Comments and Corrections

Comments and Corrections to "Expansion of the Ohm's Law in Nonsinusoidal AC Circuit"

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***Abstract*—In the paper by Jin *et al.* (*IEEE Trans. Ind. Electron.*, vol. 62, no. 3, pp. 1363–1371, Mar. 2015), the authors present an expansion of Ohm's law in the nonsinusoidal ac circuit, based on which a distinct instruction for analyzing and designing nonsinusoidal circuit including ac/dc link circuit is provided. This paper is very interesting and a good work. However, some equations and expressions may be improved. In this paper, they are discussed and modified.**

***Index Terms*—Hilbert transform (HT), instantaneous impedance/instantaneous admittance, linear capacitor, linear inductor, nonsinusoidal alternating current (ac) circuit, Ohm's law.**

I. Introduction

Jin *et al.* [1] present an expansion of Ohm's law in the nonsinusoidal ac circuit by defining instantaneous impedance, instantaneous admittance, and analytic signals of the instantaneous voltage and current. Based on the analysis, they provide a method of calculating the instantaneous parameters of linear inductors and capacitors in the nonsinusoidal ac circuit. However, the paper may be imprecise in some points, which are discussed and modified in Section II of this paper.

II. Modification

Modification A: In [1], the expression of impedance in the sinusoidal ac circuit is given as

$$X = \omega L \quad \text{or} \quad X = -1/\omega C. \tag{2}$$

Here, the formula is applicative only in the circuits with pure inductance or pure capacitance. It seems that (2) is incomplete; the impedance including both inductance and capacitance should also be considered. For instance, in an RLC series circuit, the expression of impedance should be written as

$$X = \omega L - 1/\omega C. \tag{31}$$

Modification B: Here are the citations from Section II-C of [1]:
"hence, the true instantaneous resistance is zero in Fig. 1(d)".
"hence, the true instantaneous conductance is zero in Fig. 1(f)".

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The citations' conclusions can be expressed as (32) and (33).

$$r(t) = 0 \tag{32}$$

$$g(t) = 0. \tag{33}$$

Here, $r(t)$ is defined as instantaneous resistance and $g(t)$ is defined as instantaneous conductance by Jin *et al.*

However, the waveforms of $r(t)$ in Fig. 3(c) and (d) given by [1] reveal that the values of $r(t)$ and $g(t)$ do not vanish identically, which may be neglected by Jin *et al.*, so it seems that the results of (32) and (33) are slightly inaccurate.

In order to verify the inaccuracy of (32) and (33), theory and simulation analyses are given as follows.

According to the circuit theory, in the equivalent circuit of a linear inductor, the instantaneous nonsinusoidal voltage drop on the inductor and the instantaneous current through it can be written as

$$\begin{cases} u(t) = \sum_{k=1}^{n} U_k \sin(\omega_k t + \alpha_k) \\ i(t) = -\frac{1}{L} \sum_{k=1}^{n} \frac{U_k}{\omega_k} \cos(\omega_k t + \alpha_k) \end{cases}. \tag{34}$$

Based on Hilbert transform (HT), we can obtain

$$\begin{cases} u_{\mathrm{HT}}(t) = -\sum_{k=1}^{n} U_k \cos(\omega_k t + \alpha_k) \\ i_{\mathrm{HT}}(t) = -\frac{1}{L} \sum_{k=1}^{n} \frac{U_k}{\omega_k} \sin(\omega_k t + \alpha_k) \end{cases}. \tag{35}$$

Then the derivatives of $i(t)$ and $i_{\mathrm{HT}}(t)$, $i'(t)$ and $i_{\mathrm{HT}}{}'(t)$ are defined as

$$i'(t) = \frac{1}{L} \sum_{k=1}^{n} U_k \sin(\omega_k t + \alpha_k) \tag{36}$$

$$i_{\mathrm{HT}}{}'(t) = -\frac{1}{L} \sum_{k=1}^{n} U_k \cos(\omega_k t + \alpha_k) \tag{37}$$

According to (18) in [1], the instantaneous reactance $x(t)$ can be calculated as follows:

$$\begin{aligned} x(t) = \omega_i(t) L &= \left[\arctan \frac{i_{\mathrm{HT}}(t)}{i(t)}\right]' L \\ &= \frac{1}{1 + \left[\frac{i_{\mathrm{HT}}(t)}{i(t)}\right]^2} \times \left[\frac{i_{\mathrm{HT}}(t)}{i(t)}\right]' \times L \\ &= \frac{i^2(t)}{i^2(t) + i_{\mathrm{HT}}^2(t)} \times \frac{i_{\mathrm{HT}}{}'(t) i(t) - i'(t) i_{\mathrm{HT}}(t)}{i^2(t)} \times L \\ &= \left[i_{\mathrm{HT}}{}'(t) i(t) - i'(t) i_{\mathrm{HT}}(t)\right] / \left[i^2(t) + i_{\mathrm{HT}}^2(t)\right] \cdot L. \end{aligned} \tag{38}$$

