Takagi–Sugeno Fuzzy Model Based Fault Estimation and Signal Compensation With Application to Wind Turbines

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***Abstract*—In response to the high demand of the operation reliability by implementing real-time monitoring and system health management, a robust fault estimation and fault-tolerant control approach is proposed for Takagi–Sugeno fuzzy systems in this study, by integrating the augmented system method, unknown input fuzzy observer design, linear matrix inequality optimization, and signal compensation techniques. Specifically, a fuzzy augmented system method is used to construct an augmented plant with the concerned faults and system states being the augmented states. An unknown input fuzzy observer technique is thus utilized to estimate the augmented states and decouple unknown inputs that can be decoupled. A linear matrix inequality approach is further addressed to ensure the global stability of the estimation error dynamics and attenuate the influences from the unknown inputs that cannot be decoupled. As a result, the robust estimates of the concerned faults and system states can be obtained simultaneously. Based on the fault estimates, a signal compensation scheme is developed to remove the effects of the faults on the system dynamics and outputs, leading to a stable dynamic satisfying the expected performance. Finally, the effectiveness of the proposed Takagi–Sugeno model based fault estimation and signal compensation algorithms is demonstrated by a case study on a 4.8-MW wind turbine benchmark system.**

***Index Terms*—Robust fault estimation, signal compensation, Takagi–Sugeno (T–S) fuzzy model, wind turbines.**

I. Introduction

Along with the development of advanced technologies to increase the production, the complexity and the expense of industrial systems are growing correspondingly. The components of control systems are prone to malfunction, which could bring unanticipated economic cost due to the unscheduled shutdown and repairing/maintenance. Therefore, it is of particular interest to design advanced fault diagnosis and fault-tolerant control programs to automatically monitor the behavior of industrial systems and prevent extensive damage caused by unexpected faults. In recent years, on the basis of analytical redundancy, considerable research works have been devoted to fault diagnosis and tolerant control [1]–[5].

Fault estimation/reconstruction is an advanced fault-diagnosis method that uses advanced observers or filter techniques, such as proportional integral observers [6], sliding mode observers [7], [8], and augmented observers (including descriptor observers) [9], [10]. It is paramount for fault estimators to possess a good capability to attenuate the effects from various disturbance sources. An unknown input observer (UIO) is well recognized owing to its ability to decouple the influences from the unknown inputs, resulting from the modeling errors, parameter perturbations, and exogenous disturbances. As a result, UIO-based fault reconstruction is able to provide the rich information of the concerned faults (e.g., types, sizes, and shapes) under noisy environments. In [11] and [12], UIO techniques were integrated with the augmented observer approach [9], [10] to robustly estimate faults by decoupling process disturbances. It is noticed that not all process disturbances can be decoupled by the UIO techniques in some engineering systems. Therefore, additional techniques such as robust optimal techniques are also needed to further attenuate the disturbances that cannot be decoupled. In [13], for systems corrupted by partially decoupled process disturbances, a novel robust fault-estimation approach was presented. Specifically, the augmented system approach was utilized to ensure a simultaneous estimate of the faults concerned and the system states; the UIO was employed to decouple the disturbances, which can be decoupled; whereas the linear matrix inequality (LMI) optimization was used to attenuate the disturbances, which cannot be decoupled. It is noted that the above fault-estimation techniques are mainly developed for linear systems or systems with Lipschitz nonlinear terms.

Many industrial processes are highly nonlinear, which cannot be expressed by linearized models or Lipschitz models. Therefore, the linearized or Lipschitz model based fault-diagnosis methods will become invalid to handle highly nonlinear systems. Some recent results of state estimation in general nonlinear systems were reported by using a sequential evolutionary

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filter [14] and a multivariate statistical combination forecasting method [15]. However, the fault estimation was not considered in the work above. Therefore, robust fault estimation for general nonlinear systems is still an open problem and worthy of further research. Takagi–Sugeno (T–S) fuzzy model was initialized in [16], which has been widely used to approximate a variety of highly nonlinear engineering systems. As a result, a variety of T–S fuzzy model based fault-diagnosis approaches were developed during the last decades, see, e.g., [17]–[19]. Moreover, T–S fuzzy model based UIOs were investigated in [20]–[23].

It is worthy to point out that signal compensation is a powerful fault-tolerant control technique. By using estimated faults, the effects of the faulty signals on the systems dynamics can be compensated to realize a fault-tolerant operation no matter faults occur or not [9], [10]. As a result, there is a strong motivation to develop effective fault estimation and signal compensation approaches for highly nonlinear industrial systems in partially decoupled unknown disturbances environment. To the best of our knowledge, no efforts have been paid on T–S fuzzy model based robust fault estimation and signal compensation based tolerant control for highly nonlinear systems subject to partially decoupled unknown disturbances.

Nowadays, wind turbines have drawn tremendous attention worldwide in response to the accelerative demand of sustainable energy, and considerable research works have been devoted to monitor and diagnose wind turbines [24], [25]. Recently, a benchmark wind turbine model was developed in [26], which has been widely utilized to verify/validate fault detection, fault isolation, and fault-tolerant control algorithms [17], [27], [28]. In consequence, implementing well-designed fault estimation and signal compensation algorithms to wind turbines is beneficial to maintain functionalities of the plants and improve energy revenue.

The contribution of this paper focuses on robust fault reconstruction/estimation design and fault-tolerant control for T–S fuzzy models subjected to simultaneous actuator faults, sensor faults, and partially decoupled unknown inputs. An augmented system approach combined with T–S fuzzy UIOs and robust optimization technique (e.g., linear matrix inequality) provides robust estimates of the considered faults and the system states, which are then utilized for signal compensation to remove the influences from the faults on the system dynamics and outputs. Compared with recent results about estimation-based fault-tolerant control in [29] and [30], the proposed methods can handle both actuator faults and sensor faults simultaneously. Moreover, the upper bound of unknown inputs is not necessarily required, which makes the present techniques more applicable. The real-time process of the 4.8-MW wind turbine system characterized by T–S fuzzy systems is finally utilized to demonstrate the presented fault estimation and tolerant control techniques.

The rest of paper is organized as follows. Section II introduces robust fault-estimation approach for a T–S fuzzy model corrupted by partially decoupled unknown inputs. Based on the state/fault estimations, the signal compensation technique is addressed in Section III. In Section IV, a case study is addressed to validate the proposed fault estimation and tolerant control techniques on the 4.8-MW benchmark wind turbine system. This paper ends by summarizing the conclusions in Section V.

Notation: The notations in this paper are quite standard. The superscript "T" represents the transpose of matrices or vectors. $\mathcal{R}^n$ and $\mathcal{R}^{n\times m}$ stand for the n-dimentional Euclidean space and the set of $n\times m$ real matrices, respectively. $X<0$ indicates the symmetric matrix X is negative definite, whereas the notation $X>Y$ means that $X-Y$ is positive definite. I_n denotes the identity matrix with the dimension of $n\times n$, while 0 is a scalar zero or a zero matrix with appropriate zero entries. $\forall$ means "for any".$\|\cdot\|$ refers to Euclidean norm of vectors or matrices, and $||d||_{Tf}=(\int_0^{Tf} d^T(s)d(s)ds)^{1/2}$. In a large matrix expression, we also have $\begin{bmatrix} M_1 & M_2 \\ * & M_3 \end{bmatrix} \Leftrightarrow \begin{bmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{bmatrix}$.

II. Robust Fault Estimation of T–S Fuzzy Systems

This section presents the approach for robust state/fault estimation for T–S fuzzy systems subjected to partially decoupled unknown inputs.

Consider the following nonlinear system characterized by the T–S fuzzy model:

if μ_1 is M_{1i} and $\ldots\mu_q$ is M_{qi}, then

$$\begin{cases} \dot{x}(t) &= A_i x(t) + B_i u(t) + B_{di} d(t) + B_{fi} f(t) \\ y(t) &= Cx(t) + D_f f(t) + D_d d_w(t) \end{cases} \tag{1}$$

where $x(t)\in\mathcal{R}^n$ represents the state vector; $u(t)\in\mathcal{R}^m$ stands for the control input vector and $y(t)\in\mathcal{R}^p$ is the measurement output vector; $d(t)\in\mathcal{R}^{l_d}$ and $d_w(t)\in\mathcal{R}^{l_{dw}}$ are the bounded unknown disturbance vectors from plant and sensors, respectively; $f(t)\in\mathcal{R}^{l_f}$ means the fault vector including actuator faults and sensor faults. $i=1,2,\ldots r$, with r being the total number of local models, depending on the precision requirement for modeling, the complexity of the nonlinear system, and the choice of structure of weighting functions. M_{ji} are fuzzy sets and μ denotes the decision vector containing all individual elements μ_j, $j=1,2,\ldots q$ which are known premise variables that may be the functions of the measurable state variables, external inputs, and/or time. A_i, B_i, B_{di}, B_{fi}, C, D_d, and D_f can be obtained by using the direct linearization of a nonlinear system or alternatively by using an identification procedure. For the simplification of description, in the rest of paper, the time symbol t is omitted.

From (1), the final fuzzy system is inferred as follows:

$$\begin{cases} \dot{x} = \sum_{i=1}^{r} h_i(\mu)\left(A_i x + B_i u + B_{di} d + B_{fi} f\right) \\ y = Cx + D_f f + D_d d_w \end{cases} \tag{2}$$

where $h_i(\mu)$ are weighting functions, quantifying the membership of the current operation point of the system at a zone of operation, which are selected following the convex sum properties: $\sum_{i=1}^{r} h_i(\mu)=1$ and $0\le h_i(\mu)\le 1$.

The faults concerned are assumed to be incipient or abrupt, which are two typical faults generally existing in practical processes. Therefore, the second-order derivatives of them should be zero piecewise. For faults whose second-order derivatives are not zero but bounded, the bounded signals could be regarded as a part of unknown inputs. Moreover, denote $B_{di}=[B_{di1}\ B_{di2}]$ and $d=[d_1\ d_2]^T$. We assume that $d_1\in\mathcal{R}^{l_{d1}}$ rather than $d_2\in\mathcal{R}^{l_{d2}}$ can be decoupled. In addition, we also assume that both the f and $\dot{f}$ are bounded.

