# Comments and Corrections

## Corrections to "Extended State Observer-Based Integral Sliding Mode Control for an Underwater Robot With Unknown Disturbances and Uncertain Nonlinearities"

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The purpose of this note is to correct the matching condition and stability proof in [1]. While the main results are unchanged, there should be some consequent modifications, which are shown in detail as follows:

First, [1, eq. (14)] should be modified as

$$-G_3\dot{H}_d/w_0^2 - G_2H_{un}/w_0 = P^{-1}\Theta\rho_t \tag{1}$$

where $\rho_t = [\rho_{t1}^\top, \rho_{t2}^\top, \rho_{t3}^\top]^\top \in \mathbb{R}^{18\times 1}$, $\rho_{t1}, \rho_{t2}, \rho_{t3} \in \mathbb{R}^{6\times 1}$, and the time-varying matrix $\Theta$ can be defined as

$$\Theta = \begin{bmatrix} I_{6\times 6} & -r_2 & -r_3 \\ 0_{6\times 6} & r_1 & 0_{6\times 6} \\ 0_{6\times 6} & 0_{6\times 6} & r_1 \end{bmatrix} \tag{2}$$

where $r_1 = \mathrm{diag}(\varepsilon_{11}, \ldots, \varepsilon_{16})$, $r_2 = \mathrm{diag}(\varepsilon_{21}, \ldots, \varepsilon_{26})$, $r_3 = \mathrm{diag}(\varepsilon_{31}, \ldots, \varepsilon_{36})$, $\varepsilon_1 = [\varepsilon_{11}, \ldots, \varepsilon_{16}]^\top, \varepsilon_2 = [\varepsilon_{21}, \ldots, \varepsilon_{26}]^\top, \varepsilon_3 = [\varepsilon_{31}, \ldots, \varepsilon_{36}]^\top$ are scaled estimation errors. Due to the added term $\Theta$, [1, eq. (23)] can be corrected as

$$\begin{aligned}\dot{V}_1 = &-w_0\varepsilon^\top(A_\varepsilon^\top P + PA_\varepsilon)\varepsilon + 2\varepsilon^\top PQ^{-1}\tilde{f} \\ &+ 2\varepsilon^\top PP^{-1}\Theta\rho_t - 2\varepsilon^\top P\varpi.\end{aligned} \tag{3}$$

Substituting [1, eq. (18)] into (3), we have

$$\begin{aligned}\dot{V}_1 = &-w_0\varepsilon^\top\varepsilon + 2\varepsilon^\top PQ^{-1}\tilde{f} \\ &+ 2\varepsilon^\top\Theta\rho_t - 2\varepsilon^\top P\varpi \\ \le &-w_0\|\varepsilon\|^2 + 2\|\varepsilon\|\|P\|\|Q^{-1}\tilde{f}\| \\ &+ 2\varepsilon^\top\Theta\rho_t - 2\varepsilon^\top P\varpi \\ \le &\ [-w_0 + c_2(\zeta_1+\zeta_2)/w_0]\,\|\varepsilon\|^2 \\ &+ 2\varepsilon^\top\Theta\rho_t - 2\varepsilon^\top P\varpi.\end{aligned} \tag{4}$$

Furthermore, [1, eq. (25)] should be updated as follows:

$$\dot{V}_1 \le -\beta\|\varepsilon\|^2 + 2\varepsilon^\top\Theta\rho_t - 2\varepsilon^\top P\varpi. \tag{5}$$

Since $\varepsilon^\top\Theta = [\varepsilon_1^\top, 0_{1\times 6}, 0_{1\times 6}]$, (5) can be rewritten as

$$\begin{aligned}\dot{V}_1 &\le -\beta\|\varepsilon\|^2 + 2\varepsilon_1^\top\rho_{t1} - 2\varepsilon^\top P\varpi \\ &= -\beta\|\varepsilon\|^2 + 2(C\varepsilon)^\top\rho_{t1} - 2\varepsilon^\top P\varpi\end{aligned} \tag{6}$$

where $\rho_{t1}$ is bounded and satisfies $\|\rho_{t1}\| \le \rho_2 \in \mathbb{R}^+$, and it is as same as [1, eq. (25)]; therefore, the result is unchanged.

