Memristor: Remembrance of Things Past

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Postulated in 1971, the memristor did not see the light of day until a serendipitous discovery at HP nearly four decades later. Here, I reminisce on the crisis that inspired me to develop an axiomatic nonlinear circuit theory where the memristor emerges naturally as the fourth basic circuit element.

The dawn of nonlinear electronics was ushered in during the 1960s by a Cambrian-esque explosion of newly minted two-terminal electronic devices, bearing such intimidating monikers as Esaki diode, Josephson junction, varactor diode, thyristor, impact ionization avalanche transit-time (IMPATT) diode, Gunn diode, and ovonic threshold switch. Trained in the old school of linear circuit analysis, hordes of electronics engineers were awed and shocked upon witnessing a parade of such strongly nonlinear and dynamical electronic devices unfolding at such a breathless rate. To many, the surreal proliferation of exotic devices would conjure the opening scene of Dickens's tale of yore: "It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair, we had everything before us, we had nothing before..."

The surreal proliferation of these exotic devices was an exciting time full of opportunity and challenge. At this time, I was working on my PhD research to make sense of the cornucopia of exotic nonlinear devices. Rather than charting a taxonomy to pigeonhole them, I opted for an axiomatic definition of a few basic nonlinear circuit elements that could be used to model a broad variety of nonlinear devices. I joined Purdue University upon graduation in 1964 and was assigned to revamp its outdated circuit analysis curriculum, thereby providing me an ideal launching pad for teaching nonlinear circuit theory through my device-independent black-box approach. The axioms that would predict the memristor had made their debut in the world's first textbook on nonlinear circuit theory[1] in 1969. It took a year for me to derive and prove mathematically the unique circuit-theoretic properties and memory attributes of this yet unnamed device, earning its accolade as the fourth circuit element[2,3] and its justification for submission to the *IEEE Transactions on Circuit Theory*[4] on November 25, 1970. Its publication in the following year coincided with my move to the University of California, Berkeley, to spearhead research in a new frontier dubbed nonlinear circuits and systems. The memristor was soon relegated to the back burner due to lack of research funding, where, like Rip Van Winkle, it would slumber until awakened. This was despite recognition by the IEEE of the potential of the memristor back in 1973, when they awarded me the prestigious IEEE W.R.G. Baker Prize Paper Award for the most outstanding paper reporting original work in all IEEE publications.

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To analyze circuits made of strongly nonlinear and dynamical electronic devices, it is necessary to have realistic device models made of well-defined nonlinear circuit elements as building blocks,[1,5] which did not exist then, because the electrical engineers from that bygone epoch were taught to overcome nonlinearities by expanding them in a Taylor series and then retaining only the linear term that neatly maps into a linear circuit model. But just like the ancient parable about the blind men and the elephant, such models invariably gave rise to grossly inaccurate and misleading results. Unfortunately, the "linearize then analyze" culture endowed upon the electronic engineers of the day had made it impossible to devise such generalizations due in part to the lack of a circuit-theoretic foundation for basic nonlinear circuit elements.[5] Even Richard Feynman would sloppily use the old-fashioned name condenser—instead of capacitor—in his 1963 classic, "The Feynman Lectures on Physics," when he wrote on pages 6-12, vol. II, "The coefficient of proportionality is called the Capacity, and such a system of two conductors is called a Condenser."[6] Electronic circuit theory is concerned only with the prediction of the voltage $v(t)$ and current $i(t)$ associated with the external terminals sticking out of an enclosing cocoon (henceforth called a black box) whose interior may contain some newly minted nonlinear electronic device or an interconnection of various electronic devices, batteries, and so on. And thus, basic nonlinear circuit elements must be defined from a black-box perspective, independent of their internal composition, material, geometry, and architecture.