In order to estimate faults and system states at the same time, an augmented system can be constructed as

$$\begin{cases} \dot{\bar{x}} = \sum_{i=1}^{r} h_i(\mu)\ (\bar{A}_i\bar{x} + \bar{B}_i u + \bar{B}_{di} d) \\ y\ = \bar{C}\ \bar{x} + D_d d_w \end{cases} \tag{3}$$

where $\bar{n} = n + 2l_f$, $\bar{x} = [x^T\ \dot{f}^T\ f^T]^T \in \mathcal{R}^{\bar{n}}$,

$$\bar{A}_i = \begin{bmatrix} A_i & 0_{n\times l_f} & B_{fi} \\ 0_{l_f\times n} & 0_{l_f\times l_f} & 0_{l_f\times l_f} \\ 0_{l_f\times n} & I_{l_f} & 0_{l_f\times l_f} \end{bmatrix} \in \mathcal{R}^{\bar{n}\times\bar{n}},$$

$$\bar{B}_i = \begin{bmatrix} B_i^T & 0_{m\times l_f} & 0_{m\times l_f} \end{bmatrix}^T \in \mathcal{R}^{\bar{n}\times m}$$

$$\bar{B}_{di} = \begin{bmatrix} B_{di}^T & 0_{l_d\times l_f} & 0_{l_d\times l_f} \end{bmatrix}^T \in \mathcal{R}^{\bar{n}\times l_d},$$

$$\bar{C} = \begin{bmatrix} C & 0_{p\times l_f} & D_f \end{bmatrix} \in \mathcal{R}^{p\times\bar{n}}.$$

A UIO in the following form can be designed for (3):

$$\begin{cases} \dot{\bar{z}} = \sum_{i=1}^{r} h_i(\mu)\left[R_i\bar{z} + T\bar{B}_i u + (K_{i1} + K_{i2})\, y\right] \\ \hat{\bar{x}} = \bar{z}\ + Hy \end{cases}. \tag{4}$$

Let the estimation error be $\bar{e} = \bar{x} - \hat{\bar{x}}$, leading to its derivative calculated as follows:

$$\begin{aligned} \dot{\bar{e}} = \sum_{i=1}^{r} h_i(\mu)\{&(\bar{A}_i - H\bar{C}\bar{A}_i - K_{i1}\bar{C})\,\bar{e} + (\bar{A}_i - H\bar{C}\bar{A}_i \\ &- K_{i1}\bar{C} - R_i)\bar{z} + \left[(\bar{A}_i - H\bar{C}\bar{A}_i - K_{i1}\bar{C})\, H - K_{i2}\right] y \\ &+ \left[(I_{\bar{n}} - H\bar{C}) - T\right]\bar{B}_i u + (I_{\bar{n}} - H\bar{C})\,\bar{B}_{di1} d_1 \\ &+ (I_{\bar{n}} - H\bar{C})\,\bar{B}_{di2} d_2 - K_{i1} d_s - HD_d\dot{d}_s\}. \end{aligned} \tag{5}$$

If the observer gains satisfy the following conditions:

$$(I_{\bar{n}} - H\bar{C})\ \bar{B}_{di1} = 0 \tag{6}$$

$$R_i = \bar{A}_i\ - H\bar{C}\bar{A}_i - K_{i1}\bar{C} \tag{7}$$

$$T\ = I_{\bar{n}}\ - H\bar{C} \tag{8}$$

$$K_{i2} = R_i H \tag{9}$$

the state estimation error can be reduced to

$$\begin{aligned} \dot{\bar{e}} = \sum_{i=1}^{r} h_i(\mu)\,[R_i\bar{e} + (I_{\bar{n}} - H\bar{C})\,\bar{B}_{di2} d_2 \\ - K_{i1} d_w - HD_d\dot{d}_w]. \end{aligned} \tag{10}$$

In order to meet the conditions (6)–(9), we have the following assumptions:

1) $\text{rank}\left(\bar{C}\left(\bar{B}_{d11}\ \bar{B}_{d21}\cdots\ \bar{B}_{dr1}\right)\right)$
$= \text{rank}((\bar{B}_{d11}\ \bar{B}_{d21}\cdots\ \bar{B}_{dr1}))$;

2) for $\forall\, i$, $\begin{bmatrix} A_i & B_{fi} & B_{di1} \\ C & D_{fi} & 0 \end{bmatrix}$ is of full column rank;

3) for $\forall\, i$, rank $\begin{bmatrix} sI_n - A_i & B_{di1} \\ C & 0 \end{bmatrix} = n + l_{d1}$.

According to [13], the above assumptions are to ensure that for each local model, (6) can be solved, and the model is observable.

Equation (6) implies that

$$H\bar{C}\ \left(\bar{B}_{d11}\ \bar{B}_{d21}\cdots\ \bar{B}_{dr1}\right) = \left(\bar{B}_{d11}\ \bar{B}_{d21}\cdots\ \bar{B}_{dr1}\right). \tag{11}$$

For $(\bar{B}_{d11}\ \bar{B}_{d21}\cdots\ \bar{B}_{dr1})$, there exists a nonsingular matrix M such that $(\bar{B}_{d11}\ \bar{B}_{d21}\cdots\ \bar{B}_{dr1})M = (\bar{B}_M\ 0)$, where $\bar{B}_M$ is of full column rank. Assumption (1) indicates $\text{rank}\,((\bar{C}\bar{B}_M\ 0)) = \text{rank}((\bar{B}_M\ 0))$, which implies that $\bar{C}\bar{B}_M$ is of full column rank, hence $(\bar{C}\bar{B}_M)^+$ exists. By multiplying M on the both sides of (11), we have $H\bar{C}\bar{B}_M = \bar{B}_M$. Therefore, by choosing different compatible matrix N of proper dimension, all possible solutions of H can be obtained as follows:

$$H\ = \bar{B}_M\ (\bar{C}\bar{B}_M)^+ + N\left(I - (\bar{C}\bar{B}_M)\,(\bar{C}\bar{B}_M)^+\right) \tag{12}$$

where $(\bar{C}\bar{B}_M)^+ = [(\bar{C}\bar{B}_M)^T(\bar{C}\bar{B}_M)]^{-1}(\bar{C}\bar{B}_M)^T$.

By deriving H from (12) to satisfy condition (6), a part of unknown inputs d_1 is decoupled; however, undecoupled unknown inputs d_2 and d_w still exist in error dynamic and influence the performance of the estimator. To achieve robustness to d_2 and d_w, the following theorem is addressed to attenuate their influences on estimation error.

Theorem 1: For system (3), there exists a robust UIO in the form of (4), such that $||\bar{e}||_{Tf} \le \gamma||\bar{d}||_{Tf}$, if $\forall\ i$, there exist a positive definite matrix P, matrices Y_i, and Z such that

$$\begin{bmatrix} \Lambda_i & (PF - ZG\bar{C})\,\bar{B}_{di2} & -Y_i & -(P\bar{B}_M Q - ZG)\,D_d \\ * & -\gamma^2 I_{l_{d2}} & 0 & 0 \\ * & * & -\gamma^2 I_{l_{dw}} & 0 \\ * & * & * & -\gamma^2 I_{l_{dw}} \end{bmatrix} < 0 \tag{13}$$

where $\Lambda_i = I_{\bar{n}} + \bar{A}_i^T F^T P - \bar{A}_i^T\bar{C}^T G^T Z^T + PF\bar{A}_i - ZG\bar{C}\bar{A}_i - \bar{C}^T Y_i^T - Y_i\bar{C}$, $Y_i = PK_{i1}$, $Z = PN$, $F = I_{\bar{n}} - \bar{B}_M(\bar{C}\bar{B}_M)^+\bar{C}$, $G = I - (\bar{C}\bar{B}_M)(\bar{C}\bar{B}_M)^+$, $Q = (\bar{C}\bar{B}_M)^+$, $i = 1, 2, \ldots r$, $\bar{d} = [d_2{}^T\ d_w{}^T\ \dot{d}_w^T]^T$, and γ is a performance index, standing for the magnitude of the error compared with unknown inputs. Therefore, we have $N = P^{-1}Z$ and $K_{i1} = P^{-1}Y_i$, $i = 1, 2, \ldots r$.

Proof: Choose a Lyapunov function $V(\bar{e}) = \bar{e}^T P\bar{e}$, and it is easy to verity that $V(\bar{e}) > 0$ for any nonzero $\bar{e}$. With the aid of (10), the derivative of $V(\bar{e})$ can be derived as

$$\begin{aligned} \dot{V}(\bar{e}) &= \sum_{i=1}^{r} h_i(\mu)\left\{\left[R_i\bar{e} + T\bar{B}_{di2}d_2 - K_{i1}d_w - HD_d\dot{d}_w\right]^T P\bar{e}\right. \\ &\qquad \left. +\ \bar{e}^T P\left[R_i\bar{e} + T\bar{B}_{di2}d_2 - K_{i1}d_w - HD_d\dot{d}_w\right]\right\} \\ &= \sum_{i=1}^{r} h_i(\mu)[\bar{e}^T\ (R_i^T P + PR_i)\,\bar{e} + 2\bar{e}^T PT\bar{B}_{di2}d_2 \\ &\qquad - 2\bar{e}^T PK_{i1}d_w - 2\bar{e}^T PHD_d\dot{d}_w \\ &= \sum_{i=1}^{r} h_i(\mu)[\bar{e}^T(\bar{A}_i{}^T T^T P - \bar{C}^T K_{i1}^T P + PT\bar{A}_i - PK_{i1}\bar{C})\bar{e} \\ &\qquad + 2\bar{e}^T PT\bar{B}_{di2}d_2 - 2\bar{e}^T PK_{i1}d_w - 2\bar{e}^T PHD_d\dot{d}_w] \\ &= \sum_{i=1}^{r} h_i(\mu)\,[\bar{e}^T(\bar{A}_i{}^T F^T P - \bar{A}_i{}^T\bar{C}^T G^T Z^T + PF\bar{A}_i \\ &\qquad - ZG\bar{C}\bar{A}_i - \bar{C}^T Y_i{}^T - Y_i\bar{C})\bar{e} \\ &\qquad + 2\bar{e}^T(PF - ZG\bar{C})\bar{B}_{di2}d_2 - 2\bar{e}^T PK_{i1}d_w \\ &\qquad - 2\bar{e}^T PHD_d\dot{d}_w] \end{aligned} \tag{14}$$

where $Y_i = PK_{i1}, Z = PN, F = I_{\bar{n}} - \bar{B}_M(\bar{C}\bar{B}_M)^+\bar{C}$, and $G = I - (\bar{C}\bar{B}_M)(\bar{C}\bar{B}_M)^+$.