Second, [1, eq. (29)] should be corrected as

$$\dot{s}(t) = K_p\dot{e}(t) + K_ie(t) + K_d\dot{\hat{e}}(t). \tag{7}$$

Based on the observer presented in [1, eq. (10)], we have

$$\begin{aligned}\dot{\hat{e}} = \dot{\hat{\eta}} - \ddot{\eta}_r = &-\ddot{\eta}_r - C_\eta(\eta,\hat{\nu})\hat{\dot{\eta}} - D_\eta(\eta,\hat{\nu})\hat{\dot{\eta}} \\ &- G_\eta + M_\eta LU + \hat{H}_d - 3w_0^2\tilde{x}_1 - w_0\varpi_2.\end{aligned} \tag{8}$$

Using $\dot{e} = \hat{\dot{e}} - w_0\varepsilon_2$ and (8), [1, eq. (31)] can be rewritten as

$$\begin{aligned}\dot{s} + K_ss = K_p\hat{\dot{e}} + K_ie + K_ss - w_0K_p\varepsilon_2 + K_d(-\ddot{\eta}_r \\ - C_\eta(\eta,\hat{\nu})\hat{\dot{\eta}} - D_\eta(\eta,\hat{\nu})\hat{\dot{\eta}} - G_\eta + M_\eta LU \\ + \hat{H}_d - 3w_0^2\tilde{x}_1 - w_0\varpi_2)\end{aligned} \tag{9}$$

where $\varpi = [\varpi_1^\top, \varpi_2^\top, \varpi_3^\top]^\top \in \mathbb{R}^{18\times 1}$, $\varpi_i \in \mathbb{R}^{6\times 1}$, $i = 1, 2, 3$. [1, eq. (32)] should be updated as

$$\begin{aligned}U_{\mathrm{eq}} = &-(K_dM_\eta L)^{-1}(K_p\hat{\dot{e}} + K_ie + K_ss) \\ &+ (M_\eta L)^{-1}[\ddot{\eta}_r + C_\eta(\eta,\hat{\nu})\hat{\dot{\eta}} + w_0\varpi_2 \\ &+ D_\eta(\eta,\hat{\nu})\hat{\dot{\eta}} + G_\eta - \hat{H}_d + 3w_0^2\tilde{x}_1].\end{aligned} \tag{10}$$

Equation (34) in [1] should be written as

$$U_{\mathrm{sw}} = -(K_dM_\eta L)^{-1}K_{\mathrm{sw}}\,\mathbf{sgn}(s). \tag{11}$$

Then, [1, eq. (36)] can be described as

$$U = U_{\mathrm{eq}} + U_{\mathrm{sw}}. \tag{12}$$

Compared with $U_{\mathrm{eq}}$ in the original controller, the term $(M_\eta L)^{-1}(w_0\varpi_2 + 3w_0^2\tilde{x}_1)$ are added, which will converge to zero. Then, the main experimental results are unchanged.

*Theorem 1:* Consider system [1, eq. (6)] satisfying Assumptions 1, under the designed ESO [1, eq. (10)], the tracking error and external disturbance estimation error will converge to zero under the control

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law (11), and the parameters $\beta$, $w_0$, $K_d$, $K_s$, and $K_{\text{sw}}$ satisfy following conditions: $\beta > w_0\lambda_{\max}(K_p)$, $\lambda_{\min}(K_s) > w_0\lambda_{\max}(K_p)/2$, and $\lambda_{\min}(K_{\text{sw}}) > 0$.

*Proof:* Let us choose a Lyapunov function candidate

$$V = \frac{1}{2}V_1 + \frac{1}{2}s^\top s + \frac{1}{2\gamma_2}\tilde{\rho}_2^2 \tag{13}$$

where $V_1$ is defined in [1, eq. (21)]. Based on Lemma 1, we have

$$\begin{aligned}\dot{V} \le & -\frac{\beta}{2}(\varepsilon_1^\top\varepsilon_1 + \varepsilon_2^\top\varepsilon_2 + \varepsilon_3^\top\varepsilon_3) + \varepsilon^\top C^\top \rho_{t1} \\ & - \varepsilon^\top P\varpi + s^\top \dot{s} + \frac{1}{\gamma_2}\tilde{\rho}_2\dot{\hat{\rho}}_2. \end{aligned}\tag{14}$$

