

# Fast Frequency Sweep Analysis of Passive Miniature RF Circuits Based on Analytic Extension of Eigenvalues

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**Abstract**—A fast frequency sweep approach based on analytic extension of eigenvalues (AEE) is presented and investigated for an efficient analysis of miniature passive RF circuits. In this approach, an eigenvalue decomposition is performed to the Z-parameters of the circuit components at one or a few frequencies, and analytic extension is applied to obtain the eigenvalues at all other frequencies within the band of interest. For electrically small circuit components, the frequency-independent characteristic inductances and capacitances, as well as the frequency-dependent resistances, are extracted from the eigenvalues at the sampling frequencies. The extracted characteristic parameters are then used to approximate the Z- and Y-parameters and predict the responses of the circuit over the entire frequency band. The accuracy of this approach is evaluated by comparing it with the results from a full-wave analysis. It is found that the proposed AEE is very accurate with a relative error of less than 1% for miniature RF circuits whose electrical sizes are smaller than one tenth of the wavelength and is more general and powerful than the one based on lumped equivalent circuits.

**Index Terms**—Eigenmode analysis, fast frequency sweep, quasi-static analysis, RF circuit modeling.

## I. INTRODUCTION

**F**AST and accurate numerical modeling is critical for the design and optimization of novel RF circuits and devices. This is particularly true when artificial intelligence algorithms, such as machine learning, are employed for such design and optimization because these algorithms require a large amount of data for training and validation. For RF circuits and devices, numerical methods such as the finite-element method (FEM) and the moment method (MoM), have been very well developed for accurate numerical modeling. They are most efficient when performing a single-frequency analysis; however, they become time consuming for a frequency sweep analysis over

a broad frequency band because they require solving a large numerical system repeatedly at many frequencies. To alleviate this problem, a variety of fast frequency sweeping techniques have been developed in the past, which were based on either physical or mathematical models.

The most straightforward physical-based fast frequency sweeping technique is the well-known lumped equivalent circuit (LEC) method [1]–[4], which models a small distributive RF circuit in terms of lumped equivalent elements with their capacitances, inductances, and resistances extracted from a quasi-static or full-wave numerical analysis. With the extracted lumped elements, the behavior of the RF circuit can be predicted over the desired frequency band. However, this approach has a disadvantage that the equivalent circuit model can be very complicated if the number of ports in the RF device is large, and the calculation and extraction of the characteristic parameters for each element in the model can be tedious. These challenges greatly limited the application of the LEC method.

The more powerful and generally applicable fast frequency sweeping techniques are mathematical based, such as the model-based parameter extraction (MBPE) [5], [6], the asymptotic waveform evaluation (AWE) [7]–[9], the Padé via Lanczos (PVL) [10]–[13], and the solution space projection (SSP) [14]–[16] methods. These methods employ mathematical functions such as rational functions to expand the frequency response of an RF circuit and then determine the expansion coefficients based on a full-wave numerical analysis at sampling frequencies. Because of their high efficiency and generality, the mathematical-based frequency sweeping techniques have been widely implemented in commercial simulation software. However, they are unable to produce a physical model such as an equivalent circuit model that can provide a direct physical insight for circuit designers and they have to be implemented in a specific numerical algorithm.

In this article, we present a new fast frequency sweeping technique based on analytic extension of eigenvalues (AEE) for an efficient analysis of miniature passive RF circuits over a broad frequency band. The AEE applies an eigenvalue analysis to the Z-parameters of an RF circuit at one or two frequency points and then extend the eigenvalues to other frequency points within the desired frequency band via analytic extension through a functional equation. The functional equation can

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be constructed based on an *RLC* circuit for each eigenvalue, from which the characteristic capacitance, inductance, and resistance can be extracted. With the eigenvalues calculated at all frequencies, the Z- or Y-parameters of the RF circuit can be calculated based on the expansion in terms of the corresponding eigenvectors; thus, the characteristics of the circuit over the frequency band can be predicted efficiently.