To substitute (34)–(37) into (38), the result as (39) can be obtained

$$x(t) = \frac{1}{i^2(t) + i_{\mathrm{HT}}^2(t)} \left\{ \frac{1}{L} \sum_{k=1}^{n} \frac{U_k^2}{\omega_k} + \frac{1}{L} \sum_{1 \le i < j}^{n} U_i U_j \left(\frac{1}{\omega_i} + \frac{1}{\omega_j} \right) \cos\left[(\omega_i - \omega_j)t + (\alpha_i - \alpha_j)\right] \right\}. \tag{39}$$

On the other hand, *Jin et al.* present (9) and (10) in [1] as follows:

$$x(t) = \left[u_{\mathrm{HT}}(t) i(t) - u(t) i_{\mathrm{HT}}(t)\right] / \left[i^2(t) + {i_{\mathrm{HT}}}^2(t)\right] \tag{9}$$

$$r(t) = \left[u(t) i(t) + u_{\mathrm{HT}}(t) i_{\mathrm{HT}}(t)\right] / \left[i^2(t) + {i_{\mathrm{HT}}}^2(t)\right]. \tag{10}$$

Because (9) is correct in any case, to substitute (34) and (35) into (9), we can obtain (40), which is also correct. Since the result of (40) is the same as that in (39) and (39) is derived from (38), it can be inferred that (38)—*Jin et al.*'s calculation method of instantaneous reactance $x(t)$ in [1]—is correct.

$$\begin{aligned} x(t) &= \left[u_{\mathrm{HT}}(t) i(t) - u(t) i_{\mathrm{HT}}(t)\right] / \left[i^2(t) + {i_{\mathrm{HT}}}^2(t)\right] \\ &= \frac{1}{i^2(t) + i_{\mathrm{HT}}^2(t)} \left\{ \frac{1}{L} \sum_{k=1}^{n} \frac{U_k^2}{\omega_k} + \frac{1}{L} \sum_{1 \le i < j}^{n} U_i U_j \left(\frac{1}{\omega_i} + \frac{1}{\omega_j} \right) \cos\left[(\omega_i - \omega_j)t + (\alpha_i - \alpha_j)\right] \right\}. \end{aligned} \tag{40}$$

To substitute (34) and (35) into (10), we can obtain the expression of instantaneous resistance $r(t)$ as follows:

$$r(t) = \frac{\frac{1}{L} \sum_{1 \le i < j}^{n} U_i U_j \left(\frac{1}{\omega_i} - \frac{1}{\omega_j} \right) \sin[(\omega_i - \omega_j)t + (\alpha_i - \alpha_j)]}{i^2(t) + {i_{\mathrm{HT}}}^2(t)}. \tag{41}$$

However, from (41) we can see that $r(t)$ does not vanish identically. Moreover, there is another citation in [1]: "To define, Fig. 1(c) shows the Norton equivalent circuit of a two-port network, and Fig. 1(d) shows the Thevenin equivalent circuit, and then, Norton's theorem and Thevenin's theorem are all applicable in Fig. 1(c) and (d)." Here, Jin *et al.* mean that Fig. 1(c) and (d) are equivalent and can interconvert. $i_{L.0-}(t)$ is the equivalent initial current source resulting from L with transient state procession and can be nonzero, but from (18) in [1] and (32) we can get the result that no matter $i_{L.0-}(t)$ is equal to zero or not, $u_{L.0-}(t) = i(t) \cdot r(t) = 0$. Obviously, when $i_{L.0-}(t)$ is not equal to zero, Fig. 1(c) and (d) in [1] are not equivalent. Above all, it seems that Jin *et al.* result (32) is questionable.

