Introduction to the Issue on Robust Subspace Learning and Tracking: Theory, Algorithms, and Applications

SUBSPACE learning theory for dimensionality reduction was initiated with the Principal Component Analysis (PCA) formulation proposed by Pearson in 1901. PCA was first widely used for data analysis in the field of psychometrics and chemometrics but today it is often the first step in more various types of exploratory data analysis, predictive modeling, classification and clustering problems. It finds modern applications in signal processing, biomedical imaging, computer vision, process fault detection, recommendation system design and many more domains. Since one century, numerous other subspace learning models, either reconstructive and discriminative, were developed over time in literature to address dimensionality reduction while keeping the relevant information in a different manner from PCA. However, PCA can also be viewed as a soft clustering method that seeks to find clusters in different subspaces within a dataset, and numerous clustering methods are based on dimensionality reduction. These methods are called subspace clustering methods that are extension of traditional PCA based clustering, and divide data points belonging to the union of subspaces (UoS) into the respective subspaces. In several modern applications, the main limitation of the subspace learning and clustering models are their sensitivity to outliers. Thus, further developments concern robust subspace learning which refers to the problem of subspace learning in the presence of outliers. In fact, even the classical subspace learning problem with speed or memory constraints is not a solved problem. These issues have become practically important for modern datasets because of the following reasons:

- The data and noise maybe correlated in several applications, i.e. need for correlated subspace learning.
- The data matrix is often so large that it cannot be directly stored in the computers memory, i.e. need for streaming subspace learning solutions.
- The data are held at different locations, i.e. need for distributed PCA algorithms to harness local communications and network connectivity to avoid communicating and accessing the entire array locally.
- The data are observed in high-dimension but PCA is usually linear combinations of all input variables. Thus, it encounters great fundamental challenges under high dimensionality and may produce wrong results under high-dimensionality, i.e. need for sparse PCA and scalable PCA algorithms.
- A lot of today data consist of missing entries and/or outlier corrupted entries: i.e. need for matrix completion, robust PCA and robust subspace recovery.
- A lot of data today arrive sequentially, the data subspace itself may change over time, the entire dataset cannot be stored but short buggers can be and decisions are often needed in real-time or near real-time, i.e. need for subspace tracking and dynamic RPCA.
- Many types of data are better represented as a tensor dataset rather than a vector dataset or matrix, i.e. need for tensor PCA/RPCA.

Thus, this special issue group recent works in robust subspace learning and clustering related to theory, algorithms and applications for signal processing and computer vision applications. The call for papers attracted numerous submissions worldwide attempting to address the previously mentioned key issues. After the review process, 37 papers, or 39%, were accepted for publication. Several papers concern algorithms to address *robustness* of subspace learning against different kinds of outliers. First, Markopoulos *et al.* propose an L_1-norm Principal-Component Analysis (L1-PCA) with online outlier rejection to handle outliers among the processed data showing efficiency in direction-of-arrival (DoA) estimation/tracking, image conditioning, and video surveillance. Yin *et al.* design a robust multinomial logistic regression method by solving a rank minimization problem. Performing RPCA in a supervised way, experimental analysis on synthetic and real-world data show classification accuracy. Kaloorazi and de Lamare introduce a rank-revealing matrix decomposition algorithm named Compressed Randomized UTV (CoR-UTV) decomposition with low complexity requirements. Rahmani and Atia address stable RPCA in the presence of simultaneous element-wise and column-wise corruptions, and also provide a scalable implementation of their algorithm. To handle under-determined amount of noisy linear measurements in compressed RPCA, Xue *et al.* develop a Bayesian message passing algorithm called turbo-type message passing (TMP) with low computational complexity. Kaltenstadler *et al.* address the robustness and computational efficiency of stationary subspace analysis (SSA) mainly due to the difficulty in optimizing the Kullback-Leibler which is divergence based objective. For this, a higher numerical efficiency is provided by defining analytical SSA variants while a higher robustness is obtained by utilizing