Compared to the physical- and mathematical-based fast frequency sweeping techniques mentioned earlier, the AEE method can be considered as a hybrid technique and enjoys three advantages. First, unlike the physical-based techniques, which are usually limited by the ability to construct a physical model that can accurately mimic the true characteristics of the original RF circuit, the AEE is a mathematical approach, which does not rely on an explicit equivalent circuit model and thus makes the method more generally applicable. Second, unlike the mathematical-based techniques, the AEE can automatically yield an LEC model, which not only provides important physical insights but also can be directly incorporated into a circuit analysis if necessary. Third, different from the mathematical based techniques, which are applied to the original numerical system of the employed numerical algorithm, the AEE applies to the calculated Z-parameters directly, so it is independent of the specific numerical algorithm. Furthermore, the dimension of the eigenvalue problem in the AEE is equal to the number of ports in the circuit, which is usually very small and thus makes the AEE computationally inexpensive.

We must note that although the proposed AEE method bears certain similarities to the recently developed characteristic mode analysis (CMA), which has been applied successfully to antenna analysis and design [17]–[29], the two are different both in terms of the mathematical formulation and final objectives. The CMA is formulated from integral equations and solves a generalized eigenvalue problem for the impedance matrix resultant from the MoM [17], [18]. The resulting eigenvectors represent the characteristic currents that can naturally exist on the antenna and their superposition can form the true current distribution. Since the corresponding radiation pattern can be obtained for each mode, the CMA can then be used to synthesize the desired radiation pattern by judiciously exciting necessary eigenmodes on the antenna. In the AEE for RF circuit analysis, the key idea is to provide a fast frequency sweeping technique to compute the frequency response of an RF circuit efficiently based on the AEE from the sampling frequency points to all other points via a functional equation. The types of eigenvalue problems involved and the dimensions of the eigenvalue problems are very different in the CMA and AEE.

The proposed AEE is also different from the modal analysis commonly used in microwave engineering for analyzing the crosstalk between coupled lines based on telegrapher equations [30]–[32]. This modal analysis first extracts the inductance and capacitance matrices,  $[L]$  and  $[C]$ , from a static analysis, and then performs an eigenvalue decomposition to matrices  $[L][C]$  and  $[C][L]$  to obtain the modal characteristic impedance and admittance, which is only applicable to transmission line systems consisting of  $N$  conductors. Although both approaches apply an eigenvalue decomposition, the AEE

method decomposes the extracted Z-parameters directly into eigenmodes and is applicable to any  $N$ -port RF circuits.

The objective of this article is to present the AEE method and investigate its capability for a fast frequency sweep analysis of RF circuits. Although the AEE method is a relatively general approach, we will limit to the case where the RF circuits are electrically small or the frequency band of interest is relatively low. For such a case, the characteristic capacitance and inductance in the functional equation and the eigenvectors can all be assumed invariant over the frequency band. We shall refer it to as the quasi-static AEE. In this article, we first present the formulation of the quasi-static AEE and then conduct an extensive investigation on its accuracy. Finally, we consider and discuss the modeling of various losses and draw a conclusion.

## II. FORMULATION

The proposed AEE can be used with any full-wave analysis method. Here, we describe briefly a full-wave analysis based on the FEM [33], which is used to produce all the results in this article. Consider a circuit excited by a current density  $\mathbf{J}_{\text{imp}}$ , and the electric field  $\mathbf{E}$  is governed by the wave equation

$$\nabla \times (\nu_r \nabla \times \mathbf{E}) - k_0^2 \varepsilon_r \mathbf{E} = -j\omega \mu_0 \mathbf{J}_{\text{imp}} \quad (1)$$

where  $\varepsilon_r$  denotes the relative permittivity and  $\nu_r = 1/\mu_r$  with  $\mu_r$  being the relative permeability. Testing the equation with a weighting function  $\mathbf{W}_i$  and integrating over the solution domain [33], we obtain the weak-form representation of (1) as

$$\begin{aligned} \iiint_V [(\nabla \times \mathbf{W}_i) \cdot \nu_r (\nabla \times \mathbf{E}) - k_0^2 \varepsilon_r \mathbf{W}_i \cdot \mathbf{E}] dV \\ = -j\omega \mu_0 \iiint_V \mathbf{W}_i \cdot \mathbf{J}_{\text{imp}} dV \end{aligned} \quad (2)$$

where it is assumed that the circuit is enclosed by a perfect electrically conducting (PEC) or perfect magnetically conducting (PMC) surface. Other surfaces, such as absorbing and impedance surfaces, can be modeled with a simple modification to the formulation.