In a similar way, in the equivalent circuit of a linear capacitor, the instantaneous nonsinusoidal voltage drop on the capacitor and the instantaneous current through it can be written as

$$\begin{cases} u(t) = \sum_{k=1}^{n} U_k \sin(\omega_k t + \alpha_k) \\ i(t) = C \sum_{k=1}^{n} U_k \omega_k \cos(\omega_k t + \alpha_k) \end{cases}. \tag{42}$$

Based on HT, we can obtain

$$\begin{cases} u_{\mathrm{HT}}(t) = -\sum_{k=1}^{n} U_k \cos(\omega_k t + \alpha_k) \\ i_{\mathrm{HT}}(t) = C \sum_{k=1}^{n} U_k \omega_k \sin(\omega_k t + \alpha_k) \end{cases}. \tag{43}$$

Then, the derivatives of $u(t)$ and $u_{\mathrm{HT}}(t)$, $u'(t)$ and ${u_{\mathrm{HT}}}'(t)$ are provided by

$$u'(t) = \sum_{k=1}^{n} U_k \omega_k \cos(\omega_k t + \alpha_k) \tag{44}$$

$${u_{\mathrm{HT}}}'(t) = \sum_{k=1}^{n} U_k \omega_k \sin(\omega_k t + \alpha_k). \tag{45}$$

According to (25) in [1], the instantaneous susceptance $b(t)$ can be calculated as follows:

$$\begin{aligned} b(t) &= \omega_u(t) C = \left[\arctan \frac{u_{\mathrm{HT}}(t)}{u(t)}\right]' C \\ &= \frac{1}{1 + \left[\frac{u_{\mathrm{HT}}(t)}{u(t)}\right]^2} \times \left[\frac{u_{\mathrm{HT}}(t)}{u(t)}\right]' \times C \\ &= \frac{u^2(t)}{u^2(t) + u_{\mathrm{HT}}^2(t)} \times \frac{{u_{\mathrm{HT}}}'(t) u(t) - u'(t) u_{\mathrm{HT}}(t)}{u^2(t)} \times C \\ &= \left[{u_{\mathrm{HT}}}'(t) u(t) - u'(t) u_{\mathrm{HT}}(t)\right] / \left[u^2(t) + u_{\mathrm{HT}}^2(t)\right] \cdot C. \end{aligned} \tag{46}$$

To substitute (42), (43)–(45) into (46), the result can be obtained as

$$b(t) = \frac{1}{u^2(t) + u_{\mathrm{HT}}^2(t)} \left\{ C \sum_{k=1}^{n} U_k^2 \omega_k + C \sum_{1 \le i < j}^{n} U_i U_j (\omega_i + \omega_j) \cos[(\omega_i - \omega_j)t + (\alpha_i - \alpha_j)] \right\}. \tag{47}$$

On the other hand, Jin *et al.* present (12) and (13) in [1] as follows:

$$g(t) = [u(t) i(t) + u_{\mathrm{HT}}(t) i_{\mathrm{HT}}(t)] / [u^2(t) + {u_{\mathrm{HT}}}^2(t)] \tag{12}$$

$$b(t) = [u(t) i_{\mathrm{HT}}(t) - u_{\mathrm{HT}}(t) i(t)] / [u^2(t) + {u_{\mathrm{HT}}}^2(t)]. \tag{13}$$

Because (13) is correct in any case, to substitute (42) and (43) into (13), we can obtain (48), which is also correct. Since the result of (48) is the same as that in (47) and (47) is derived from (46), it can be inferred that (46)—Jin *et al.*'s calculation method of instantaneous susceptance $b(t)$ in [1]—is right.

$$\begin{aligned} b(t) &= \left[u(t) i_{\mathrm{HT}}(t) - u_{\mathrm{HT}}(t) i(t)\right] / \left[u^2(t) + {u_{\mathrm{HT}}}^2(t)\right] \\ &= \frac{1}{u^2(t) + u_{\mathrm{HT}}^2(t)} \left\{ C \sum_{k=1}^{n} U_k^2 \omega_k + C \sum_{1 \le i < j}^{n} U_i U_j (\omega_i + \omega_j) \cos[(\omega_i - \omega_j)t + (\alpha_i - \alpha_j)] \right\}. \end{aligned} \tag{48}$$

To substitute (42) and (43) into (12), we can obtain the expression of the instantaneous conductance $g(t)$ as

$$g(t) = \frac{C \sum_{1 \le i < j}^{n} U_i U_j (\omega_i - \omega_j) \sin[(\omega_i - \omega_j)t + (\alpha_i - \alpha_j)]}{u^2(t) + u_{\mathrm{HT}}^2(t)}. \tag{49}$$