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the Wasserstein-2 distance. To deal with slow changes in the data, Linh-Trung *et al.* propose low-complexity and adaptive algorithms for robust subspace tracking in certain adverse scenarios of noisy data. These algorithms are analyzed in terms of convergence properties and are effective in the case where the data is affected by sparse outliers. For abrupt changes in the data, Jiao *et al.* design a subspace change-point detection where a stream of high-dimensional data points lie on a low-dimensional subspace. Wang *et al.* present a rigorous analysis and comparison of three popular subspace tracking algorithms namely, Oja's method, GROUSE and PETRELS in the case of streaming and highly incomplete observations. Many problems in signal processing and machine learning can be posed as the problem of learning lower dimensional representation of the data. Kadioglu *et al.* employ a robust subspace approach for Brain Computer Interface (BCI) application where electroencephalography (EEG) data are used to determine user's intent to type letters through stimulation with rapid serial visual presentations (RSVP). For network anomaly detection problem, Yang *et al.* provide a successive convex approximation framework for sparse optimization with flexibility, fast convergence, low complexity and guaranteed convergence to a stationary point. For fuzzy extreme learning machine, Kale and Sonavan employ a Robust PCA feature selection which is able to handle a weighted classification problem in downy mildew and nitrogen deficiency of foliage. Computer vision applications are also addressed in several papers. First, Rezaei and Ostadabbas design a fast robust matrix completion (fRMC) algorithm for moving object detection. Moschoglou *et al.* propose a Multi-Attribute RPCA (MA-RCA) technique suitable for facial UV maps containing a considerable amount of missing information and outliers, while additionally, elegantly incorporates knowledge from various available attributes, such as age and identity. By learning a low-dimensional subspace, Liu *et al.* design a new method that enhances the neighbor-reversibility correlation of two images that describe similar content without utilizing the whole database images for training. To be robust against spectral variability in inverse problems of hyperspectral images unmixing, Hong and Zhu propose a subspace-based unmixing model using low-rank learning strategy named subspace unmixing with low-rank attribute embedding (SULoRA). Tensors based approaches are also addressed in this special issue. First, Chen *et al.* propose a Tensor Nuclear Norm (TNN)-based low-rank approximation with total variation regularization (TLR-TV) for color and multispectral image denoising. By using a new tensor nuclear norm that extends the conventional TNN, Liu *et al.* better extract the low-rank tensor components in multi-way data by investigating the low-rank structure for core tensor in t-SVD framework. By also using TNN, Jiang *et al.* propose to solve low-rank tensor completion and robust tensor PCA based on some novel notion of even-order tensor ranks named the M-rank, the symmetric M-rank, and the strongly symmetric M-rank. In a disruptive manner, Kong *et al.* propose a new definition of tensor Schatten-p norm (t-Schatten-p norm) based on t-SVD which can better approximate the l_1-norm of tensor multi-rank than TNN. Therefore it is used for the Low-Rank Tensor Recovery problems as a tighter regularizer, and is applied with success on face image denoising and image and video inpainting. To handle sparse noise in high dimensional data, Dong *et al.* design a robust tensor approximation (RTA) framework with Laplacian Scale Mixture (LSM) modeling for multi-dimensional data computationally efficient denoising techniques for multi-frame images and video. In most of these TRPCA and robust tensor completion methods, missing entries are assumed to be randomly distributed, and thus the low-rank prior are used to well pose the problem. But, in real applications, missing entries may not only be randomly but also structurally distributed. To address this, Yang *et al.* develop a robust tensor completion method with double priors on the latent tensor, named tensor completion from structurally missing entries by low tensor train (TT) rankness and fiber-wise sparsity (TranSpa). In order to increase privacy in distributed algorithms, Imtiaz and Sarwate design two novel distributed and differentially private algorithms for two popular matrix and tensor factorization methods: principal component analysis (PCA) and orthogonal tensor decomposition (OTD). To remedy high computation complexity in graph-based methods, Yu *et al.* propose an unsupervised graph-based dimensionality reduction method named Fast and Flexible Large Graph Embedding based on anchors (FFLGE). FFLGE constructs an anchor-based graph and designs similarity matrix and then perform the dimensionality reduction efficiently. Furthermore, Locality Preserving Projection (LPP) and Principal Component Analysis (PCA) are two special cases of FFLGE. Liu *et al.* design a semi-supervised tensorial locally linear embedding (STLLE) for Polarimetric Synthetic Aperture Radar (PolSAR) data to keep the spatial structure and correlation among pixels. In practice, STLLE outperforms matrix-based methods for area's classification in images. Liu *et al.* propose an unsupervised feature extraction method using a collaboration-competition preserving graph embedding for hyper-spectral imagery which incorporates collaborative representation using l_2-norm regularization with locality-constrained property into graph construction. The constructed graph reveal local intrinsic manifold and global geometry information of hyper-spectral data. Ramirez develops two binary matrix factorization methods based on a binary adaptation of the dictionary learning paradigm to binary matrices. Qi *et al.* propose an unsupervised joint subspace and dictionary learning for cross-domain person re-identification.