From the circuit-theoretic point of view, the three basic two-terminal circuit elements are defined in terms of a relationship between two of the four fundamental circuit variables, namely, the current i, the voltage v, the charge q, and the flux φ. Out of the six possible combinations of these four variables, five have led to well-known relationships.[1] Two of these relationships are already given by $q(t) = \int_{-\infty}^{t} i(\tau)\, d\tau$ and $\varphi(t) = \int_{-\infty}^{t} v(\tau)\, d\tau$. Three other relationships are given, respectively, by the axiomatic definition of the three classical circuit elements, namely, the resistor (defined by a relationship between v and i), the inductor (defined by a relationship between φ and i), and the capacitor (defined by a relationship between q and v). Only one relationship remains undefined: the relationship between φ and q. From the logical as well as axiomatic points of view, it is necessary for the sake of completeness to postulate the existence of a fourth basic two-terminal circuit element, which is characterized by a $\varphi - q$ relationship (see Figure1(a)). This element is christened the memristor because it behaves like a nonlinear resistor with memory, remembering things past.

As a proof of principle, three memristors were built in 1969 using operational amplifiers and off-the-shelf electronic components. They demonstrated three distinct $\varphi - q$ curves on a bespoke memristor curve tracer that I had designed for this purpose.[4] However, the challenge of fabricating a passive monolithic memristor remained unfulfilled for 37 years until a team of scientists from the HP lab, under the leadership of Dr. R. Stanley Williams, reported in the May 1, 2008 issue of *Nature* the world's first operational memristor made by sandwiching a thin film of titanium dioxide between platinum electrodes.[7,8] The pinched hysteresis loop fingerprint of this seminal HP memristor is reproduced below the four-element quartet in Figure1(a).[9] It is truly remarkable that the pinched hysteresis loop is in fact an endearing signature that nature endowed upon all basic nonlinear circuit elements[9,10] capable of remembering their past, including the memcapacitor and the meminductor that I identified to be lossless at the opening lecture of the first UC Berkeley Memristor and Memristive Systems Symposium in November 2008.[11]

Prior to Dr. Williams's *Nature* paper, no one understood how certain experimental two-terminal solid state devices could remember and switch between two or more values of resistance without a power supply. The seminal *Nature* publication has provided a unifying foundation for all non-volatile memory devices, which go by such names as ReRAM, PCRAM, STT-RAM, RRAM, MRAM, FRAM, and PCM, where the device's high and low resistance states are used to code the 0 and 1 binary bits, instead of the conventional voltage or current, which collapses to zero when power is interrupted.

News of the nanoscale HP memristor has breathed new life into a device of antiquity that had been relegated to the dustbin of history. Overnight, it has triggered a torrent of research and development activities on memristors worldwide in both industry and academia (see Figure 1(b) and Figure 1(c)), in anticipation of its disrupting potential in AI and neuromorphic computing. Companies such as Panasonic and Fujitsu have sold several hundred million chips with embedded memristor memory based on Tungsten oxide since 2013, and Taiwan Semiconductor Manufacturing Company (TSMC) has announced that it has developed a 22-nm memristor crossbar process for ASIC embedded memories that will be available for mass production in 2019. Aside from its predicted eventual replacement of the over-extended flash memories, DRAMs, and even hard drives[12] with disrupting non-volatile memristor technology, memristors can emulate synapses and ion channels in neurons and muscle fibers,[13,14] sweat ducts in human skin,[9] and even the primitive amoeba's amazing counting ability.[9] Moreover, the memristor's scalable diminutive physical size makes it the right stuff for building brain-like intelligent machines. Furthermore, because even plants can remember events and communicate through memristors,[15] could memristors in fact be the *sine qua non* for emulating life itself?