However, from (49) we can see that $g(t)$ does not vanish identically. Moreover, there is another citation in [1]: "To define, Fig. 1(e) shows the Norton equivalent circuit of a two-port network, and Fig. 1(f) shows the Thevenin equivalent circuit, and then, Norton's theorem and Thevenin's theorem are all applicable in Fig. 1(e) and (f)." Here, Jin *et al.* mean that Fig. 1(e) and (f) are equivalent and can interconvert. $u_{C.0-}(t)$ is the equivalent initial voltage source resulting from C with transient state

TABLE X
RESULTS OF SIMULATION ANALYSIS IN FIG. 9

Component	Average value	RMS value
L	$r_{\mathrm{AVG}} = 1.38 \times 10^{-5}\,\Omega$	$r_{\mathrm{AVG}} = 0.641\,\Omega$
C	$g_{\mathrm{AVG}} = -1.95 \times 10^{-5}\,S$	$g_{RMS} = 0.112S$

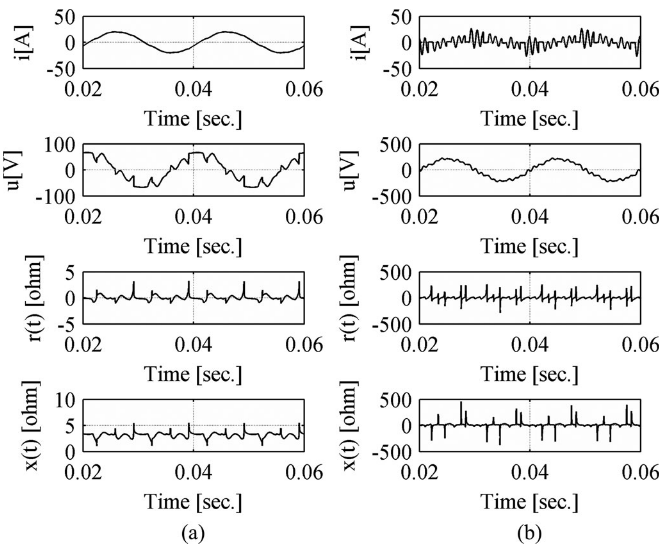

Fig. 9. Analysis results of simulations according to Fig. 3(a) and Table I in [1]. (a) Results for linear inductance. (b) Results for linear capacitance.

procession and can be nonzero, but from (25) in [1] and (33) we can get the result that no matter $u_{C.0-}(t)$ is equal to zero or not, $i_{C.0-}(t) = u(t) \cdot g(t) = 0$. Obviously when $u_{C.0-}(t)$ is not equal to zero, Fig. 1(e) and (f) in [1] are not equivalent. Above all, it seems that the Jin *et al.* result (33) is also questionable.

In order to verify the discussion, the same simulations according to Fig. 3(a) and Table I in [1] are made and the results are given in Table X and Fig. 9.

The waveforms of $r(t)$ in Fig. 9(a) and (b) show that under the Jin *et al.* definitions, the instantaneous resistance $r(t)$ of L and the instantaneous conductance $g(t)$ of C do not vanish identically in the nonsinusoidal ac circuit. And the results of rms value in Table X also declare that $r(t)$ and $g(t)$ cannot be omitted since the value of R_{HVS} is 1 Ω in theory.

Furthermore, simulations according to Fig. 7 and Table VIII in [1] are also made and the results are given in Table XI.

We can see that the rms values in Table XI further prove that $r(t)$ and $g(t)$ do not vanish identically.

Based on the above conclusions, $r(t)$ and $g(t)$ cannot be omitted, so the models in Fig. 1 (c)–(f) given by [1] are not exact. Besides, according to (8) and (11) given by [1], (19) and (26) in [1] should be replaced by (50) and (51), respectively,

$$u_{\mathrm{HT}}(t) = -i_{\mathrm{HT}}(t) \cdot \omega_i(t) L + r(t) \cdot i(t) \quad (50)$$

$$i_{\mathrm{HT}}(t) = u_{\mathrm{HT}}(t) \cdot \omega_c(t) C + u(t) \cdot g(t). \quad (51)$$

TABLE XI
RESULTS OF SIMULATION ACCORDING TO FIG. 7 AND TABLE VIII IN [1]