It can be seen from the condition that $\Lambda_i < 0$, which implies $\bar{A}_i^T F^T P - \bar{A}_i^T \bar{C}^T G^T Z^T + PF\bar{A}_i - ZG\bar{C}\bar{A}_i - \bar{C}^T Y_i^T - Y_i\bar{C} < 0$. Apparently, when $d_2 = 0$ and $d_w = 0$, $\dot{V}(\bar{e}) < 0$. Based on the Lyaponov stability theory, the error dynamic system (10) is stable.

Now, let us verify the robustness of the estimator. Let $\Gamma = \int_0^{Tf}(\bar{e}^T\bar{e} - \gamma^2\bar{d}^T\bar{d})dt$, and by adding and subtracting $\int_0^{Tf}\dot{V}(\bar{e})dt$ into Γ, we can have

$$\Gamma = \int_0^{Tf}\left(\bar{e}^T\bar{e} - \gamma^2\bar{d}^T\bar{d} + \dot{V}(\bar{e})\right)dt - \int_0^{Tf}\dot{V}(\bar{e})\,dt$$
$$= \int_0^{Tf}\sum_{i=1}^{r} h_i(\mu)[\bar{e}^T(\bar{A}_i^T F^T P - \bar{A}_i^T\bar{C}^T G^T Z^T + PF\bar{A}_i - ZG\bar{C}\bar{A}_i - \bar{C}^T Y_i^T - Y_i\bar{C})\bar{e} + 2\bar{e}^T(PF - ZG\bar{C})\bar{B}_{di2}d_2 - 2\bar{e}^T PK_{i1}d_w - 2\bar{e}^T P\left(\bar{B}_M Q + NG\right)D_d\dot{d}_w - \gamma^2\bar{d}^T\bar{d}]dt - \int_0^{Tf}\dot{V}(\bar{e})\,dt$$
$$= \int_0^{Tf}\begin{bmatrix}\bar{e}^T & \bar{d}^T\end{bmatrix}\left[\sum_{i=1}^{r} h_i(\mu)\Phi_i\right]\begin{bmatrix}\bar{e}\\ \bar{d}\end{bmatrix}dt - \int_0^{Tf}\dot{V}(\bar{e})\,dt \tag{15}$$

where

$$\Phi_i = \begin{bmatrix}\Lambda_i & \left(PF - ZG\bar{C}\right)\bar{B}_{di2} & -Y_i & -\left(P\bar{B}_M Q - ZG\right)D_d\\ * & -\gamma^2 I_{l_{d2}} & 0 & 0\\ * & * & -\gamma^2 I_{l_{dw}} & 0\\ * & * & * & -\gamma^2 I_{l_{dw}}\end{bmatrix} \tag{16}$$

$$\Lambda_i = I_{\bar{n}} + \bar{A}_i^T F^T P - \bar{A}_i^T\bar{C}^T G^T Z^T + PF\bar{A}_i - ZG\bar{C}\bar{A}_i - Y_i\bar{C} - \bar{C}^T Y_i^T,$$

and $Q = (\bar{C}\bar{B}_M)^+$. Under zero initial condition $\bar{e}(0) = 0$, therefore

$$\int_0^{Tf}\dot{V}(\bar{e})\,dt = V(\bar{e}) \geq 0. \tag{17}$$

Since $\Phi_i < 0$ in terms of (13), then $\sum_{i=1}^{r} h_i(\mu)\Phi_i < 0$. From (15) to (17), one has $\Gamma < 0$, which indicates $||\bar{e}||_{Tf} \leq \gamma||\bar{d}||_{Tf}$. As a result, for any given performance index γ, the estimation error can be reduced to be less than certain value by choosing observer gains according to LMIs (13).

Remark 1: In the aforementioned fault-estimation approach design, we consider system for all local models with $C_1 = C_2 = \ldots = C_r$, $D_{f1} = D_{f2} = \ldots = D_{fr}$, and $D_{d1} = D_{d2} = \ldots = D_{dr}$. These kinds of models are widely used to represent real industrial systems, such as wind turbines, robotic systems, and electrical models. For more general situations, when C_i, D_{fi}, and D_{di}, $i = 1, 2, \cdots r$, are not equal among different local models, the system can be represented as follows:

$$\begin{cases}\dot{x} = \sum_{i=1}^{r} h_i(\mu)(A_i x + B_i u + B_{di}d + B_{fi}f)\\ y = \sum_{i=1}^{r} h_i(\mu)\ (C_i x + D_{fi}f + D_{di}d_w)\end{cases}. \tag{18}$$

To make the problem easier to tackle, the following augmented system can be constructed:

$$\begin{cases}\dot{\bar{x}} = \sum_{i=1}^{r} h_i(\mu)\ \left(\bar{A}_i\bar{x} + \bar{B}_i u + \bar{B}_{di}d\right)\\ y = \bar{C}\,\bar{x} + d_s\end{cases} \tag{19}$$

where $\bar{C}$ is chosen from any $\bar{C}_i, \bar{C}_i = [\,C_i\ 0_{p\times l_f}\ D_{fi}\,] \in \mathcal{R}^{p\times\bar{n}}$, and $d_s = \sum_{i=1}^{r} h_i(\mu)[(\bar{C}_i - \bar{C})\bar{x} + D_{di}d_w\ ]$. As a matter of fact, a pre-designed controller (see (20)) can make the system dynamics stable; therefore, the influences from the system states due to different system output matrices can be regarded as the disturbances. In this case, d_w in Theorem 1 should be replaced by d_s, and D_d should be replaced by I_p. In the stability analysis, when $d_s = 0$, it means $d_w = 0$, and $\bar{C}_1 = \bar{C}_2 = \ldots = \bar{C}_r$. When $d_s \neq 0$, it implies that $d_w \neq 0$ and/or $\bar{C}_i$, $i = 1, 2, \cdots\ldots r$, are not equal. As a result, the methods used in Theorem 1 can be directly applied to prove the robust stability of the estimation error dynamics.

Now, the design procedure of the T–S fuzzy model based fault estimation for general nonlinear systems can be summarized as follows.

Procedure 1: State/fault estimation

1) Construct the augmented system in the form of (3) for the T–S fuzzy model (2).
2) For system with fuzzy output, rewrite the augmented system in the form of (19).
3) Solve the LMIs (13) to obtain the matrices P, Y_i, and Z, and calculate the gain $K_{i1} = P^{-1}Y_i$ and $N = P^{-1}Z$.
4) Use N to solve H from (12).
5) Calculate the other gain matrices R_i, T, and K_{i2} following the formulae (6)–(9), respectively.
6) Obtain the augmented estimate $\hat{\bar{x}}$ by implementing UIO (4), leading to the simultaneous estimates of state and fault as $\hat{x} = [\,I_n\ 0_{n\times 2l_f}\,]\hat{\bar{x}}$ and $\hat{f} = [\,0_{l_f\times(n+l_f)}\ I_{l_f}\,]\hat{\bar{x}}$.

III. Tolerant Design With Signal Compensation

In the last section, a robust fault-estimation technique was proposed. In this section, we will apply the obtained estimates to compensate faulty signals. Assume there is a pre-existing nonlinear dynamic output feedback controller characterized by a T–S fuzzy model, designed for normal operating conditions (i.e., fault-free scenario), in the following format:

$$\begin{cases}\dot{x}_c = \sum_{l=1}^{r_c} h_{cl}(\mu_c)\ (A_{cl}x_c + B_{cl}y)\\ u = \sum_{l=1}^{r_c} h_{cl}(\mu_c)\ C_{cl}x_c\end{cases} \tag{20}$$

where $x_c \in \mathcal{R}^{n_c}$ is the state of dynamic controller (20), $l = 1, 2, \cdots\ldots r_c$, $h_{cl}(\mu_c)$ are membership functions satisfying $\sum_{l=1}^{r_c} h_{cl}(\mu_c) = 1$, and $0 \leq h_{cl}(\mu_c) \leq 1$, A_{cl}, B_{cl}, and C_{cl} are control gains of appropriate dimensions which are pre-designed in the absence of faults, whose designs are beyond the concern in this study.

On the basis of the estimation of augmented state $\hat{\bar{x}}$, the fault term can be reconstructed as

$$\hat{f} = \left[\,0_{l_f\times\bar{n}}\ 0_{l_f\times l_f}\ I_{l_f}\,\right]\hat{\bar{x}}. \tag{21}$$

The measurement output can thus be compensated as follows:

$$\begin{aligned} y_c &= y - \sum_{i=1}^{r} h_i(\mu) D_{fi} J\hat{\bar{x}} \\ &= Cx + \sum_{i=1}^{r} h_i(\mu) D_{fi} J\bar{e} + d_c \\ &= Cx + \bar{e}_1 + d_c \end{aligned} \tag{22}$$

where $J = [0_{l_f\times\bar{n}}\ 0_{l_f\times l_f}\ I_{l_f}]$, $C = \bar{C}\ [I_n\ 0_{n\times l_f}\ 0_{n\times l_f}]^T$, $d_c = \sum_{i=1}^{r} h_i(\mu)[(C_i - C)x + D_{di}d_w]$, and $\bar{e}_1 = \sum_{i=1}^{r} h_i(\mu)D_{fi}J\bar{e}$.