For subspace clustering, Meng *et al.* propose a general framework for analyzing the performance of various subspace clustering algorithms when applied to the compressed data. Thus, the most commonly used subspace clustering algorithms, i.e. sparse SC (SSC), SSC-Orthogonal Matching Pursuit (SSC-OMP), and thresholding based SC (TSC) are investigated in terms of robustness to noise. In literature, recent studies have established the correctness of the SSC by showing that it produces subspace-preserving affinities under broad geometric conditions. In this context, Li *et al.* study the correctness of affine SSC (ASSC). Hashemi and Vikalo introduce an evolutionary subspace clustering method whose the aim is to cluster a collection of evolving data points that lie on a union of low-dimensional evolving subspaces. Generalizing from both quantized matrix recovery and subspace clustering, Wang *et al.* study the problem of combined data recovery and subspace clustering based on the quantized measurements of data points following the UoS model. Salehani *et al.* first investigate the non-negative block value decomposition (NBVD) approach through graph based representation for clustering. Gitlin *et al.* improve K-Subspaces clustering algorithm with a robust subspace recovery (RSR) method known as Coherence Pursuit (CoP) to handle low-rank outliers with low computational complexity. Hinojosa *et al.* design a set of coding patterns such that inter-class and intra-class data structure is preserved after the Compressive Spectral Imaging (CSI) acquisition in order to improve clustering results directly on the compressed domain. To validate the coding pattern design, an algorithm based on sparse subspace clustering (SSC) is proposed to perform clustering on the compressed measurements. For unsupervised multi-modal subspace clustering, Abavisani and Patel propose to employ a convolutional neural network (CNN) approach which consists of a multimodal encoder, a self-expressive layer, and a multimodal decoder. The network uses the distance between the decoders reconstruction and the original input in its training. Extensive experiments on three datasets show that the proposed methods significantly outperform the state-of-the-art multimodal subspace clustering methods. Finally, Wang *et al.* explore two modifications of the variational autoencoder (VAE) that dresses traditional autoencoders with probabilistic attire.

The Guest Editors acknowledge all the authors for their valuable contributions to this special issue and encourage them to continue the research in this field by investigating the key issues pointed out in this editorial. We would also like to thank all the reviewers for their thorough and timely effort in evaluating the papers and their valuable constructive comments to the authors, that have contributed to reach high quality papers for this special issue. Finally, we would like to thank the Editorial Board, Editor-in-Chief Professor Shri Narayanan and the journal coordinator Allison Fleisher for their valuable support throughout the entire process.

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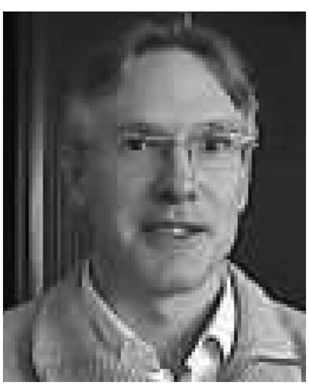

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