Substituting [1, eq. (13)] and [1, eq. (15)] into (14), we have

$$\begin{aligned}\dot{V} \le & -\beta\varepsilon^\top\varepsilon/2 + s^\top\dot{s} + \|\tilde{Y}\|\rho_2 - \varepsilon^\top P\varpi + \|\tilde{Y}\|\tilde{\rho}_2 \\ = & -\beta\varepsilon^\top\varepsilon/2 + s^\top\dot{s} + \|\tilde{Y}\|\hat{\rho}_2 \\ & - \frac{\|\tilde{Y}\|^2\hat{\rho}_2 - c_1\|\tilde{Y}\|^2\dot{h}_1\hat{\rho}_2^2/\|\tilde{Y}\|}{\|\tilde{Y}\| - c_1\dot{h}_1\hat{\rho}_2} \\ = & -\beta\varepsilon^\top\varepsilon/2 + s^\top\dot{s}. \end{aligned}\tag{15}$$

Substituting (12) into (9), we have

$$\dot{s} = -K_s s - w_0 K_p\varepsilon_2 - K_{\text{sw}}\mathbf{sgn}(s). \tag{16}$$

Substituting (16) into (15), we see that the derivative of $V$ can be described as

$$\dot{V} \le -\frac{\beta}{2}\varepsilon^\top\varepsilon - s^\top K_s s - w_0 s^\top K_p\varepsilon_2 - s^\top K_{\text{sw}}\mathbf{sgn}(s). \tag{17}$$

Since $-w_0 s^\top K_p\varepsilon_2 \le w_0\lambda_{\max}(K_p)(\varepsilon^\top\varepsilon + s^\top s)/2$, we have

$$\begin{aligned}\dot{V} \le & -\frac{\beta}{2}\varepsilon^\top\varepsilon - s^\top K_s s - \lambda_{\min}(K_{\text{sw}})\|s\| \\ & + w_0\lambda_{\max}(K_p)(\varepsilon^\top\varepsilon + s^\top s)/2 \\ \le & -\xi^\top\Lambda\xi - \lambda_{\min}(K_{\text{sw}})\|s\| \end{aligned}\tag{18}$$

where $\xi = [\varepsilon^\top, s^\top]^\top$, $\Lambda = \begin{bmatrix}\Lambda_1 & 0_{18\times6} \\ 0_{6\times18} & \Lambda_2\end{bmatrix}$, $\Lambda_1 = \left(\frac{\beta}{2} - \frac{w_0\lambda_{\max}(K_p)}{2}\right) I_{18\times18}$, $\Lambda_2 = \left(\lambda_{\min}(K_s) - \frac{w_0\lambda_{\max}(K_p)}{2}\right)I_{6\times6}$.

Because the parameters $\beta$, $w_0$, $K_s$, and $K_{\text{sw}}$ satisfy related conditions mentioned in Theorem 1, we know that $\beta > w_0\lambda_{\max}(K_p)$, $\lambda_{\min}(K_s) > w_0\lambda_{\max}(K_p)/2$, $\lambda_{\min}(K_{\text{sw}}) > 0$; therefore, $\lambda_{\min}(\Lambda) > 0$.

Equation (17) implies that $\dot{V} < 0$ for $\xi \ne 0$, and the signals $s$, $\varepsilon$, and $\tilde{\rho}_2$ are bounded. Based on (18), we have $\dot{V} \le -\xi^\top\Lambda\xi$. Then, we have $\lim_{t\to\infty}\int_0^t(\xi^\top\Lambda\xi)d\tau \le V(0) - V(\infty)$. Because $V(0)$ and $V(\infty)$ are bounded, $s$ and $\varepsilon$ are square integrable. According to (16) and the boundedness of $s$, we can conclude that $\dot{s}$ is bounded.