Suppose

$$\text{rank}\ [B_h\ B_{fh}] = \text{rank}\ B_h \tag{23}$$

where $B_h = \sum_{i=1}^{r} h_i(\mu)B_i$, $B_{fh} = \sum_{i=1}^{r} h_i(\mu)B_{fi}$. The compensated signal for the actuator is designed as $u_f = K_f\hat{f}$, where

$$K_f = B_h^+ B_{fh}. \tag{24}$$

Therefore, it is clear that

$$\sum_{i=1}^{r} h_i(\mu)(B_{fi} - B_i K_f) = B_{fh} - B_h B_h^+ B_{fh} = 0. \tag{25}$$

Subtracting u_f from the actuator input, and using the compensated measurement output y_c to replace the actual measurement y, the controller can be compensated as follows:

$$\begin{cases} \dot{x}_c = \sum_{l=1}^{r_c} h_{cl}(\mu_c)\ (A_{cl}x_c + B_{cl}y_c) \\ u = \sum_{l=1}^{r_c} h_{cl}(\mu_c)\ \left(C_{cl}x_c - K_f J\hat{\bar{x}}\right) \end{cases} \tag{26}$$

Substituting (26) into (2), the following closed-loop system can be formulated:

$$\begin{cases} \dot{\tilde{x}} = \sum_{i=1}^{r} h_i(\mu) \sum_{l=1}^{r_c} h_{cl}(\mu_c)\left(\tilde{A}_{il}\tilde{x} + \tilde{B}_{dil}\tilde{d} + B_{eil}\tilde{e}\right) \\ y_c = \tilde{C}\,\tilde{x} + J_e\tilde{e} + J_c\tilde{d} \end{cases}. \tag{27}$$

where

$$\tilde{x} = \begin{bmatrix} x^T & x_c^T \end{bmatrix}^T,\quad \tilde{e} = \begin{bmatrix} \bar{e}_1^T & \bar{e}_2^T \end{bmatrix}^T, \bar{e}_2 = K_f\,J\bar{e}$$

$$\tilde{A}_{il} = \begin{bmatrix} A_i & B_iC_{cl} \\ B_{cl}C & A_{cl} \end{bmatrix},\quad \tilde{B}_{dil} = \begin{bmatrix} B_{di} & 0 \\ 0 & B_{cl} \end{bmatrix},\quad \tilde{C} = \begin{bmatrix} C & 0_{p\times n_c} \end{bmatrix}$$

$$\tilde{d} = \begin{bmatrix} d^T & d_c^T \end{bmatrix}^T,\quad B_{eil} = \begin{bmatrix} 0 & B_i \\ B_{cl} & 0 \end{bmatrix},\quad J_e = \begin{bmatrix} I_p & 0_{p\times m} \end{bmatrix}$$

and

$$J_c = \begin{bmatrix} 0_{p\times l_d} & I_p \end{bmatrix}.$$

Now, it is ready to discuss the stability and robustness of the dynamic system (27).

Theorem 2: If there is a preexisting controller in the form of (20) to ensure plant (2) to be stable and satisfy the following robust performance index:

$$||y||_{Tf}^2 \le \gamma_p^2 ||\tilde{d}||_{Tf}^2 \tag{28}$$

in the fault-free case, where γ_p is a positive scalar, then, based on the robust fault-estimation scheme designed following Theorem 1, the controller (26) can drive the trajectories of compensated system (27) to be stable and satisfy the following robust performance index:

$$||y_c||_{Tf}^2 \le \gamma_0^2 ||\tilde{d}||_{Tf}^2 + \gamma_{0e}^2 ||\tilde{e}||_{Tf}^2 \tag{29}$$

where $\gamma_0^2 > \gamma_p^2 + \alpha_2 + \alpha_0(\alpha_3 + \alpha_4)$, $\gamma_{0e}^2 > \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$, $\alpha_1 = ||J_e^T J_e||$, $\alpha_2 = ||J_e^T J_c||$, $\alpha_3 = ||\tilde{C}^T J_e||$, α_4 is a positive scalar such that $\sum_{i=1}^{r} h_i(\mu) \sum_{l=1}^{r_c} h_{cl}(\mu_c)\|\tilde{P}B_{eil}\| \le \alpha_4$.

Proof: 1) Proof of the closed-loop stability by assuming $d = 0$ and $d_c = 0$.

Choose Lyapunov function as

$$\tilde{V} = V_c + \xi V = \tilde{x}^T\tilde{P}\tilde{x} + \xi\bar{e}^T P\bar{e} \tag{30}$$

where ξ is a positive scalar, $\tilde{P}$ and P are positive-definite matrices with appropriate dimensions. From Theorem 1, there exists a positive scalar φ_0 such that

$$\dot{V} \le -\varphi_0\|\bar{e}\|^2. \tag{31}$$

It can be derived that

$$\|\tilde{e}\|^2 = \|\bar{e}_1\|^2 + \|\bar{e}_2\|^2 \le \varphi_e\|\bar{e}\|^2 \tag{32}$$

where φ_e is a positive scalar such that $\varphi_e \ge (\sum_{i=1}^{r} h_i(\mu)||D_{fi}J||)^2 + ||K_fJ||^2$.

Therefore,

$$\dot{V} \le -\varphi_0||\bar{e}||^2 \le -\frac{\varphi_0}{\varphi_e}||\tilde{e}||^2. \tag{33}$$

Using (27), (30), and (33), one can yield

$$\begin{aligned} \dot{\tilde{V}} \le \sum_{i=1}^{r} h_i(\mu) \sum_{l=1}^{r_c} h_{cl}(\mu_c)\,[\tilde{x}^T\left(\tilde{A}_{il}^T\tilde{P} + \tilde{P}\tilde{A}_{il}\right)\tilde{x} \\ +2\tilde{x}^T\tilde{P}B_{eil}\tilde{e}] - \xi\tfrac{\varphi_0}{\varphi_e}||\tilde{e}^2||. \end{aligned} \tag{34}$$

Since system (2) is stable under controller (20) in the fault-free case, there is a positive scalar φ_c such that $\forall\, i, l$, there is a positive scalar φ_c such that

$$\sum_{i=1}^{r} h_i(\mu) \sum_{l=1}^{r_c} h_{cl}(\mu_c)\,\tilde{x}^T\left(\tilde{A}_{il}^T\tilde{P} + \tilde{P}\tilde{A}_{il}\right)\tilde{x} \le -\varphi_c||\tilde{x}||^2. \tag{35}$$

As a result, we have

$$\begin{aligned} \dot{\tilde{V}} &\le \sum_{i=1}^{r} h_i(\mu) \sum_{l=1}^{r_c} h_{cl}(\mu_c)\left(2\tilde{x}^T\tilde{P}B_{eil}\tilde{e}\right) \\ &\quad - \varphi_c\tilde{x}^2 - \xi\frac{\varphi_0}{\varphi_e}||\tilde{e}^2|| \\ &\le -\varphi_c||\tilde{x}^2|| + \xi_f||\tilde{x}||||\tilde{e}|| - \xi\frac{\varphi_0}{\varphi_e}||\tilde{e}^2|| \end{aligned} \tag{36}$$

where $\xi_f \ge 2\sum_{i=1}^{r} h_i(\mu)\sum_{l=1}^{r_c} h_{cl}(\mu_c)||\tilde{P}B_{eil}||$. Selecting

$$\xi \ge \frac{\xi_f^2\varphi_e}{\varphi_0\varphi_c} \tag{37}$$

it follows that

$$\dot{\tilde{V}} \leq -\frac{\varphi_c}{2}||\tilde{x}||^2 - \frac{\xi\varphi_0}{2\varphi_e}||\tilde{e}||^2 \tag{38}$$

which indicates the compensated system (27) is stable.

2) Proof of the robust performance when $d \neq 0$ and $d_c \neq 0$.

From (27) and (30), one has

$$\begin{aligned}\dot{V}_c &= \sum_{i=1}^{r} h_i(\mu) \sum_{l=1}^{r_c} h_{cl}(\mu_c) \Big[\tilde{x}^T \left(\tilde{A}_{il}^T \tilde{P} + \tilde{P}\tilde{A}_{il}\right)\tilde{x} \\ &\quad + 2\tilde{x}^T \tilde{P}\tilde{B}_{dil}\tilde{d} + 2\tilde{x}^T \tilde{P} B_{eil}\tilde{e}\Big] \\ &= \sum_{i=1}^{r} h_i(\mu) \sum_{l=1}^{r_c} h_{cl}(\mu_c) \Big[\tilde{x}^T \left(\tilde{A}_{il}^T \tilde{P} + \tilde{P}\tilde{A}_{il}\right)\tilde{x} + 2\tilde{x}^T \tilde{P}\tilde{B}_{dil}\tilde{d} \\ &\quad + 2\tilde{x}^T \tilde{P} B_{eil}\tilde{e}\Big] + {y_c}^T y_c - \gamma_0^2 \tilde{d}^T\tilde{d} - \gamma_{0e}^2 \tilde{e}^T\tilde{e} \\ &\quad - {y_c}^T y_c + \gamma_0^2 \tilde{d}^T\tilde{d} + \gamma_{0e}^2 \tilde{e}^T\tilde{e} \\ &= \sum_{i=1}^{r} h_i(\mu) \sum_{l=1}^{r_c} h_{cl}(\mu_c) \Big[\tilde{x}^T \left(\tilde{A}_{il}^T \tilde{P} + \tilde{P}\tilde{A}_{il}\right)\tilde{x} + 2\tilde{x}^T \tilde{P}\tilde{B}_{dil}\tilde{d} \\ &\quad + 2\tilde{x}^T \tilde{P} B_{eil}\tilde{e}\Big] + y^T y + \tilde{e}^T J_e^T J_e \tilde{e} + 2\tilde{e}^T J_e^T J_c \tilde{d} \\ &\quad + 2\tilde{x}^T \tilde{C}^T J_e \tilde{e} - \gamma_0^2 \tilde{d}^T\tilde{d} - \gamma_{0e}^2 \tilde{e}^T\tilde{e} \\ &\quad - {y_c}^T y_c + \gamma_0^2 \tilde{d}^T\tilde{d} + \gamma_{0e}^2 \tilde{e}^T\tilde{e} \\ &\leq \sum_{i=1}^{r} h_i(\mu) \sum_{l=1}^{r_c} h_{cl}(\mu_c) \Big[\tilde{x}^T \left(\tilde{A}_{il}^T \tilde{P} + \tilde{P}\tilde{A}_{il}\right)\tilde{x} + 2\tilde{x}^T \tilde{P}\tilde{B}_{dil}\tilde{d}\Big] \\ &\quad + ||y||^2 + ||J_e^T J_e|| ||\tilde{e}||^2 + 2||J_e^T J_c|| ||\tilde{e}|| ||\tilde{d}|| \\ &\quad + 2\sum_{i=1}^{r} h_i(\mu) \sum_{l=1}^{r_c} h_{cl}(\mu_c) \left(||\tilde{C}^T J_e|| + ||\tilde{P} B_{eil}||\right) ||\tilde{e}|| ||\tilde{x}|| \\ &\quad - \gamma_0^2 \tilde{d}^T\tilde{d} - \gamma_{0e}^2 \tilde{e}^T\tilde{e} \\ &\quad - {y_c}^T y_c + \gamma_0^2 \tilde{d}^T\tilde{d} + \gamma_{0e}^2 \tilde{e}^T\tilde{e}. \end{aligned} \tag{39}$$

