Corrections to "HTGCE: An Graph-Based and Classifier-Oriented Dimension Reduction Method for Multisource Heterogeneous Feature Tensors"

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I. Equations

IN THE above article [1], some errors appear in equations due to incorrect fonts. First, the matrix variables should be denoted by uppercase boldface letters, e.g., $\boldsymbol{U}_l^m$ and $\boldsymbol{S}_g$. The vector variables should be denoted as lowercase boldface letters, e.g., $\boldsymbol{c}$ and $\boldsymbol{c}'$.

In (7), the font of S_w should be changed to $\boldsymbol{S}_w$. Therefore, the corrected version is shown as follows:

$$\boldsymbol{S}_w = \sum_{l\in\{-1,1\}} \sum_{y_i=l} (\boldsymbol{x}_i - \overline{\boldsymbol{x}_l}) \times (\boldsymbol{x}_i - \overline{\boldsymbol{x}_l})^T = \boldsymbol{X}\boldsymbol{L}\boldsymbol{X}^T. \tag{7}$$

In (9)–(12), the variables $\boldsymbol{U}_l^m$ to be optimized should be changed to $\mathcal{Y}_1^{(im)}$ and $\mathcal{Y}_g^{(im)}$, respectively. In (5) and (9)–(12), the font of variables S_1, S_g, c, and c' should be bold italic. The corrected equations are shown as follows:

$$\left(\sum_i \sum_j \boldsymbol{S}(i,j) \times \left(\boldsymbol{Y}_{i,j} \times \boldsymbol{Y}_{i,j}^T\right)\right)\boldsymbol{u} = \lambda \times \left(\sum_i \boldsymbol{D}(i,i) \times \mathrm{mat}_{(k)}\left(X_i \prod_{l\neq k} \times_l \boldsymbol{U}_l\right) \times \mathrm{mat}_{(k)}\left(X_i \prod_{l\neq k} \times_l \boldsymbol{U}_l\right)^T\right)\boldsymbol{u} \tag{5}$$

$$\min_{\mathcal{Y}_1^{(im)}} \sum_{m=1}^{M_0} \sum_{i\neq j} \left\| \mathcal{Y}_1^{(im)} - \mathcal{Y}_1^{(jm)} \right\|_F^2 \boldsymbol{S}_1^m(i,j) \tag{9}$$

$$\min_{\mathcal{Y}_1^{(im)}} \sum_{m=1}^{M_0} \sum_{i\neq j} \left\| \mathcal{Y}_1^{(im)} - \mathcal{Y}_1^{(jm)} \right\|_F^2 \boldsymbol{S}_1(i,j) \tag{10}$$

$$\min_{\boldsymbol{S}_1, \mathcal{Y}_1^{(im)}} \sum_{i\neq j} \sum_{m=1}^{M_0} \left\| \mathcal{Y}_1^{(im)} - \mathcal{Y}_1^{(jm)} \right\|_F^2 \boldsymbol{S}_1(i,j) \tag{11}$$

$$\text{s.t. } 0 \le \boldsymbol{S}_1(i,j) \le 1, \quad \sum_j \boldsymbol{S}_1(i,j) \ge k_n$$
$$\boldsymbol{S}_1(i,j) = \boldsymbol{S}_1(j,i) \quad \forall i,j$$

$$\min_{\boldsymbol{S}_g, \mathcal{Y}_g^{(im)}} \sum_{g=1}^{3} \left(\boldsymbol{c}(g) \cdot \sum_{m=1}^{M_0} \boldsymbol{c}'(m)^2 \cdot \sum_{i\neq j} \left\| \mathcal{Y}_g^{(im)} - \mathcal{Y}_g^{(jm)} \right\|_F^2 \boldsymbol{S}_g(i,j) \right)$$
$$\text{s.t. } 0 \le \boldsymbol{S}_g(i,j) \le 1, \quad \sum_j \boldsymbol{S}_g(i,j) \ge k_n$$
$$\boldsymbol{S}_g(i,j) = \boldsymbol{S}_g(j,i)$$
$$\sum_m \boldsymbol{c}'(m) = 1, \quad \boldsymbol{c}'(m) \ge 0 \quad 1 \le m \le M_0. \tag{12}$$

In (13)–(16), (18), (19), (23), (28), (29), (34), and (35), S_g should be corrected as $\boldsymbol{S}_g$, U_l^m should be corrected as $\boldsymbol{U}_l^m$, I should be corrected as $\boldsymbol{I}$, c should be corrected as $\boldsymbol{c}$, a should be corrected as $\boldsymbol{a}$, and c' should be corrected as $\boldsymbol{c}'$. Furthermore, the operator "min" in (16) should be changed to "max." The corrected equations are given as follows, (13)–(16), (18), (19), (23), as shown at the of the next page, (28), (29), (34), and (35), as shown at the top of page 3.

