Letters

Discussion on "Semi-Implicit Euler Digital Implementation of Conditioned Super-Twisting Algorithm With Actuation Saturation"

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***Abstract*—Theorem 1 in the discussed paper is shown to be incorrect by means of a simple counterexample. The example shows that the semi-implicit conditioned super-twisting algorithm does not admit a general tuning rule for its parameter κ_1 that can guarantee the same control accuracy as in the absence of actuator saturation.**

***Index Terms*—Counterexample, input saturation.**

Theorem 1 in [1] is incorrect. The theorem claims boundedness of control error e_k and unsaturated control input u_k by $|e_k| \leq h^2 L_1$ and $|u_k| \leq F$, respectively, after a finite number of time steps, if the controller parameters κ_1 and κ_2 satisfy $\kappa_1 > \sqrt{2\kappa_2 F/(F - L_0)}$ and $\kappa_2 > L_1$, wherein h is the sampling period, L_0 and L_1 are amplitude and slope of a disturbance acting on the plant, respectively, and $F > L_0$ is the control saturation level. A counterexample to this claim is shown in the following.

Consider, for simplicity, the sampling period[1] $h = 1$, parameters

$$L_0 = \tfrac{1}{2}, \qquad L_1 = \tfrac{1}{2}, \qquad F = 1, \qquad \kappa_2 = 1, \tag{1}$$

and an *arbitrary* $\kappa_1 > \sqrt{2\kappa_2 F/(F - L_0)} = 2$. Define a periodic disturbance sequence[2] (ε_k), for $k = 0, 1, 2, \ldots$, as

$$\varepsilon_k = \begin{cases} 0 & \text{if k is even} \\ \frac{1}{2}(-1)^{\frac{k+1}{2}} & \text{if k is odd,} \end{cases} \tag{2}$$

which satisfies $|\varepsilon_k| \leq L_0$ and $|\Delta_{k+1}| = \frac{1}{h}|\varepsilon_{k+1} - \varepsilon_k| \leq L_1$ for all integers k.

Applying (19) or, equivalently, [1, Algorithm 1] to the discrete-time plant $e_{k+1} = e_k + hu_k^* + h\varepsilon_{k+1}$ with initial values $e_0 = 0$, $v_0 = -\frac{1}{2}$ then yields periodic sequences

$$(u_k) = \left(-\frac{1}{2}, \frac{3}{2}, \frac{1}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{3}{2}, \ldots\right) \tag{3a}$$

$$(u_k^*) = \left(-\frac{1}{2}, 1, \frac{1}{2}, -1, -\frac{1}{2}, 1, \ldots\right) \tag{3b}$$

$$(v_k) = \left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots\right) \tag{3c}$$

$$(e_k) = (0, -1, 0, 1, 0, -1, \ldots) \tag{3d}$$

for unsaturated control input u_k, saturated control input u_k^*, controller state v_k, and control error e_k. Obviously, $|e_k| = 1$ and $|u_k| = \frac{3}{2}$ hold for every odd integer k, contrary to the claim $|e_k| \leq h^2 L_1 = \frac{1}{2}$, $|u_k| \leq F = 1$ from Theorem 1. Simulation results depicted in Fig. 1 furthermore show that the same effect also occurs with initial condition $v_0 = 0$ and large e_0, as it is typically the case in practical application of the semi-implicit conditioned super-twisting algorithm.

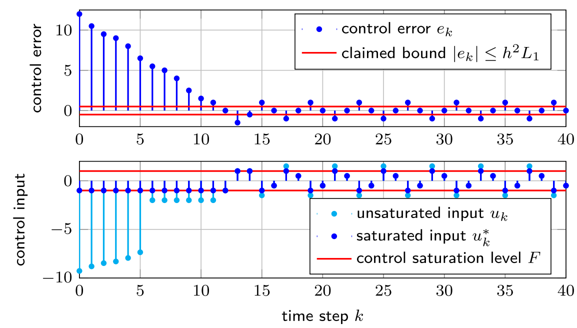

Fig. 1. Simulation results with parameters (1) and $\kappa_1 = 2.1 > 2$, disturbance given by (2), and the initial condition $e_0 = 12$, $v_0 = 0$.

A reason for the invalidity of Theorem 1 is that its proof incorrectly concludes forward invariance of $|u_k| \leq F$ from the fact that $|u_k| = F$ implies $|u_{k+1}| - |u_k| \leq 0$. In the continuous-time case, which is considered in [2], an analogous reasoning is indeed enough to show forward invariance due to continuity of the trajectory. In the discrete-time case, this reasoning fails, however, because a discrete-time trajectory may skip the case $|u_k| = F$. The counterexample exhibits such a trajectory where this effect additionally deteriorates accuracy.

It is noteworthy that the presented counterexample is independent of parameter κ_1, because its value becomes relevant in [1, Algorithm 1] only for $|e_k| > h^2\kappa_2 = 1$, which is never the case in (3d). Hence, Theorem 1 stays false also if the condition on the controller gain κ_1 is replaced by any other, more restrictive tuning rule.

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[1]Analogous examples for arbitrary other sampling periods may be obtained by means of time-scaling.

[2]Note that $\varepsilon_k = \varepsilon(kh)$ holds with a continuous-time sawtooth signal ε with period $4h = 4$ satisfying $|\dot{\varepsilon}| \leq L_1$ almost everywhere and $|\varepsilon| \leq L_0$.

References

[1] X. Yang, X. Xiong, Z. Zou, and Y. Lou, "Semi-implicit Euler digital implementation of conditioned super-twisting algorithm with actuation saturation," *IEEE Trans. Ind. Electron.*, vol. 70, no. 8, pp. 8388–8397, Aug. 2023.

[2] R. Seeber and M. Reichhartinger, "Conditioned super-twisting algorithm for systems with saturated control action," *Automatica*, vol. 116, 2020, Art. no. 108921.