Since (28) holds by a pre-designed controller in the fault-free case, we have

$$\sum_{i=1}^{r} h_i(\mu) \sum_{l=1}^{r_c} h_{cl}(\mu_c) [\tilde{x}^T \left(\tilde{A}_{il}^T \tilde{P} + \tilde{P}\tilde{A}_{il}\right)\tilde{x} + 2\tilde{x}^T \tilde{P}\tilde{B}_{dil}\tilde{d}] + ||y||^2 \leq \gamma_p^2 ||\tilde{d}||^2. \tag{40}$$

Substituting (40) into (39), one can have

$$\begin{aligned}\dot{V}_c &\leq \gamma_p^2 ||\tilde{d}||^2 + ||J_e^T J_e|| ||\tilde{e}||^2 + 2||J_e^T J_c|| ||\tilde{e}|| ||\tilde{d}|| \\ &\quad + 2\sum_{i=1}^{r} h_i(\mu) \sum_{l=1}^{r_c} h_{cl}(\mu_c) \left(||\tilde{C}^T J_e|| + ||\tilde{P} B_{eil}||\right) ||\tilde{e}|| ||\tilde{x}|| \\ &\quad - \gamma_0^2 \tilde{d}^T\tilde{d} - \gamma_{0e}^2 \tilde{e}^T\tilde{e} - {y_c}^T y_c + \gamma_0^2 \tilde{d}^T\tilde{d} + \gamma_{0e}^2 \tilde{e}^T\tilde{e} \\ &\leq \gamma_p^2 ||\tilde{d}||^2 + ||J_e^T J_e|| ||\tilde{e}||^2 \\ &\quad + ||J_e^T J_c|| \left(||\tilde{e}||^2 + ||\tilde{d}||^2\right) + \sum_{i=1}^{r} h_i(\mu) \sum_{l=1}^{r_c} h_{cl}(\mu_c) (||\tilde{C}^T J_e|| \\ &\quad + ||\tilde{P} B_{eil}||) \left(||\tilde{e}||^2 + ||\tilde{x}||^2\right) - \gamma_0^2 \tilde{d}^T\tilde{d} - \gamma_{0e}^2 \tilde{e}^T\tilde{e} \\ &\quad - {y_c}^T y_c + \gamma_0^2 \tilde{d}^T\tilde{d} + \gamma_{0e}^2 \tilde{e}^T\tilde{e} \\ &\leq [(\gamma_p^2 + ||J_e^T J_c||) ||\tilde{d}||^2 + (||J_e^T J_e|| + ||J_e^T J_c|| \\ &\quad + ||\tilde{C}^T J_e|| + \sum_{i=1}^{r} h_i(\mu) \sum_{l=1}^{r_c} h_{cl}(\mu_c) ||\tilde{P} B_{eil}||) ||\tilde{e}||^2 \\ &\quad + \left(||\tilde{C}^T J_e|| + \sum_{i=1}^{r} h_i(\mu) \sum_{l=1}^{r_c} h_{cl}(\mu_c) ||\tilde{P} B_{eil}||\right) ||\tilde{x}||^2 \\ &\quad - \gamma_0^2 \tilde{d}^T\tilde{d} - \gamma_{0e}^2 \tilde{e}^T\tilde{e} - {y_c}^T y_c + \gamma_0^2 \tilde{d}^T\tilde{d} + \gamma_{0e}^2 \tilde{e}^T\tilde{e} \\ &\leq (\gamma_p^2 + \alpha_2 - \gamma_0^2) ||\tilde{d}||^2 + \left(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - \gamma_{0e}^2\right) ||\tilde{e}||^2 \\ &\quad + (\alpha_3 + \alpha_4) ||\tilde{x}||^2 - {y_c}^T y_c + \gamma_0^2 \tilde{d}^T\tilde{d} + \gamma_{0e}^2 \tilde{e}^T\tilde{e} \end{aligned} \tag{41}$$

where $\alpha_1 = ||J_e^T J_e||$, $\alpha_2 = ||J_e^T J_c||$, $\alpha_3 = ||\tilde{C}^T J_e||$, and α_4 is a positive scalar such that $\sum_{i=1}^{r} h_i(\mu) \sum_{l=1}^{r_c} h_{cl}(\mu_c) ||\tilde{P} B_{eil}|| \leq \alpha_4$.

From (28), we know $||y||^2 \leq \gamma_p ||\tilde{d}||^2$, which means

$$\tilde{x}^T \tilde{C}^T \tilde{C}\tilde{x} + \tilde{d}^T J_c^T J_c \tilde{d} + 2\tilde{x}^T \tilde{C}^T J_c \tilde{d} \leq \gamma_p ||\tilde{d}||^2. \tag{42}$$

From (42), we can have

$$\begin{aligned}&\lambda_{\min}\left(\tilde{C}^T \tilde{C}\right) ||\tilde{x}||^2 + \lambda_{\min}\left(J_c^T J_c\right) ||\tilde{d}||^2 - 2||\tilde{C}^T J_c|| ||\tilde{x}|| ||\tilde{d}|| \\ &\quad \leq \tilde{x}^T \tilde{C}^T \tilde{C}\tilde{x} + \tilde{d}^T J_c^T J_c \tilde{d} + 2\tilde{x}^T \tilde{C}^T J_c \tilde{d} \leq \gamma_p ||\tilde{d}||^2 \end{aligned} \tag{43}$$

which implies there exists a positive scalar calculated as $\alpha_0 = (\frac{(\sqrt{||\tilde{C}^T J_c||^2 + \lambda_{\min}(\tilde{C}^T \tilde{C})\gamma_p} + ||\tilde{C}^T J_c||)}{\lambda_{\min}(\tilde{C}^T \tilde{C})})^2$ such that $||\tilde{x}||^2 \leq \alpha_0 ||\tilde{d}||^2$ by simple calculation. Therefore, (41) indicates

$$\begin{aligned}\dot{V}_c &\leq [\gamma_p^2 + \alpha_2 + \alpha_0(\alpha_3 + \alpha_4) - \gamma_0^2] ||\tilde{d}||^2 \\ &\quad + \left(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - \gamma_{0e}^2\right) ||\tilde{e}||^2 \\ &\quad - {y_c}^T y_c + \gamma_0^2 \tilde{d}^T\tilde{d} + \gamma_{0e}^2 \tilde{e}^T\tilde{e}. \end{aligned} \tag{44}$$

From (44), we have

$$\begin{aligned}0 \leq V_c &\leq \int_0^{Tf} \{[\gamma_p^2 + \alpha_2 + \alpha_0(\alpha_3 + \alpha_4) - \gamma_0^2] ||\tilde{d}||^2 \\ &\quad + \left(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - \gamma_{0e}^2\right) ||\tilde{e}||^2 - {y_c}^T y_c \\ &\quad + \gamma_0^2 \tilde{d}^T\tilde{d} + \gamma_{0e}^2 \tilde{e}^T\tilde{e}\} dt. \end{aligned} \tag{45}$$

If we make $\gamma_0^2 \geq \gamma_p^2 + \alpha_2 + \alpha_0(\alpha_3 + \alpha_4)$ and $\gamma_{0e}^2 \geq \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$, (45) can be reduced to

$$0 \leq V_c \leq \textstyle\int_0^{Tf} (-{y_c}^T y_c + \gamma_0^2 \tilde{d}^T\tilde{d} + \gamma_{0e}^2 \tilde{e}^T\tilde{e}) dt \tag{46}$$

which means

$$\textstyle\int_0^{Tf} {y_c}^T y_c \, dt \leq \int_0^{Tf} \gamma_0^2 \tilde{d}^T\tilde{d} \, dt + \int_0^{Tf} \gamma_{0e}^2 \tilde{e}^T\tilde{e} dt. \tag{47}$$

Therefore, we can derive $||y_c||^2_{Tf} \leq \gamma_0^2||\tilde{d}||^2_{Tf} + \gamma_{0e}^2||\tilde{e}||^2_{Tf}$, which completes the proof.

Now, the procedure of signal compensation can be summarized as follows.

Procedure 2: Tolerant control with signal compensation

1) Obtain the estimates of the augmented state vector $\widehat{\bar{x}}$ from the robust estimation algorithm described in Procedure 1;
2) Implement the sensor compensation in terms of (22);
3) Based on a pre-existing controller (20) implement a compensation controller in the form (26) to plant (2).