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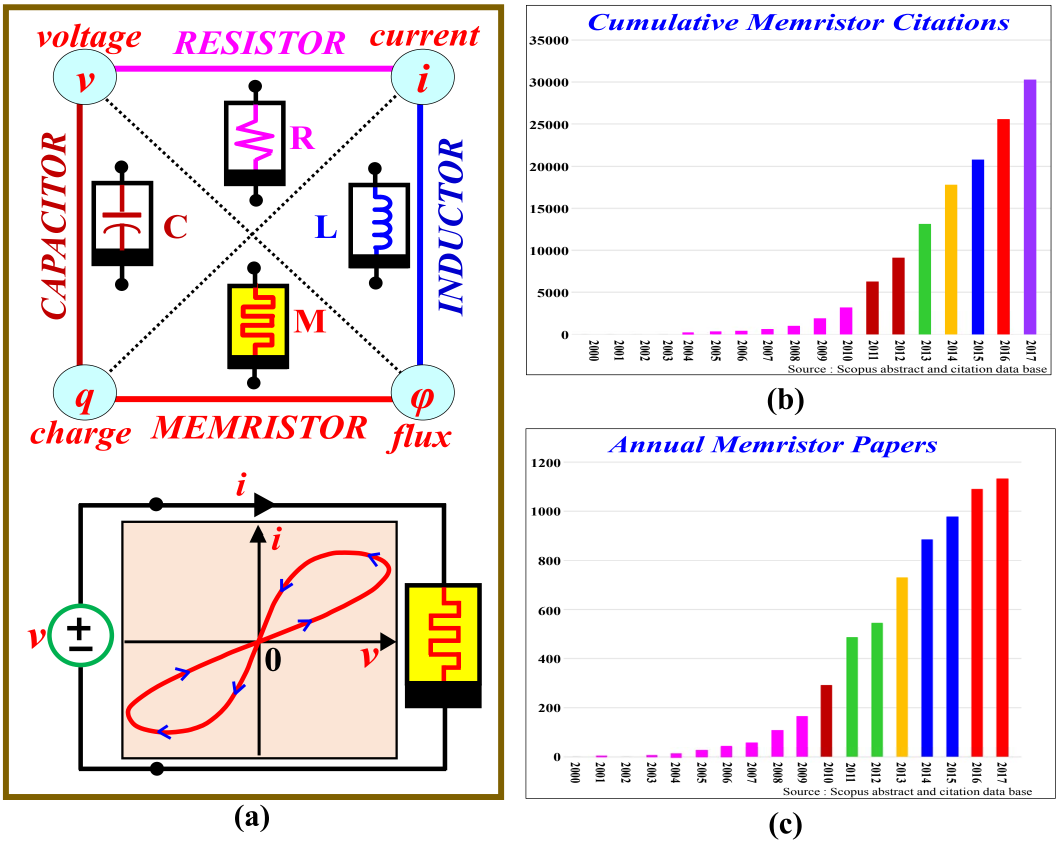

Figure 1. (a) Four-element quartet, along with the pinched hysteresis loop of the HP titanium dioxide memristor. (b) Bar graph depicting the cumulative number of papers citing the memristor between 2004 and 2017. Source: Scopus abstract and citation database. (c) Bar graph depicting the total number of papers on memristors published during each year between 2003 and 2017. Source: Scopus abstract and citation database.

PINCHED HYSTERESIS LOOP

The term “pinched hysteresis loop” was first introduced in a figure in my 2003 article, “Nonlinear circuit foundations for nano devices,”[5] which I’ve reproduced in Figure 2. The earliest known published pinched hysteresis loop dates back to 1939,[16] which I retrieved and re-packaged in Figure 3. The emergence of two symmetrical lobes through the origin must have mystified the authors. For a historical perspective, I reproduced another published pinched hysteresis loop by Kikuchi *et al*.[17] in the left image of Figure 4. The right image of Figure 4 shows information about their paper. The “LETTER ‘8’” in the title shows the authors clearly recognized the distinctive “pinched” characteristic of a new class of two-terminal device. Considering that the Kikuchi *et al*. paper was published merely four months after my paper,[4] the world’s first memristor could have been identified back in 1971—a lamentable missed opportunity in science.

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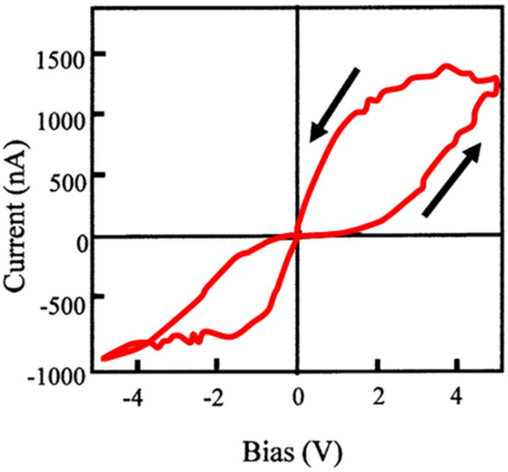

Figure 2. Reproduction of a figure from “Nonlinear circuit foundations for nano devices.”[5]