To discretize (2), we first expand the electric field using the curl-conforming basis functions as

$$\mathbf{E} = \sum_j \mathbf{N}_j e_j \quad (3)$$

where  $\mathbf{N}_j$  denotes the edge-based vector basis functions. The tree-cotree splitting algorithm [34]–[36] is applied to enforce the gauge condition and avoid the low-frequency breakdown problem. By following Galerkin's method, (2) can be discretized to yield a linear algebraic system as:

$$[K]\{e\} = \{b\} \quad (4)$$

where

$$[K]_{ij} = \iiint_V [(\nabla \times \mathbf{N}_i) \cdot \nu_r (\nabla \times \mathbf{N}_j) - k_0^2 \varepsilon_r \mathbf{N}_i \cdot \mathbf{N}_j] dV \quad (5)$$

$$\{b\}_i = -j\omega \mu_0 \iiint_V \mathbf{N}_i \cdot \mathbf{J}_{\text{imp}} dV. \quad (6)$$

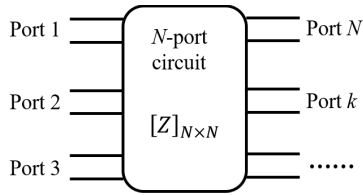


Fig. 1.  $N$ -port circuit network and  $Z$ -matrix description.

For a line current source,  $\{b\}$  is given by

$$\{b\}_i = -j\omega\mu_0 I \int_C \mathbf{N}_i \cdot \hat{l} dl \quad (7)$$

where  $\hat{l}$  denotes the direction of the current. By solving (4) for a given frequency, we obtain an accurate full-wave solution to the electric field. To extract the  $Z$ -parameters, the circuit is excited at one port a time with a lumped element current source. By exciting the ports in turn, the voltage and current values at all the ports can be readily calculated and transformed into the  $Z$ -parameters.

Once the  $Z$ -parameters are obtained with a full-wave analysis at a chosen frequency, the AEE is applied to perform an eigenvalue analysis. However, different from the CMA, where a generalized eigenvalue problem is solved by splitting the real and imaginary parts of the MoM impedance matrix to guarantee the orthogonality of the radiation patterns, here, a standard eigenvalue problem is solved to diagonalize the circuit's  $Z$ -parameters as this would provide a more straightforward insight into the physical meaning of the eigenvalues. Furthermore, by diagonalizing  $Z$ -parameters, complex eigenvalues are obtained whose values can be modeled as an impedance of a lumped circuit model where the characteristic resistance and reactance have different variations with frequency and can be analyzed separately. Otherwise, if a generalized eigenvalue problem is formulated as is done in the CMA, the frequency behavior of the eigenvalues becomes difficult to model and certain approximations have to be employed for the extension to other frequencies [37].

Consider an  $N$ -port circuit as shown in Fig. 1, and the  $Z$ -parameters are defined as

$$[Z]\{I\} = \{V\} \quad (8)$$

where  $[Z] = [R] + j[X]$  is the  $N$ -port impedance matrix,  $\{I\}$  is the current vector flowing into the ports, and  $\{V\}$  is the voltage vector on the ports. Now, consider the standard eigenvalue problem

$$[Z]\{v_n\} = \lambda_n \{v_n\} \quad (9)$$

where  $\lambda_n$  denotes the eigenvalues and  $\{v_n\}$  denotes the corresponding eigenvectors. Since  $[Z]$  is not Hermitian,  $\lambda_n$  and  $\{v_n\}$  may be complex. However, for reciprocal circuits, the impedance matrix is symmetric, and we have the orthogonality relations

$$\{v_m\}^T \{v_n\} = \delta_{mn}. \quad (10)$$

When  $\{v_n\}$  are calculated, any current vector  $\{I\}$  can then be represented by an eigen expansion using eigenvectors  $\{v_n\}$  as the basis functions

$$\{I\} = \sum_n \alpha_n \{v_n\}. \quad (11)$$

Depending on whether the circuit is excited by a current or voltage source, the coefficients  $\alpha_n$  can be determined by utilizing the orthogonality from (10) in a slightly different manner. For example, when the  $Y$ -parameters are of interest, the voltage excitation  $\{V_{\text{src}}\}$  should be applied by definition. Substituting (11) into (8) and testing it with  $\{v_m\}$  yields

$$\sum_n \alpha_n \{v_m\}^T [Z] \{v_n\} = \{v_m\}^T \{V_{\text{src}}\}. \quad (12)$$