IV. Case Study: Benchmark Wind Turbine System

A. T–S Fuzzy Modeling of Benchmark Wind Turbines

A benchmark model was designed in [27], based on a generic three blade horizontal wind turbine driven by variable speeds, with a full converter coupling and a rated power of 4.8 MW. The overall benchmark model is obtained by interconnecting the models of the individual subsystems, including blade and pitch systems, drive train, generator and convertor, and controller. By integrating the subsystems together, the global wind turbine model can be described in the state-space form

$$\begin{cases} \dot{x} = A(x)\,x + Bu \\ y = Cx \end{cases} \tag{48}$$

where $x = [w_r \quad w_g \quad \theta_\Delta \quad \dot{\beta} \quad \beta \quad \tau_g\,]^{\mathrm{T}}$ is the state vector, and $u = [\tau_{g,r} \quad \beta_r\,]^{\mathrm{T}}$ is the control input vector obtained from the pre-designed controller

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \alpha_{\rm gc} \\ 0 & 0 & 0 & \omega_n & 0 & 0 \end{bmatrix}^T, C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A(x) = \begin{bmatrix} A_{11} & \frac{B_{dt}}{N_g J_r} & -\frac{K_{dt}}{J_r} & 0 & 0 & 0 \\ \frac{\eta_{dt} B_{dt}}{N_g J_g} & \frac{-\frac{\eta_{dt} B_{dt}}{N_g^2} - B_g}{J_g} & \frac{\eta_{dt} K_{dt}}{N_g J_g} & 0 & 0 & -\frac{1}{J_g} \\ 1 & -\frac{1}{N_g} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2\varsigma\omega_n & -\omega_n^2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\alpha_{\rm gc} \end{bmatrix}$$

where $A_{11} = -\frac{B_{dt}+B_r}{J_r} + \frac{1}{2J_r\lambda^2}\rho\pi R^5 C_q(\lambda,\beta) w_r$. The physical meanings of the wind turbine parameters are given in Table I.

TABLE I
Parameter Symbols of Benchmark Wind Turbines

w_r	Rotor angular speed	β_r	Pitch reference
θ_Δ	Torsion angle	N_g	Gear ratio
w_g	Generator rotating speed	J_r	Rotor moment of inertia
β	Pitch angle	K_{dt}	Torsion stiffness
α_{gc}	Generator and converter parameter	J_g	Generator moment of inertia
B_g	Generator external damping	η_{dt}	Efficiency of drive train
ω_n	Natural frequency	τ_g	Generator torque
λ	Tip-speed-ratio	R	Rotor radius
ς	Damping ration	C_q	Torque coefficient
$\tau_{g,r}$	Generator torque reference	B_{dt}	Torsion damping coefficient
B_r	Rotor external damping	ρ	Air density

In order to handle the nonlinearities in system (48), three parameters w_r, β, and λ are selected as the premise variables for building the fuzzy wind turbine model. A set of local operation mode $[W_l\ N_\tau\ V_k]^T$ are chosen, where $l = 1,2, \tau = 1,2,3,4$, $k = 1,2,$ 3, and $W_1 = 0.0591$, $W_2 = 2.5$, $N_1 = -2$, $N_2 = 5$, $N_3 = 35$, $N_4 = 70$, $V_1 = 1$, $V_2 = 7$, $V_3 = 13$. Therefore, the nonlinear model (48) can be approximated by a set of if-then rules as follows:

If w_r is W_l, β is N_τ, and λ is V_k, then

$$\begin{cases} \dot{x} = A_{l\tau k}\, x + B_{l\tau k} u \\ y = C_{l\tau k}\, x \end{cases} \tag{49}$$

where $A_{l\tau k}$ is $A(x)$ by replacing A_{11} with $A_{l\tau k 11} = -\frac{B_{dt}+B_r}{J_r} + \frac{1}{2J_r\lambda^2}\rho\pi R^5 C_q(M_{3l\tau k}, M_{2l\tau k}) M_{1l\tau k}$, $B_{l\tau k} = B$, and $C_{l\tau k} = C$.

Now, let us consider to define proper membership functions to make the T–S fuzzy model approximate the original plant accurately, which can be accessed by defining the submembership functions for the three decision parameters separately, and the global membership functions can be yielded by multiplying them together. Specifically, $a_1(w_r)$ and $a_2(w_r)$ are two membership functions representing the possibility in the range of two local models W_1 and W_2. They can be defined as $a_1(w_r) = \frac{-w_r+W_2}{W_2-W_1}$, and $a_2(w_r) = \frac{-W_1+w_r}{W_2-W_1}$. We can easily verify that $a_1(w_r), a_2(w_r) \in [0,1]$, and $a_1(w_r) + a_2(w_r) = 1$. As a result, $a_1(w_r)$ and $a_2(w_r)$ can be the submembership functions of any given w_r in terms of W_1 and W_2.

Since four values of β are chosen to build a T–S fuzzy model, we have $b_1(\beta)$, $b_2(\beta)$, $b_3(\beta)$, and $b_4(\beta)$ as the membership functions representing the possibility in the range of four local models N_1, N_2, N_3, and N_4, which are defined as follows:

$$\begin{aligned} b_1(\beta) &= \frac{-\beta+N_2}{2(N_2-N_1)}\cdot \mathrm{sgn}(-\beta+N_2) + \frac{-\beta+N_2}{2(N_2-N_1)} \\ b_2(\beta) &= \frac{-N_1+\beta}{2(N_2-N_1)}\cdot \mathrm{sgn}(-\beta+N_2) + \frac{-N_1+\beta}{2(N_2-N_1)} \\ &\quad + \frac{-\beta+N_3}{2(N_3-N_2)}\cdot \mathrm{sgn}(-N_2+\beta)\cdot\ \mathrm{sgn}(-\beta+N_3) \\ &\quad + \frac{-\beta+N_3}{2(N_3-N_2)} \\ b_3(\beta) &= \frac{-N_2+\beta}{2(N_3-N_2)}\cdot \mathrm{sgn}(-\beta+N_3)\cdot \mathrm{sgn}(-N_2+\beta) \\ &\quad + \frac{-N_2+\beta}{2(N_3-N_2)} + \frac{-\beta+N_4}{2(N_4-N_3)}\cdot \mathrm{sgn}(-N_3+\beta) \\ &\quad + \frac{-\beta+N_4}{2(N_4-N_3)} \\ b_4(\beta) &= \frac{-N_3+\beta}{2(N_4-N_3)}\cdot \mathrm{sgn}(-N_3+\beta) + \frac{-N_3+b_3}{2(N_4-N_3)} \end{aligned}$$

where

$$\mathrm{sgn}\ (\sigma) = \begin{cases} -1 & \sigma < 0 \\ 0 & \sigma = 0 \\ 1 & \sigma > 0. \end{cases} \tag{50}$$

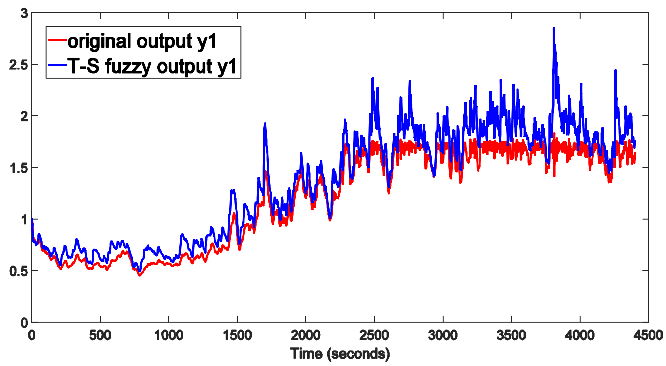

Fig. 1. T–S fuzzy modeling performance of y_1.

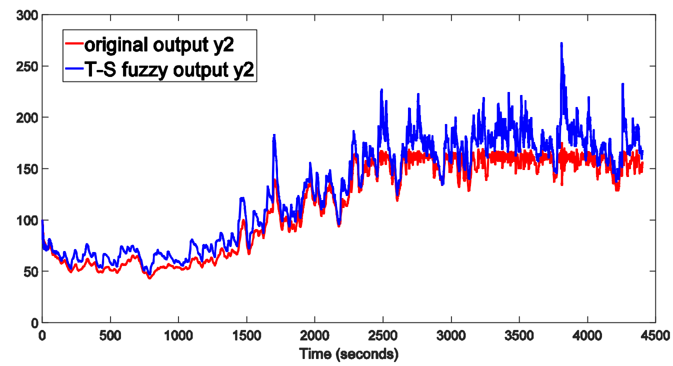

Fig. 2. T–S fuzzy modeling performance of y_2.

It is easy to verity that $b_1(\beta)$, $b_2(\beta)$, $b_3(\beta)$, and $b_4(\beta)$ match convex sum properties.

Similarly, we can have the membership functions of V_1, V_2, and V_3 in the following form:

$$\begin{aligned} c_1(\lambda) &= \frac{-\lambda + V_2}{2(V_2 - V_1)} \cdot \operatorname{sgn}(-\lambda + V_2) + \frac{-\lambda + V_2}{2(V_2 - V_1)} \\ c_2(\lambda) &= \frac{-V_1 + \lambda}{2(V_2 - V_1)} \cdot \operatorname{sgn}(-\lambda + V_2) + \frac{-V_1 + \lambda}{2(V_2 - V_1)} \\ &\quad + \frac{-\lambda + V_3}{2(V_3 - V_2)} \cdot \operatorname{sgn}(-V_2 + \lambda) + \frac{-\lambda + V_3}{2(V_3 - V_2)} \\ c_3(\lambda) &= \frac{-V_2 + \lambda}{2(V_3 - V_2)} \cdot \operatorname{sgn}(-V_2 + \lambda) + \frac{-V_2 + \lambda}{2(V_3 - V_2)}. \end{aligned}$$

Finally, we can define $h_{l\tau k}(\mu) = a_l \cdot b_\tau \cdot c_k$ as global membership functions of the T–S fuzzy models and it is easy to verify that $h_{l\tau k}(\mu)$ also satisfy the convex sum conditions $0 \le h_{l\tau k}(\mu) \le 1$ and $\sum_{l=1}^{2} \sum_{\tau=1}^{4} \sum_{k=1}^{3} h_{l\tau k}(\mu) = 1$.