From [1, eq. (12)], we know that $\|\tilde{f}\| = \|\tilde{\varphi}\|$. Furthermore, we have $\tilde{f}$ is bounded according to (1). The boundedness of $\dot{H}_d$ and $H_{un}$ implies that $\rho_t(t)$ is bounded from (1). Because $\varepsilon$, $\hat{\rho}_2$, and $\dot{h}(t)$ are bounded, from [1, eq. (13)], we can obtain $\varpi$ is bounded. Then, $\dot{\varepsilon}$ is bounded from [1, eq. (16)]. The boundedness of $\dot{s}$ and $\dot{\varepsilon}$ implies that $\dot{\xi}$ is bounded. According to Lemma 2, we have $\lim_{t\to\infty}\xi(t) = 0$, i.e., $\lim_{t\to\infty}s(t) = 0$ and $\lim_{t\to\infty}\varepsilon(t) = 0$.

Defining that

$$z(t) = s(t) - w_0 K_d\varepsilon_2(t) + K_p e(0) + K_d\hat{\dot{e}}(0) + K_d e(0) \tag{19}$$

where $z(t) = [z_1(t), \ldots, z_6(t)]^\top \in \mathbb{R}^{6\times1}$.

Substituting $\hat{\dot{e}} = \hat{\dot{\eta}} - \dot{\eta}_r = \hat{\dot{e}} - w_0\varepsilon_2$ into [1, eq. (27)], we have

$$K_p e(t) + K_i\int_0^t e(\tau)d\tau + K_d\dot{e}(t) + K_d e(0) = z(t). \tag{20}$$

Because $K_p$, $K_i$, and $K_d$ are positive definite diagonal matrices, we have

$$z_i(t) = K_{pi}e_i(t) + K_{ii}\int_0^t e_i(\tau)d\tau + K_{di}\dot{e}_i(t) + K_{di}e_i(0) \tag{21}$$

where $z_i(t)$ is the $i$th element of $z(t)$, and $i = 1, \ldots, 6$.

Then, take Laplace transformation of (21), we have

$$\frac{e_i(p)}{z_i(p)} = \frac{p}{K_{di}p^2 + K_{pi}p + K_{ii}} \tag{22}$$

where $p$ is the Laplace transformation operator, $e_i(p)$ and $z_i(p)$ are the Laplace transformations of $e_i(t)$ and $z_i(t)$, respectively.

Using the final value theorem, we have

$$e(\infty) = \lim_{p\to0}\frac{p^2 z_i(p)}{K_{di}p^2 + K_{pi}p + K_{ii}}. \tag{23}$$

Since the initial error $e(0)$ and $\hat{\dot{e}}(0)$ are bounded, and $\varepsilon_2(t)$ is bounded, from (19), $z_i(t)$ is bounded. $z_i(t)$ can converge to $K_p e(0) + K_d\hat{\dot{e}}(0) + K_d e(0)$ as time goes to infinity. Then, we have $|z_i(t)| \le z_{i\max} < \infty$. The Laplace transformation of $z_i(t)$ satisfies

$$\begin{aligned}|z_i(p)| = & \left|\int_0^\infty e^{-p\tau}z_i(\tau)d\tau\right| \le \int_0^\infty |e^{-p\tau}z_i(\tau)|d\tau \\ \le & z_{i\max}\int_0^\infty |e^{-p\tau}|dt \le \frac{z_{i\max}}{p}. \end{aligned}\tag{24}$$

Then, we have

$$\lim_{p\to0}|p^2 z_i(p)| = 0. \tag{25}$$

Hence, it can be induced from (25) that $\lim_{p\to0}p^2 z_i(p) = 0$, and then we have

$$e_i(\infty) = \lim_{p\to0}\frac{p^2 z_i(p)}{K_{di}p^2 + K_{pi}p + K_{ii}} = 0. \tag{26}$$

The system described by (21) and (22) is stable if the parameters $K_{di}$, $K_{pi}$, and $K_{ii}$ are chosen as positive constants to satisfy Hurwitz stability criterion. According to (23) and (26), we have $\lim_{t\to\infty}e(t) = 0$. ■

## Reference

[1] R. Cui, L. Chen, C. Yang, and M. Chen, “Extended state observer-based integral sliding mode control for an underwater robot with unknown disturbances and uncertain nonlinearities,” *IEEE Trans. Ind. Electron.*, vol. 64, no. 8, pp. 6785–6795, Aug. 2017.