b})/\partial b_j = \sum\nolimits_{i\in\mathcal{B}} x_{ij}(a_i, b_j)\gamma_{ij} - \gamma_j^{\min}. \tag{21}$$

Since $x_{ij} \in \{0, 1\}$, $\sum_{j\in\mathcal{U}} x_{ij}(a_i, b_j)$ is bounded. In our primal problem, $K_i = \sum_{j\in\mathcal{U}} x_{ij} \le |\mathcal{U}|$, and thus $K_i(a_i)$ is bounded. Hence, we can conclude that the subgradients of dual objective function are also bounded.

$$\sup_t \{\|\partial g(\mathbf{a}, \mathbf{b})/\partial a_i\|\} \le c, \tag{22}$$

$$\sup_t \left\{\left\|\partial g(\mathbf{a}, \mathbf{b})/\partial b_j\right\|\right\} \le c, \tag{23}$$

where c is a scalar. In this sense, our problem satisfies the necessary conditions of the convergence proof in [12], whereby we can prove **Lemma 1**. □

After the dual solution is obtained for (14), the primal variable **x** can be recovered according to (15) and (16). There is a possibility that a user may have more than one BS with the same maximal value for $(\log(c_{ij}) - a_i + b_j\gamma_{ij})$. Such ties can be solved using heuristics. In general, we would like to keep $\sum_{j\in\mathcal{U}} x_{ij}$ as close to the K_i calculated according to (17) as possible. In our simulation experiment, only a small number of users are involved in ties, so tie-breaking via exhaustive search is feasible.

Note that since the primal optimization problem (8)–(12) is discrete in nature, solving the dual problem (14) may not be the same as solving the primal problem. As such, a positive duality gap may exist. Nevertheless, the optimum of the dual problem often leads to good primal solutions [3], and we have proven that the gap between primal and dual problems is bounded.

Algorithm 1 At user side

---

1. **if** $t = 0$, **then**
2. Initialize $b_j(t+1)$.
3. **else**
4. Updates $b_j(t+1)$ according to (19).
5. **end if**
6. $t \leftarrow t+1$.
7. Each user measures SINR via pilot signal from all BSs, and receives the values of $a_i(t)$, $\forall\, i$ via BS broadcast.
8. User j determines the serving BS i according to $i^*(t) = \arg\max_i(\log(c_{ij}) + b_j(t)\gamma_{ij} - a_i(t))$.
9. Each user feedbacks the user association request to the chosen BS.

Algorithm 2 At BS side

---

1. **if** $t = 0$, **then**
2. Initialize $a_i(t+1)$.
3. **else**
4. Each BS calculates the value of $K_i(t)$ according to (17).
5. Receives the updated user association matrix $x_{ij}(t)$ from users.
6. Updates $a_i(t+1)$ as (18).
7. **end if**
8. $t \leftarrow t+1$.
9. Broadcasts the value of $a_i(t)$.

IV. Proposed Distributed Energy Efficient Fair User Association Algorithm

The primal optimization problem (8) is combinational due to the binary variable x_{ij}. The complexity of the centralized brute force algorithm is $\mathcal{O}(|\mathcal{B}|^{|\mathcal{U}|})$, which is impossible even for a modest-sized network. Furthermore, centralized algorithm requires global network information and centralized controller for coordination, and the amount of exchanged information is proportional to $(|\mathcal{B}| \times |\mathcal{U}|)$. Based on the previous analysis, we propose a distributed energy efficient fair user association algorithm, which involves the algorithm at user side and the algorithm at BS side as shown in **Algorithm 1** and **Algorithm 2**, respectively. The user and BS sides implement **Algorithm 1** and **Algorithm 2** iteratively until convergence. In each iteration, the complexity of the distributed algorithm is $\mathcal{O}(|\mathcal{B}||\mathcal{U}|)$. As for the exchanged information, in each iteration, each BS

TABLE I
SIMULATION PARAMETERS

Parameter	Value	Parameter	Value
Bandwidth	10 MHz	Inter Site Distance	500m
Noise power	-174 dBm/Hz	Transmit power of PBS	30 dBm
S	10 [5]	Static power consumption of PBS	13.6 W [9]
$\gamma^{\min}$	0dB	Efficiency of power amplifier	0.3 [10]

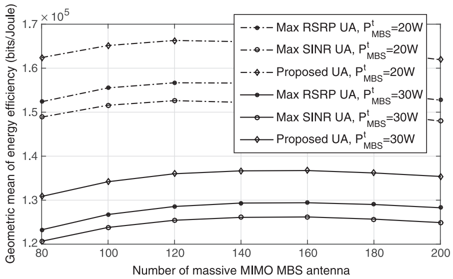

Fig. 1. Geometric mean of energy efficiency versus number of antennas and transmit powers of MBS.

broadcasts a_i, and each user feedbacks its association request only to the chosen BS. Thus the exchanged information in the proposed distributed algorithm is $(|\mathcal{B}| + |\mathcal{U}|)$ for each iteration. The simulation shows that when the step size is set as $\delta(t) = 1/t$, where t is the number of iteration, the proposed distributed algorithm converges reasonably quick with less than 50 iterations.

V. SIMULATION RESULTS

To evaluate the performance of the proposed user association algorithm, we simulate a downlink HetNet composed of 19 macrocells. In each macrocell, 3 PBSs are symmetrically located along a circle with radius 200 m and MBS in the center. In each macrocell, 30 users are randomly distributed. The basic simulation parameters are shown in Table I. The pathloss between MBS and user, PBS and user is $128.1+37.6\log_{10} d$ (km) and $140.7+36.7\log_{10} d$ (km), respectively. We set the coefficients for power consumption under LZFBF precoding as $C_{0,0} = 4, C_{1,0} = 4.8, C_{2,0} = 0, C_{3,0} = 2.08 \times 10^{-8}, C_{0,1} = 1$, $C_{1,1} = 9.5 \times 10^{-8}$ and $C_{2,1} = 6.25 \times 10^{-8}$ [10].