The final T–S fuzzy model can be inferred as

$$\begin{cases} \dot{x} = \sum_{l=1}^{2} \sum_{\tau=1}^{4} \sum_{k=1}^{3} h_{l\tau k}(\mu)\left(A_{l\tau k}\, x + B_{l\tau k} u\right) \\ y = \sum_{l=1}^{2} \sum_{\tau=1}^{4} \sum_{k=1}^{3} h_{l\tau k}(\mu)\; C_{l\tau k} x \end{cases}. \tag{51}$$

In order to evaluate the modeling performance, we can compare the outputs of a T–S fuzzy model with the original plant outputs. Figs. 1–4 show the consistency between original outputs and T–S fuzzy outputs in the fault-free case. We can see that the T–S fuzzy model built by the presented approach can track all the four original system outputs well, in spite of some modeling errors. It should be noted that, practically, system (51) cannot be exactly the same as the actual plant (48) due to modeling error. Let ε_1 and ε_2 be the errors of state and output equations, respectively. The system (48) is thus described by the T–S fuzzy model with uncertainties

$$\begin{cases} \dot{x} = \sum_{l=1}^{2} \sum_{\tau=1}^{4} \sum_{k=1}^{3} h_{l\tau k}(\mu)\left(A_{l\tau k}\, x + B_{l\tau k} u + E_1 \varepsilon_1\right) \\ y = \sum_{l=1}^{2} \sum_{\tau=1}^{4} \sum_{k=1}^{3} h_{l\tau k}(\mu)\;\left(C_{l\tau k} x + E_2 \varepsilon_2\right) \end{cases} \tag{52}$$

where E_1 and E_2 are constant matrices composed of 0 and 1, with 0 denoting no error, whereas 1 denoting an error on a variable. It is noticed that in the benchmark model, the nonlinear parts only exist in the first state w_r thus the modeling errors influence w_r directly. In other words, for this model, the error only exists in the first state w_r, thus $E_1 = [1\ 0\ 0\ 0\ 0\ 0]^T$, and $E_2 = 0$.

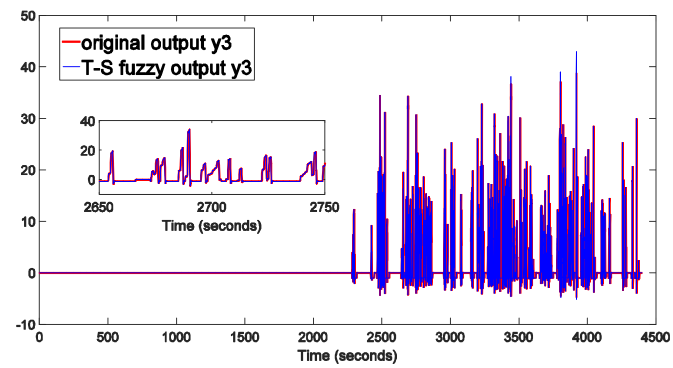

Fig. 3. T–S fuzzy modeling performance of y_3.

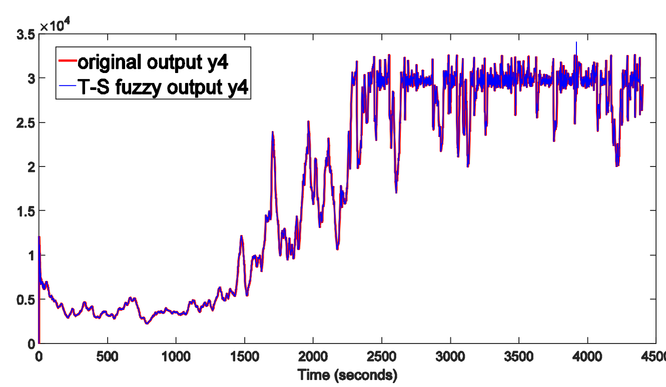

Fig. 4. T–S fuzzy modeling performance of y_4.

Remark 2: In the wind turbine model, we use multi subscripts "$l\tau k$" to denote parameters of local model. By using single "i" to cover all different combinations of the subscripts, the fuzzy system can be rewritten in the following form:

$$\begin{cases} \dot{x} = \sum_{i=1}^{r} h_i(\mu)\;\left(A_i x + B_i u + B_{ui} d_u + B_{fi} f + E_1 \varepsilon_1\right) \\ y = \sum_{i=1}^{r} h_i(\mu)\;\left(C_i x + D_{ui} d_{su} + D_{fi} f + E_2 \varepsilon_2\right) \end{cases} \tag{53}$$

where $r = 24$, $d_u \in \mathcal{R}^{l_{du}}$, and $d_{su} \in \mathcal{R}^{l_{dsu}}$ are extra perturbations of the plant and measurement, respectively, B_{ui} and D_{ui} are the coefficients with appropriate dimensions. The other symbols are the same as before.

As the T–S fuzzy representation is an approximate of the real model, the modeling error can be regarded as a part of unknown inputs. In other words, by defining $d = [\varepsilon_1^T \; d_u^T]^T$, $d_w = [\varepsilon_2^T \; d_{su}^T]^T$, $B_{di} = [E_1 \; B_{ui}]$, and $D_{di} = [E_2 \; D_{ui}]$, the original nonlinear systems can be represented by plant (2).

Until now, the procedures to build the T–S fuzzy models for nonlinear systems can be summarized as follows.

Procedure 3: T–S fuzzy modeling

1) Select variables $\{\mu_j, \; j = 1, 2, .., q\}$ as premise decision variables that play the dominant roles on the nonlinear behaviors of the system.
2) Choose a set of valid operating points $M_{ji}, i = 1, 2, .., r$, $j = 1, 2, .., q$, to represent the several working conditions of the system, depending on the decision vector $\mu = \{\mu_j, \; j = 1, 2, .., q\}$.
3) Determine the membership functions $h_i(\mu)$, $i = 1, 2, .., r$, for any given working condition, which represent their weights in each space divided by the chosen premise variables.
4) Determine the local linear models around each operating point.
5) Obtain the global model (53) by combining local models weighted by the corresponding membership functions, and find where the modeling errors exist, such that E_1 and E_2 can be determined.
6) Set $B_{di} = [E_1 \; B_{ui}]$ and $D_{di} = [E_2 \; D_{ui}]$, where B_{ui} and D_{ui} are the distribution matrices of the external disturbances, which are determined in the linearized models. As a result, the T–S fuzzy model in the format of (18) can thus be established.

B. Fault Estimation and Signal Compensation of Wind Turbines

In this section, we use the 4.8-MW wind turbine benchmark model to demonstrate the effectiveness of the proposed fault reconstruction/estimation and tolerant control algorithms.

1) Fault Estimation/Reconstruction: The actuator of the power converter for controlling the generator torque could be faulty due to either faults in the converter electronics or an off-set on the converter torque. In this case study, the fault is considered to be 50% loss of actuation effectiveness from 1000 to 2000 s. In this case, the coefficient of actuator fault is $B_{fa} = [0\;0\;0\;0\;0\;\alpha_{gc}]^{\mathrm{T}}$. For an actual wind turbine, the sensors can be faulty resulting from electrical or mechanical failures. Here, the sensor fault is assumed to be 50% deviation of the real output of sensor 1. Thus, the coefficient of the sensor fault should be $D_{fs} = [1\;0\;0\;0]^{\mathrm{T}}$. Consequently, the fault vector considered is $f = [f_a \; f_s]^{\mathrm{T}}$ with $B_f = [B_{fa} \; 0]$ and $D_f = [0 \; D_{fs}]$. The unknown inputs and the corresponding coefficients are given as $d_{u1} = 0.01\sin(t)$, $d_{u2} = 0.03\sin(2t)$, $d_{su1} = \mathrm{rand}(-0.02, 0.02)$, $d_{su2} = \mathrm{rand}(-0.2, 0.2)$, $d_{su3} = 0.01\sin(t)$, $d_{su4} = 0.2\sin(5t)$, $B_{ui} = \begin{bmatrix} 0.2 & 0.25 & 0 & 0 & -0.3 & 0.4 \\ 0.4 & 0.5 & 0 & 0 & -0.6 & 0.8 \end{bmatrix}^T$, and $D_{ui} = I$. Choosing $\gamma = 1.25$, we can obtain the observer gains by solving LMIs (13) so that the modeling error ε_1 is decoupled and the influences of d_u and d_w are attenuated.

The curves displayed in Figs. 5–12 exhibit the estimation performance for all system states, actuator fault and sensor fault, respectively. From these figures, one can see the estimation performance is excellent. The T–S fuzzy modeling error and external disturbances have been attenuated effectively. Hence the

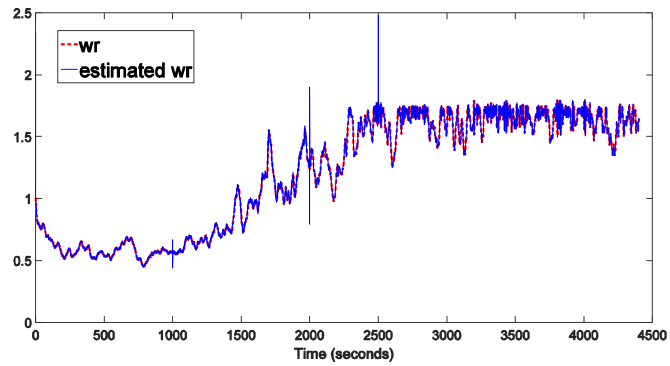

Fig. 5. w_r and its estimation.

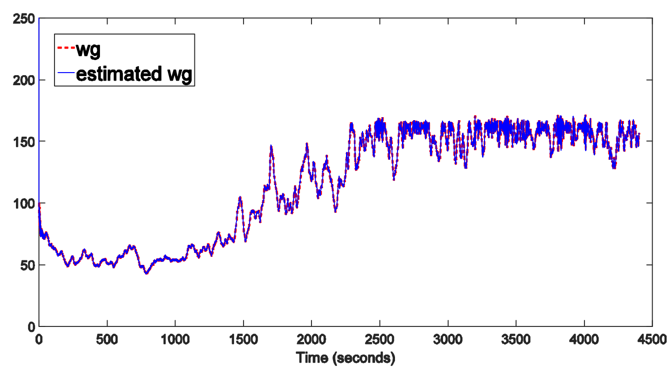

Fig. 6. w_g and its estimation.