II. Sentences

In Section III-B, several equations were misnumbered in the text discussion.

- Equation (34) should be corrected to (9), i.e., "it is effective to use the same adjacency matrix to share the graph structure of all the M_0 types of feature tensors. In this way, (9) can be revised as."
- Equation (36) should be corrected as (11), and the corrected version is shown as "In (11), the adjacency matrix $\boldsymbol{S}_1$ is regarded as variables, and the optimal values will be updated during the solving of (11). In addition, it is noted that the optimization problem of (11) learns the projection matrices only using the samples from case 1, and ignores the samples from cases 2 and 3."
- Equation (37) should be corrected as (12), and the corrected version is given as follows.
  - "In (12), the term $\boldsymbol{S}_g(i,j) = \boldsymbol{S}_g(j,i)$ is used to determine the symmetry of the adaptive adjacency matrix."
  - "The purpose of the optimization problem in (12) is to make the reduced features preserve the local structure of the paired multisource samples and the local structure of single-source samples."

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$$\begin{aligned}
&\min_{\boldsymbol{S}_g,\boldsymbol{U}_l^m,\mathcal{Y}_g^{(im)},\mathcal{W}_k^{(m)},\boldsymbol{c}'} \sum_{g=1}^{3}\left(\boldsymbol{c}(g)\sum_{m=1}^{M_0}\left(\boldsymbol{c}'(m)^2\sum_{i\neq j}\left\|\mathcal{Y}_g^{(im)}-\mathcal{Y}_g^{(jm)}\right\|_F^2\boldsymbol{S}_g(i,j)\right)\right)\\
&\quad+\delta_1\sum_{k=1}^{K}\sum_{m=1}^{M_0}\left(\left\langle\overline{\mathcal{Y}}^{(km)},\mathcal{W}_k^{(m)}\right\rangle-\left\langle\overline{\mathcal{Y}'}^{(km)},\mathcal{W}_k^{(m)}\right\rangle\right)\\
&\quad+\delta_2\sum_{g,i,m}\left\|\mathcal{Y}_g^{(im)}\prod_l\times_l\boldsymbol{U}_l^{mT}-\mathcal{X}_g^{(im)}\right\|_F^2\\
&\text{s.t. } \boldsymbol{U}_l^m\boldsymbol{U}_l^{mT}=\boldsymbol{I},\ \left\|\mathcal{W}_k^{(m)}\right\|_F^2=1\quad 1\le m\le M_0\\
&\quad\sum_m\boldsymbol{c}'(m)=1\\
&\quad\boldsymbol{c}'(m)\ge 0\quad 1\le m\le M_0\\
&\quad 0\le\boldsymbol{S}_g(i,j)\le 1,\sum_j\boldsymbol{S}_g(i,j)\ge k_n\ \forall i,j\ 1\le g\le 3\\
&\quad\boldsymbol{S}_g(i,j)=\boldsymbol{S}_g(j,i)\quad\forall i,j\ 1\le g\le 3
\end{aligned}\tag{13}$$

$$\begin{aligned}
&\min_{\boldsymbol{U}_{m_{\text{iter}}}^{(m)}\big|_{m=1}^{M_0}}\sum_{g,i,m}\left\|\mathcal{Y}_g{}^{(im)}\prod_l\times_l\boldsymbol{U}_l^{mT}-\mathcal{X}_g^{(im)}\right\|_F^2\\
&\text{s.t. } \boldsymbol{U}_l^m\boldsymbol{U}_l^{mT}=\boldsymbol{I}
\end{aligned}\tag{14}$$

$$\begin{aligned}
&\min_{\boldsymbol{U}_{m_{\text{iter}}}^{(m)}\big|_{m=1}^{M_0}}\sum_{g,i}\left\|\boldsymbol{U}_{m_{\text{iter}}}^{mT}\ \text{Mat}_{(m_{\text{iter}})}\left(\mathcal{Y}_g^{(im)}\prod_{l\neq m_{\text{iter}}}\times_l\boldsymbol{U}_l^{mT}\right)-Mat_{(m_{\text{iter}})}\left(\mathcal{X}_g^{(im)}\right)\right\|_F^2\\
&\text{s.t. } \boldsymbol{U}_l^m\boldsymbol{U}_l^{mT}=\boldsymbol{I}
\end{aligned}\tag{15}$$