Because of the orthogonality relationship (10), (12) reduces to

$$\alpha_n = \frac{\{v_n\}^T \{V_{\text{src}}\}}{\lambda_n}. \quad (13)$$

If the  $Z$ -parameters are to be computed, the current source should be impressed instead. The derivation of the coefficients then becomes much easier, and one only needs to take the inner product of (11) with  $\{v_m\}$ , which gives

$$\alpha_n = \{v_n\}^T \{I_{\text{src}}\}. \quad (14)$$

Next, we examine the physical meaning of the eigenvalues to formulate a functional equation to extend the eigenvalues from one or two sampling frequencies to all other frequencies within the frequency band of interest. Since the standard eigenvalue problem diagonalizes the impedance matrix into

$$[Z] = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_N \end{bmatrix} \quad (15)$$

the eigenvalue  $\lambda_n$  corresponds to the impedance for the  $n$ th eigenmode. In the antenna community, it is found that the input impedance for each characteristic mode can be represented by a series  $RLC$  model [38]–[40]. Therefore, it is intuitive to express the functional equation for the AEE as

$$\lambda_n = R_n + j\omega L_n + \frac{1}{j\omega C_n} \quad (16)$$

where  $R_n$ ,  $L_n$ , and  $C_n$  are the characteristic resistance, inductance, and capacitance, respectively. Although this representation is purely mathematical, it implicitly provides an LEC model and gives physical meaning to the eigenvalues. However, it should be pointed out that  $\lambda_n$  is not an input impedance at any port although the impedance of a lumped circuit is used to express the eigenvalues. Instead, the eigenvalue together with its corresponding eigenvector contributes to the self-impedance and mutual impedance between all ports for each mode. By superposing the contributions of all modes, the  $Z$ -parameters can be obtained. For RF circuits that are electrically very small, a quasi-static approximation can be made in which  $L_n$  and  $C_n$  as well as the eigenvectors  $\{v_n\}$  are assumed to be frequency independent over the frequency band [23]. Note that the characteristic resistance  $R_n$  is not

necessarily constant. However, over the frequency band where the quasi-static analysis is valid, the behavior of  $R_n$  as a function of frequency is supposed to be *a priori* knowledge. Specifically, the frequency-dependent resistance varies exponentially as  $R_n \sim f^\beta$ . For example, the sheet resistance from the skin effect gives  $\beta = 0.5$ , and the resistance due to the dielectric loss gives  $\beta = -1$ .

Once the eigenvalues at two frequency points are obtained by the AEE, the imaginary parts can be taken to solve for the constant  $L_n$  and  $C_n$  based on (16). Through extensive numerical experiments, it is found that for electrically small circuits, all modes are either capacitively or inductively dominant, and there are no zero crossings and, therefore, no resonances. A further approximation can be made to reduce the computation to one single-frequency point [37]. For an eigenvalue with a positive imaginary part, it is an inductively dominant mode and the inductance  $L_n$  contributes more to the total impedance. Therefore, the functional equation (16) can be approximated as

$$\text{Im}(\lambda_n) \approx \omega L_n. \quad (17)$$

Similarly, for an eigenvalue with a negative imaginary part, the capacitance  $C_n$  dominates and only the capacitive term is to be considered as

$$\text{Im}(\lambda_n) \approx -\frac{1}{\omega C_n}. \quad (18)$$

Utilizing the results from two frequency points to consider both  $L_n$  and  $C_n$  will improve the accuracy, and hence, a tradeoff has to be made between the accuracy and the cost. Once  $R_n$ ,  $L_n$ , and  $C_n$  are determined, the eigenvalues at all frequencies can be predicted by (16)–(18).

