# Comments on "Influence of the Statistical Properties of Phase and Intensity on Closure Phase"

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***Abstract*— A recent publication claims that closure phases in SAR interferometry bear no relationship to physical changes of the scatterer, but only to the statistical properties of the averaged pixels. We disprove this claim with a simple counterexample and remind the reader of cases in which closure phases indicate a clear physical content, including the exploitation of closure phases in other fields.**

***Index Terms*— Closure phase, SAR interferometry.**

## I. Introduction

In the above article [1], we claim to demonstrate that closure phases in SAR interferometry do not carry any physical information but are only related to the dispersion of phase and amplitude. To our knowledge, the first physical model explicitly predicting the presence of closure phases was in [2]. Implicitly, closure phases are in the standard models for volumetric scattering and decorrelation, as in [3].

In particular, we want to disprove the following statements.

1) We show that the nonzero phase triplet is only related to the statistical properties of the pixels within the multilooked window.
2) We showed that closure phase [...] similar to InSAR coherence, [it] contains no information about the magnitude of physical changes.
3) We showed that phase closure [...] does not relate to the magnitude of physical, deforming, and nondeforming changes.

These are general statements of the authors of [1], and we are going to disprove them with a counterexample.

## II. Counterexample

Let us take a scatterer made of two subpopulations, represented as the stochastic variables $a$ and $b$, with $\mathrm{E}[a] = \mathrm{E}[b] = 0$ and $\mathrm{E}[ab^*] = 0$. If we consider the following three SAR data sets, comprising, for instance, laid over returns, with some relative motion:

$$y_1 = a + b \cdot e^{j\varphi_1} \tag{1}$$

$$y_2 = a + b \cdot e^{j\varphi_2} \tag{2}$$

$$y_3 = a + b \cdot e^{j\varphi_3} \tag{3}$$

where $\varphi_1$, $\varphi_2$, and $\varphi_3$ are constants, then the expected values of the resulting interferograms are

$$i_{12} = \mathrm{E}[y_2 y_1^*] = \sigma_a^2 + \sigma_b^2 e^{+j(\varphi_2-\varphi_1)} \tag{4}$$

$$i_{23} = \mathrm{E}[y_3 y_2^*] = \sigma_a^2 + \sigma_b^2 e^{+j(\varphi_3-\varphi_2)} \tag{5}$$

$$i_{31} = \mathrm{E}[y_1 y_3^*] = \sigma_a^2 + \sigma_b^2 e^{+j(\varphi_1-\varphi_3)}. \tag{6}$$

For $\sigma_a^2 \neq \sigma_b^2$ and $\varphi_n \neq \varphi_k$, it is immediate to verify that the closure phase is not zero [4]. We can take, for instance, $\sigma_a^2 = 1$, $\sigma_b^2 = 0.5$, $\varphi_1 = 0°$, $\varphi_2 = 90°$, and $\varphi_3 = 180°$. The three interferograms are

$$i_{12} = 1 + 0.5j \tag{7}$$

$$i_{23} = 1 + 0.5j \tag{8}$$

$$i_{31} = 0.5 + 0j \tag{9}$$

and the closure phase is $\angle i_{12} i_{23} i_{31} = \arctan(0.5) + \arctan(0.5) + \arctan(0) \neq 0$.

Finally, if the $\varphi$s are proportional to moisture levels (or any physical quantity, for what it matters), then the magnitude of the closure phase will obviously reflect the magnitude of the moisture variations (or of any physical quantity). For example, a constant moisture will produce a zero closure phase, whereas if moisture levels and, consequently, the $\varphi$'s are changing, the closure phase will be typically different from zero. The relation might be complex, but it exists since $\angle i_{12} i_{23} i_{31}$ is clearly a function of the $\varphi$'s.

In order to be more precise as for the minimal number of looks to be used, closure phases are already evident with two looks. With three looks, the $3 \times 3$ covariance matrix that yields the three interferograms needed for a closure phase can reach the full rank. With an increasing number of looks, the dispersion of the phase closure will get smaller (see [4, eq. (23)]). Incidentally, in [1, eq. (15)], the covariance terms are missing.

## III. Discussion

The crucial point of the model presented in Section II, ignored by Molan *et al.* [1], is that we are considering two different populations of scatterers, each with a distinct phase history. This is not contemplated in [1, eq. (20)], which shows only one scatterer population with varying phase and intensity. With such a model, it is not surprising that the simulations consistently give zero-average closure phases (see [1, Fig. 2]) and the only visible effects are those of noise. The critic here is that the assumed model is not general enough.