$$\begin{aligned}
&\max_{\boldsymbol{U}_{m_{\text{iter}}}^{(m)}\big|_{m=1}^{M_0}}\sum_{g,i}tr\left(\boldsymbol{U}_{m_{\text{iter}}}^{mT}\ Mat_{(m_{\text{iter}})}\left(\mathcal{Y}_g^{(im)}\prod_{l\neq m_{\text{iter}}}\times_l\boldsymbol{U}_l^{mT}\right)Mat_{(m_{\text{iter}})}^T\left(\mathcal{X}_g^{(im)}\right)\right)\\
&\text{s.t. } \boldsymbol{U}_l^m\boldsymbol{U}_l^{mT}=\boldsymbol{I}
\end{aligned}\tag{16}$$

$$\begin{aligned}
&\min_{\mathcal{Y}_g^{(im)}}\sum_{g=1}^{3}\left(\boldsymbol{c}(g)\times\sum_{m=1}^{M_0}\left(\boldsymbol{c}'(m)^2\sum_{i\neq j}\left\|\mathcal{Y}_g^{(im)}-\mathcal{Y}_g^{(jm)}\right\|_F^2\times\boldsymbol{S}_g(i,j)\right)\right)\\
&\quad+\delta_1\sum_{k=1}^{K}\sum_{m=1}^{M_0}\left(\left\langle\overline{\mathcal{Y}}^{(km)},\mathcal{W}_k^{(m)}\right\rangle-\left\langle\overline{\mathcal{Y}'}^{(km)},\mathcal{W}_k^{(m)}\right\rangle\right)\\
&\quad+\delta_2\sum_{g,i,m}\left\|\mathcal{Y}_g^{(im)}\prod_l\times_l\boldsymbol{U}_l^{mT}-\mathcal{X}_g^{(im)}\right\|_F^2
\end{aligned}\tag{18}$$

$$\begin{aligned}
&\min_{\mathcal{Y}_g^{(im)}}\sum_{g=1}^{3}\boldsymbol{c}(g)\times\left(\sum_{m=1}^{M_0}\boldsymbol{c}'(m)^2\times\sum_{i,j}\boldsymbol{L}_g(i,j)\times\left\langle\mathcal{Y}_g^{(im)},\mathcal{Y}_g^{(jm)}\right\rangle\right)\\
&\quad+\delta_1\sum_{k=1}^{K}\sum_{m=1}^{M_0}\left(\boldsymbol{c}'(m)\times\sum_i\boldsymbol{a}_{(m,k)}(i)\times\left\langle\mathcal{Y}^{(im)},\mathcal{W}_k^{(m)}\right\rangle\right)\\
&\quad+\delta_2\sum_{g,i,m}\left\|\mathcal{Y}_g^{(im)}\prod_l\times_l\boldsymbol{U}_l^{mT}-\mathcal{X}_g^{(im)}\right\|_F^2
\end{aligned}\tag{19}$$

$$\begin{aligned}
&\min_{\boldsymbol{S}_g}\sum_{m=1}^{M_0}\boldsymbol{c}'(m)^2\sum_{i\neq j}\left\|\mathcal{Y}_g^{(im)}-\mathcal{Y}_g^{(jm)}\right\|_F^2\boldsymbol{S}_g(i,j)\\
&\text{s.t. } 0\le\boldsymbol{S}_g(i,j)\le 1\quad\forall i,j\ 1\le g\le 3\\
&\quad\sum_j\boldsymbol{S}_g(i,j)\ge k_n\quad\forall i\ 1\le g\le 3\\
&\quad\boldsymbol{S}_g(i,j)=\boldsymbol{S}_g(j,i)\quad\forall i,j\ 1\le g\le 3
\end{aligned}\tag{23}$$