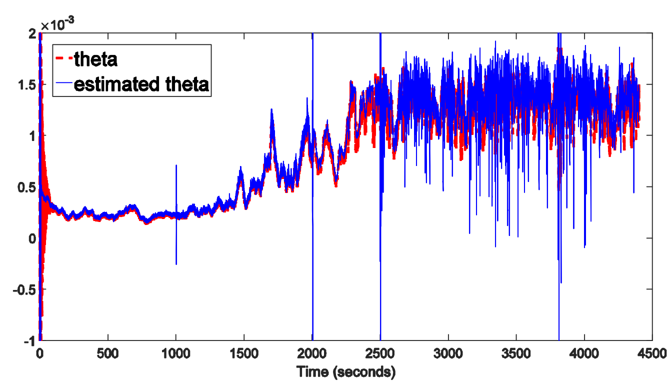

Fig. 7. θ_Δ and its estimation.

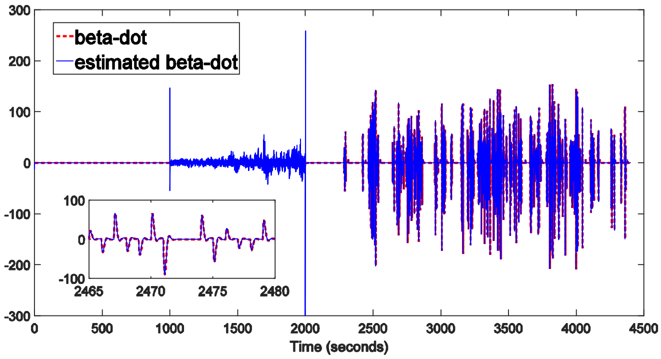

Fig. 8. $\dot{\beta}$ and its estimation.

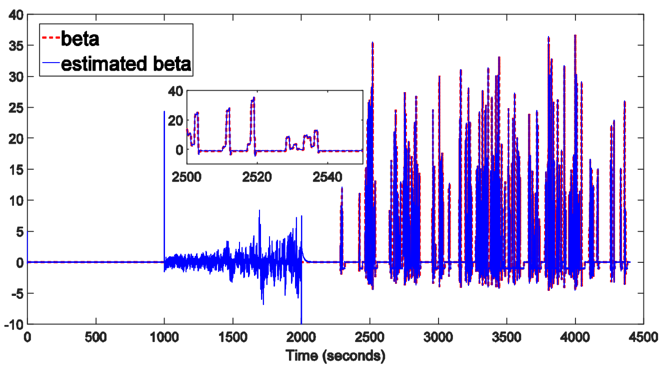

Fig. 9. β and its estimation.

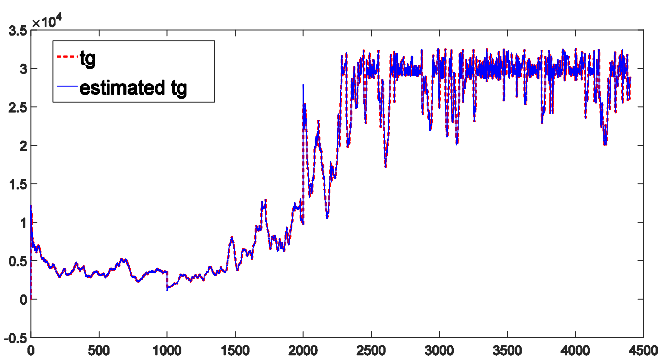

Fig. 10. τ_g and its estimation.

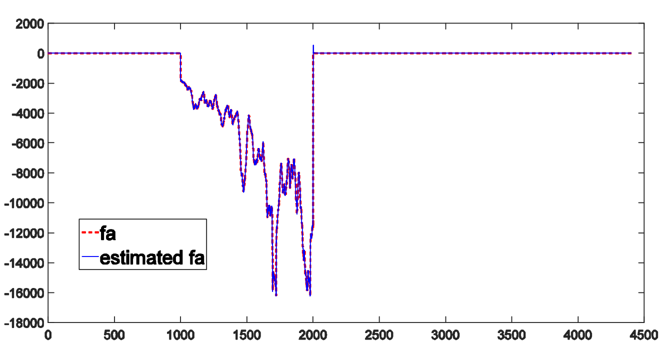

Fig. 11. Generator torque fault f_a and its estimation.

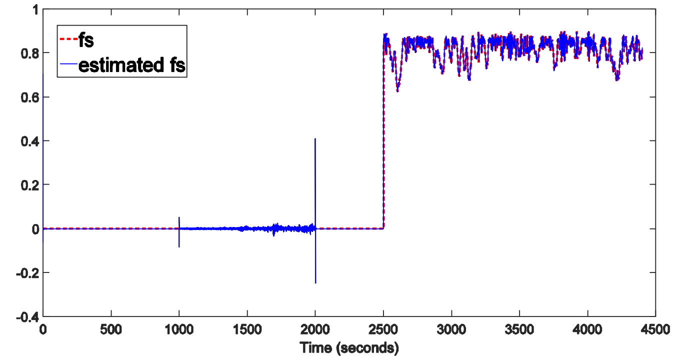

Fig. 12. Rotor speed sensor fault f_s and its estimation.

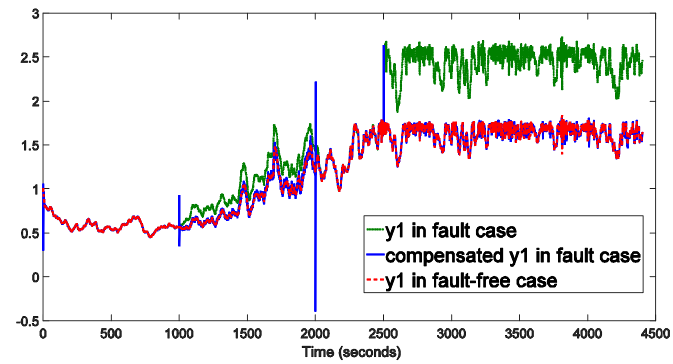

Fig. 13. First wind turbine output y_1 : with and without compensation.

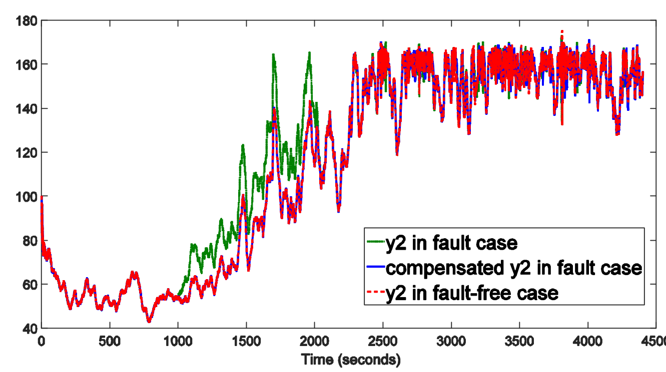

Fig. 14. Second wind turbine output y_2: with and without compensation.

proposed fault-estimation approach can successfully estimate the faults and system states robustly.

2) Fault Tolerance by Signal Compensation: Based on the robust fault estimations, signal compensation techniques are applied to remove effects caused by faults. Figs. 13–16 show the simulated results, which compare the outputs with and without signal compensation. One can see that the compensated outputs for the wind turbines under faulty conditions can track the system output without faults. As a result, the tolerance of the wind turbine systems is realized.

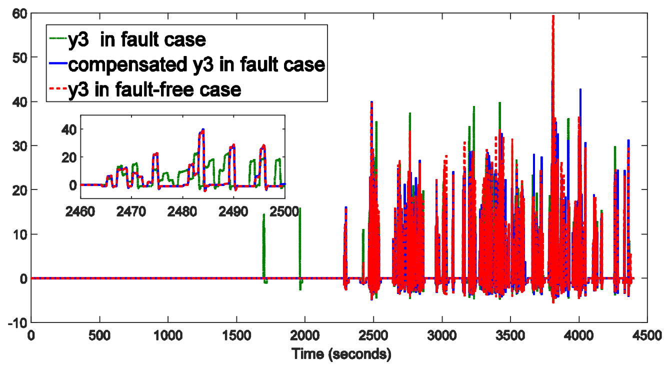

Fig. 15. Third wind turbine output y_3 : with and without compensation.

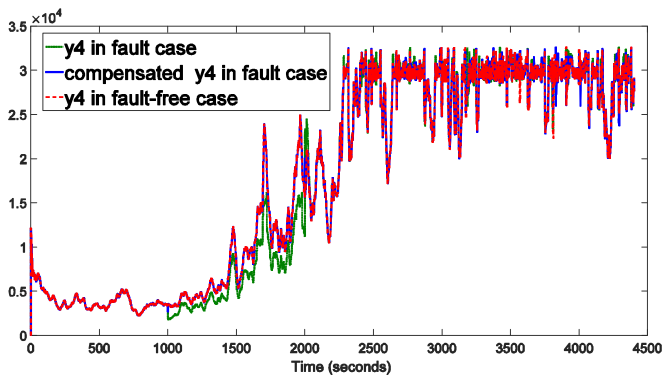

Fig. 16. Fourth wind turbine output y_4 : with and without compensation.

V. Conclusion

In this paper, integrated robust fault estimation and signal compensation techniques for tolerant control were addressed for T–S fuzzy systems. Moreover, a case study for a 4.8-MW wind turbine benchmark system was carried out including T–S fuzzy modeling of the wind turbine, and the implementation and verification of the proposed fault estimation and tolerant control algorithms. It is worth to point out that the proposed methods have been developed for a general nonlinear T–S fuzzy system corrupted by partially decoupled unknown uncertainties, which would find applications in wide industrial systems. A remarkable advantage of the proposed fault-tolerant control approach is that the pre-existing controller can work well by integrating the proposed signal compensation technique under both faulty and fault-free scenarios. In the future, it is of interest to extend the proposed methods to more complex systems, such as stochastic nonlinear systems subjected to Brownian motions, and apply to the real-time industrial processes.

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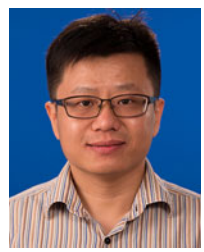

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