In our example, the two populations are statistically present in every single look pixel and one could wonder if this is

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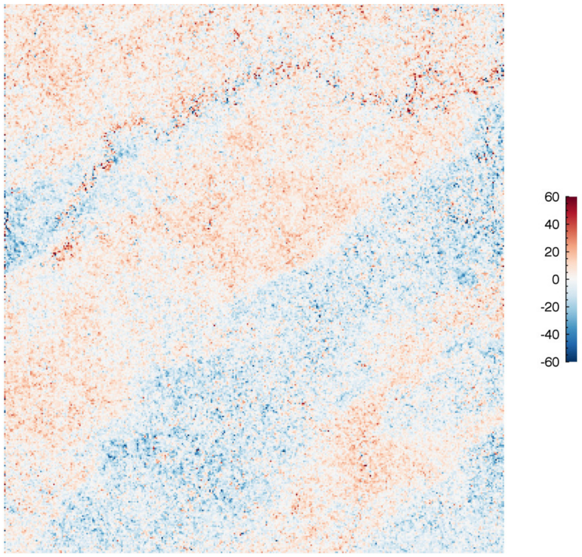


Fig. 1. Closure phase observed over North Carolina/Virginia, from L-band images acquired by PALSAR-2. The color scale is in degrees. Acquisition dates: June 19, 2017; July 3, 2017; and November 6, 2017. The scene size is approximately 70 km × 70 km.

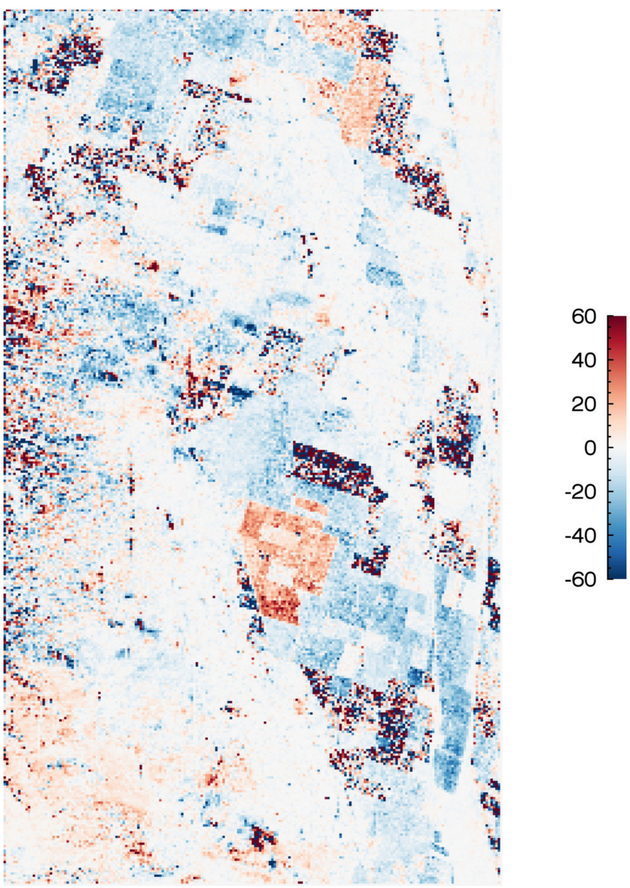


Fig. 2. Closure phase observed over San Joaquin County, California. The L-band SAR images were acquired by JPL with UAVSAR. The color scale is in degrees. Acquisition dates: April 4, 2017; October 4, 2017; and November 7, 2017. The scene size is approximately 40 km × 20 km.

necessary. Indeed, the spatial segregation of the two populations to neighboring pixels will not change the conclusion, considering that the mixing at the multilooking stage has an equivalent impact.

These closure phases carry physical information, which is often evident from the observations, when they display prominent areas of uniform values (see Figs. 1 and 2). A meta-argument in favor of the potential for physical content of closure phases is the fact that they are routinely exploited in astronomical interferometric imaging [5]. There are also applications in seismic imaging [6]. We have shown the possibility to invert moisture series from closure phases in L-band in [7]. The inversion algorithm is complex and has some limitations, but forward modeling from ground truth is not difficult to realize. This is shown, for example, in [8]. A newly submitted manuscript (see [9]) is also presenting observations related to deformation biases, which can only be explained by nonzero physical closure phases.

One could eventually ask where is the logical pitfall of the demonstration in [1]. It looks like the thesis that *the closure phases are zero when using the expected values of the interferograms* is inadvertently introduced between (13) and (14) when one reads "By considering $\varphi_{0,1} + \varphi_{0,2} - \varphi_{0,3} = 0 \ldots$." The assumption might be true for single pixels but is not valid for average phases after a multilooking process.

## IV. Conclusion

Physical closure phases are mathematically possible and exist beyond doubt in the real world too. They are quantitatively predictable with more realistic scattering models than have been used in the past, and they can indeed be used for the successful retrieval of those scattering mechanisms. The readers are encouraged to look through the telescope for themselves, besides considering the mathematical evidence.

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