$$\begin{aligned}&\min_{S_g}\sum_{g=2}^{3}\sum_{m}\boldsymbol{c}'(m)^2\sum_{i\neq j}\left\|\mathcal{X}_g^{(im)}\prod_l\times_l\boldsymbol{U}_l^m-\mathcal{X}_g^{(jm)}\prod_l\times_l\boldsymbol{U}_l^m\right\|_F^2\boldsymbol{S}_g(i,j)\\&\text{s.t. } \left\|\boldsymbol{U}_l^m\right\|_F^2=1\quad 1\le m\le M_0\quad 1\le l\le l_m\\&\quad 0\le\boldsymbol{S}_g(i,j)\le1,\sum_j\boldsymbol{S}_g(i,j)\ge k_n\ \forall i\ 2\le g\le3\\&\quad \boldsymbol{S}_g(i,j)=S_g(j,i)\quad\forall i,j\ 2\le g\le3\end{aligned}\tag{28}$$

$$\begin{aligned}&\min_{S_1}\sum_{m=1}^{M_0}\boldsymbol{c}'(m)^2\sum_{i\neq j}\left\|\mathcal{X}_1^{(im)}\prod_l\times_l\boldsymbol{U}_l^m-\mathcal{X}_1^{(jm)}\prod_l\times_l\boldsymbol{U}_l^m\right\|_F^2\boldsymbol{S}_1(i,j)\\&\text{s.t. } 0\le\boldsymbol{S}_1(i,j)\le1\quad\forall i,j\\&\quad\sum_j\boldsymbol{S}_1(i,j)\ge k\quad\forall i\\&\quad\boldsymbol{S}_1(i,j)=\boldsymbol{S}_1(j,i)\quad\forall i,j\end{aligned}\tag{29}$$

$$\begin{aligned}&\min_{c'}\sum_{g=1}^{3}\left(\boldsymbol{c}(g)\times\sum_{m=1}^{M_0}\left(\boldsymbol{c}'(m)^2\sum_{i\neq j}\left\|\mathcal{Y}_g^{(im)}-\mathcal{Y}_g^{(jm)}\right\|_F^2\times\boldsymbol{S}_g(i,j)\right)\right)\\&\text{s.t. }\sum_m\boldsymbol{c}'(m)=1\quad\boldsymbol{c}'(m)\ge0\quad1\le m\le M_0\end{aligned}\tag{34}$$

$$\begin{aligned}&\mathcal{L}(\boldsymbol{c}',\alpha,\boldsymbol{\mu})\\&\quad=\sum_{g=1}^{3}\left(\boldsymbol{c}(g)\times\sum_{m=1}^{M_0}\left(\boldsymbol{c}'(m)^2\sum_{i\neq j}\left\|\mathcal{Y}_g^{(im)}-\mathcal{Y}_g^{(jm)}\right\|_F^2\boldsymbol{S}_g(i,j)\right)\right)\\&\qquad+\alpha\left(\sum_m\boldsymbol{c}'(m)-1\right)-\sum_m\boldsymbol{\mu}(m)\times\boldsymbol{c}'(m)\end{aligned}\tag{35}$$

  - "The optimization problem in (12) only focuses on the local structure preservation during the dimension reduction."
- Equation (30) should be written as (13), and the corrected version is shown as follows.
  - "Equation (13) is the final optimization problem of the proposed HTGCE method."
- Equation (34) should be corrected as (14), and (35) should be corrected as (15), and the corrected version is given as follows.
  - "Equation (14) can be transformed into matrix form as."
  - "In addition, the objective function of (15) can be recast as."

In Section IV-A, several equations were misnumbered in the text discussion. Equation (37) should be corrected as (29) and (36) should be corrected as (28). The corrected sentences are as follows.

- "For the optimal $\boldsymbol{S}_1^*$ and optimal objective function value f_1^* of (29), we have $f^*\le f_1^*$."
- "Since the adjacency matrices $\{\boldsymbol{S}_g\}_{g=1}^3$ are limited by the same constraints, the optimal solution $\boldsymbol{S}_1^*$ for (29) can be considered the feasible solution $\{\boldsymbol{S}_2=\boldsymbol{S}_3=\boldsymbol{S}_1^*\}$ for (28). Thus, it is possible to obtain a smaller value for the optimization problem in (28). Therefore, Proposition 1 holds."

REFERENCES

[1] T. Gao, H. Chen, and W. Chen, "HTGCE: An graph-based and classifier-oriented dimension reduction method for multisource heterogeneous feature tensors," *IEEE Trans. Geosci. Remote Sens.*, vol. 60, pp. 1–18, 2022, Art no. 5625918, doi: 10.1109/TGRS.2022.3186320.