Simulation parameters	Sampling frequency	Average value	rms value
$L_f = 3$ mH	50 kHz	$L : r_{\mathrm{AVG}} = -0.0012\,\Omega$ $C : g_{\mathrm{AVG}} = 7.72 \times 10^{-4}$ S	$L : r_{\mathrm{RMS}} = 0.1128\,\Omega$ $C : g_{\mathrm{RMS}} = 0.0107$ S
$L_f = 3$ mH	100 kHz	$L : r_{\mathrm{AVG}} = 0.043\,\Omega$ $C : g_{\mathrm{AVG}} = 4.01 \times 10^{-5}$ S	$L : r_{\mathrm{RMS}} = 4.5951\,\Omega$ $C : g_{\mathrm{RMS}} = 0.0124$ S
$L_f = 2 * 3$ mH	100 kHz	$L : r_{\mathrm{AVG}} = 0.0081\Omega$ $C : g_{\mathrm{AVG}} = 4.26 \times 10^{-5}$ S	$L : r_{\mathrm{RMS}} = 0.2183\,\Omega$ $C : g_{\mathrm{RMS}} = 0.0032$ S
$R = 20\,\Omega$	25 kHz	$L : r_{\mathrm{AVG}} = 0.0046\,\Omega$ $C : g_{\mathrm{AVG}} = 3.89 \times 10^{-4}$ S	$L : r_{\mathrm{RMS}} = 0.1663\,\Omega$ $C : g_{\mathrm{RMS}} = 0.0076$ S
$R = 0.5 * 20\,\Omega$	100 kHz	$L : r_{\mathrm{AVG}} = 2.49 \times 10^{-4}\,\Omega$ $C : g_{\mathrm{AVG}} = 1.23 \times 10^{-4}$ S	$L : r_{\mathrm{RMS}} = 0.0672\,\Omega$ $C : g_{\mathrm{RMS}} = 0.0047$ S
C $= 3 * 40\,\mu F$	100 kHz	$L : r_{\mathrm{AVG}} = -0.1424\,\Omega$ $C : g_{\mathrm{AVG}} = -0.0014$ S	$L : r_{\mathrm{RMS}} = 14.9651\,\Omega$ $C : g_{\mathrm{RMS}} = 0.3987$ S
C $= 3 * 40\,\mu$F	50 kHz	$L : r_{\mathrm{AVG}} = -3.37 \times 10^{-5}\,\Omega$ $C : g_{\mathrm{AVG}} = 2.12 \times 10^{-4}$ S	$L : r_{\mathrm{RMS}} = 0.2243\,\Omega$ $C : g_{\mathrm{RMS}} = 0.0106$ S

Even so, Tables X and XI show that in most cases, the average values of $r(t)$ and $g(t)$ are very small, which do not cause a great error in calculating results of R in Tables III–V given by [1], that is why the Jin *et al.* can get the values of R with high accuracy. Thus, in most cases we can regard taking the values of $r(t)$ and $g(t)$ as zero as an approximate treatment method, while the conclusions of the discuss ion can make the mathematical models and theory analyses of [1] more precise.

Modification C: The aim of (30) in [1] is first to calculate the admittance of L_{load} in series with R_{load}, and then, in parallel with C_{load} and second to obtain the one phase equivalent value. Nevertheless, there may be some clerical errors and the modified expression is shown as follows:

$$\begin{cases} a = (R + j2\pi f_f L)/(j2\pi f_f C) \\ b = (R + j2\pi f_f L) + 1/(j2\pi f_f C) \\ b/a = G_{\mathrm{eq}} + jB_{\mathrm{eq}} \end{cases} . \quad (52)$$

In order to verify (52), the second sets of data of Z_{load} in Table II given by [1] are selected and presented as

$$\begin{cases} L_{\mathrm{load}.Y} = 10\,\mathrm{mH} \\ R_{\mathrm{load}.Y} = 10\,\Omega \\ C_{\mathrm{load}.Y} = 1000\,\mu\mathrm{F} \end{cases} . \quad (53)$$

To substitute (53) into (52), we can get the result that the value of b/a is $(0.091 + j0.2856)S$.

Based on (29) in [1], C_{eq} and R_{eq} can be calculated as

$$\begin{cases} C_{\mathrm{eq}} = 0.2856/(2\pi \times 50) \times 10^6\,\mu\mathrm{F} = 909.09\,\mu\mathrm{F} \\ R_{\mathrm{eq}} = 1/0.091\,\Omega = 10.99\,\Omega \end{cases} . \quad (54)$$

From (54), we can obtain the same results as those in Table II in [1], demonstrating that (52) is a reasonable substitution.

REFERENCES

[1] G. Jin, A. Luo, Y. Chen, and H. Xiao, "Expansion of the Ohm's law in nonsinusoidal AC Circuit," *IEEE Trans. Ind. Electron.*, vol. 62, no. 3, pp. 1363–1371, Mar. 2015.