In particular, there is no "standard" commonly accepted method for user association in massive MIMO enabled network. We then compare the performance of the proposed user association algorithm (proposed UA) with the simple max Reference Signal Received Power user association (max RSRP UA), and max SINR user association (max SINR UA). In max RSRP UA, user associates with the BS from which it receives the highest RSRP, while in max SINR UA, user chooses the BS with the highest SINR, where the SINR received from massive MIMO MBS is scaled by the array gain.

Fig. 1 demonstrates the geometric mean of energy efficiency versus number of antennas and transmit powers of massive MIMO MBS. We observe that regardless of the number of antennas and the transmit power of MBS, the proposed UA achieves better energy efficiency than the max SINR UA and max RSRP UA. For the same transmit power of MBS, the energy efficiency increases and then decreases with increasing number of antennas. This is attributed to the fact that when the number of antennas exceeds a critical value (approximately 120 when $P^t_{MBS} = 20$ W), adding more antennas improves spectrum efficiency, but degrades energy efficiency due to more power consumption. It also illustrates that with the same number of antennas, the relatively lower transmit power of MBS achieves better energy efficiency, which exhibits the superiority of massive MIMO in fulfilling user QoS requirement with lower transmit powers.

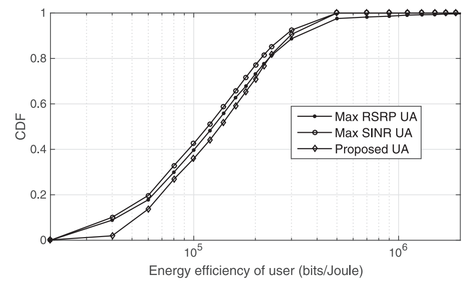

Fig. 2. CDF of user energy efficiency.

We then set transmit power of MBS as 30 W and number of MBS antenna as 100. Fig. 2 shows the cumulative distribution function (CDF) of the user energy efficiency in different UAs. We observe that the CDF of the proposed UA improves significantly at low energy efficiency versus the CDFs of max SINR UA and max RSRP UA. The CDF of max RSRP UA catches up at the energy efficiency of 2×10^5 bits/Joule. The reason is that the proposed UA takes advantage of the energy efficiency proportional fairness criterion which provides a more uniform energy efficiency by taking resources from strong users. As such, the proposed UA improves the user fairness in terms of energy efficiency compared to other UAs.

VI. CONCLUSION

We proposed distributed energy efficient fair user association in massive MIMO enabled HetNets. We formulated a network-wide logarithmic utility maximization problem, under user QoS constraint. The proposed algorithm has low complexity of $\mathcal{O}(|\mathcal{B}||\mathcal{U}|)$ in each iteration. Simulation results verify the effectiveness of the proposed algorithm, and provide insights into the effect of massive MIMO on energy efficient user association.

REFERENCES

[1] J. Andrews *et al.*, "What will 5G be?" *IEEE J. Sel. Area Commun.*, vol. 32, no. 6, pp. 1065–1082, Jun. 2014.
[2] D. W. K. Ng, E. S. Lo, and R. Schober, "Energy-efficient resource allocation in OFDMA systems with large numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 11, no. 9, pp. 3292–3304, Sep. 2012.
[3] K. Shen and W. Yu, "Distributed pricing-based user association for downlink heterogeneous cellular networks," *IEEE J. Sel. Area Commun.*, vol. 32, no. 6, pp. 1100–1113, Jun. 2014.
[4] D. Liu, Y. Chen, K. K. Chai, T. Zhang, and M. Elkashlan, "Opportunistic user association for multi-service HetNets using nash bargaining solution," *IEEE Commun. Lett.*, vol. 18, no. 3, pp. 463–466, Mar. 2014.
[5] D. Bethanabhotla, O. Y. Bursalioglu, H. C. Papadopoulos, and G. Caire, "Optimal user-cell association for massive MIMO wireless networks," 2014, arXiv:1407.6731.
[6] E. Björnson, M. Kountouris, and M. Debbah, "Massive MIMO and small cells: Improving energy efficiency by optimal soft-cell coordination," in *Proc. 20th Int. Conf. Telecommun.*, May 2013, pp. 1–5.
[7] Q. Ye *et al.*, "User association for load balancing in heterogeneous cellular networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 6, pp. 2706–2716, Jun. 2013.
[8] K. Hosseini, W. Yu, and R. S. Adve, "Large-scale MIMO versus network MIMO for multicell interference mitigation," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 930–941, Oct. 2014.
[9] G. Auer *et al.*, "How much energy is needed to run a wireless network?" *IEEE Wireless Commun.*, vol. 18, no. 5, pp. 40–49, Oct. 2011.
[10] E. Björnson, L. Sanguinetti, J. Hoydis, and M. Debbah, "Designing multi-user MIMO for energy efficiency: When is massive MIMO the answer?" in *Proc. IEEE WCNC*, Apr. 2014, pp. 1–6.
[11] A. Garcia-Saavedra, P. Serrano, A. Banchs, and M. Hollick, "Energy-efficient fair channel access for IEEE 802.11 WLANs," in *Proc. IEEE WoWMoM*, Jun. 2011, pp. 1–9.
[12] S. Boyd, L. Xiao, and A. Mutapcic, *Subgradient Methods*. Stanford, CA, USA: Stanford Univ., 2003.