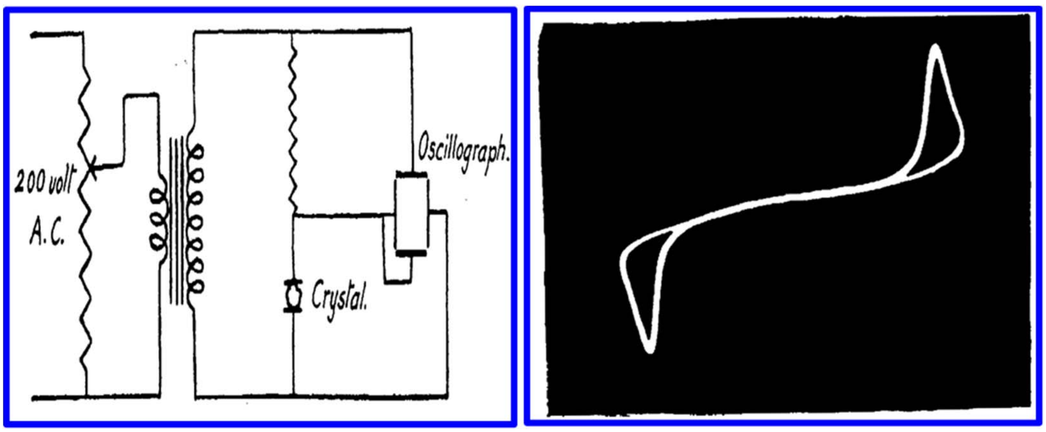

Figure 3. The first published pinched hysteresis loop of a two-terminal solid state device in 1939 was measured from a boron crystal driven by a 200-volt sinusoidal voltage generator.[16]

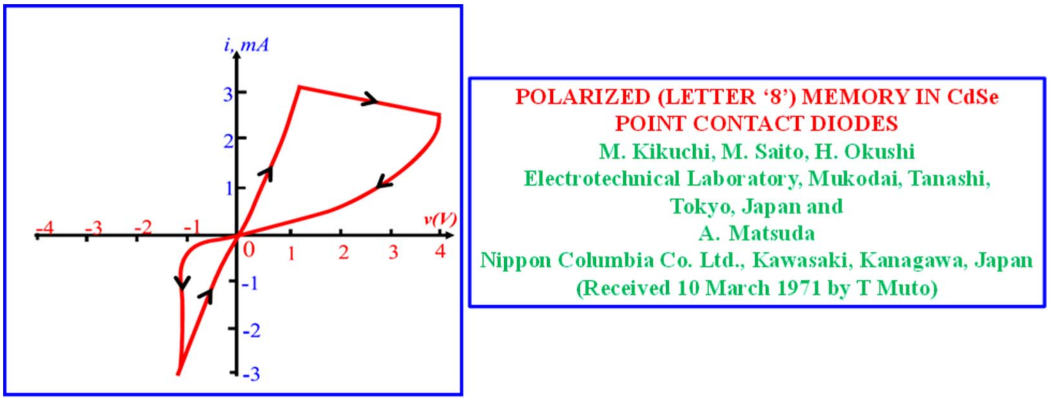

Figure 4. Letter "8" pinched hysteresis loop of CdSe point contact diodes.[17]

CONCLUSION

I close my above reminiscence (which is an embellishment on a previous work[18]) by reproducing the last paragraph of my 1971 article, "Memristor—the missing circuit element."[4] In hindsight, this passage foreshadowed the pinched hysteresis loop fingerprints not only of the ideal memristor predicted in the article, but also of all memristors cited in my later works.[9-11,13-15,19,20]

> *Although no physical memristor has yet been discovered in the form of a physical device without internal power supply, the circuit-theoretic and quasi-static electromagnetic analyses presented in Sections III and IV make plausible the notion that a memristor device with a monotonically increasing $\varphi - q$ curve could be invented, if not discovered accidentally. It is perhaps not unreasonable to suppose that such a device might already have been fabricated as a laboratory curiosity, but was improperly identified! After all, a memristor with a simple $\varphi - q$ curve will give rise to a rather peculiar—if not complicated hysteretic—v-i curve when erroneously traced in the current-versus-voltage plane. (Moreover, such a curve will change with frequency as well as with the tracing waveform.) Perhaps, our perennial habit of tracing the v-i curve of any new two-terminal device has already misled some of our device-oriented colleagues and prevented them from discovering the true essence of some new device, which could very well be the missing memristor.*

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