Once the eigenvalues are computed at all the frequency points, the Z- and Y-parameters can be calculated very efficiently. By definition,  $Z_{ij}$  can be found by driving port  $j$  with the current  $I_j = 1$ , with all other ports open, and measuring the voltage at port  $i$ , so that

$$Z_{ij} = \left. \frac{V_i}{I_j} \right|_{I_k=0 \text{ for } k \neq j} = V_i |_{I_j=1, I_k=0 \text{ for } k \neq j}. \quad (19)$$

Therefore, we first excite port  $j$  and leave all other ports open-circuited, which gives  $\{I_{\text{src}}^{(j)}\} = [0 \dots 0, 1, 0, \dots, 0]^T$  with only the  $j$ th element to be one and all others to be zero. By substituting the current source vector  $\{I_{\text{src}}^{(j)}\}$  into the modal expansion in (11), the voltages on the ports can be computed as

$$\begin{aligned} \{V^{(j)}\} &= [Z] \{I_{\text{src}}^{(j)}\} = \sum_n \alpha_n^{(j)} [Z] \{v_n\} \\ &= \sum_n \alpha_n^{(j)} \lambda_n \{v_n\} \end{aligned} \quad (20)$$

where  $\alpha_n^{(j)}$  are obtained by (14) as  $\alpha_n^{(j)} = \{v_n\}_j^T$ .  $Z_{ij}$  can finally be written in a compact form as

$$[Z] = \sum_n \lambda_n \{v_n\} \{v_n\}^T. \quad (21)$$

Similarly, to compute  $Y_{ij}$ , which are defined as

$$Y_{ij} = I_i |_{V_j=1, V_k=0 \text{ for } k \neq j} \quad (22)$$

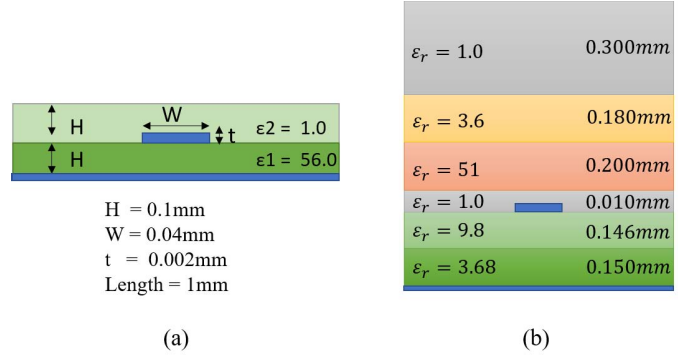


Fig. 2. Test problems for the quasi-static AEE. (a) Two-layer microstrip line. (b) Five-layer microstrip line.

we can set the voltage source to  $\{V_{\text{src}}^{(j)}\} = [0 \dots 0, 1, 0, \dots, 0]^T$  with only the  $j$ th element to be one and all others to be zero and then find the modal solution of the current at port  $i$ . Finally, we have

$$[Y] = \sum_n \frac{\{v_n\} \{v_n\}^T}{\lambda_n}. \quad (23)$$

In (21) and (23),  $\{v_n\}$  represents the current flowing into the ports for the  $n$ th mode. The eigenvalue together with the eigenvector gives the self-impedance and mutual impedance or admittance for each mode.

### III. NUMERICAL RESULTS

In this section, we present the comparison between the quasi-static AEE and full-wave solutions for several testing problems to investigate the range of validity and errors of the AEE-based quasi-static analysis.

#### A. Single Frequency Versus Two Frequencies

Two test problems are designed to examine the behavior of the eigenvalues in the quasi-static AEE and compare the accuracy between the single- and two-frequency computations. The first consists of a conducting strip, which is 0.04 mm wide, 0.002 mm thick, and 1.0 mm long, and is embedded in a two-layer medium shown in Fig. 2(a). The whole structure is bounded by a conducting shield, which is 0.36 mm wide. The conducting strip is excited by a 1.0-A current source at one end and terminated by a 50-Ω resistor at the other end. The second one consists of the same conducting strip as in the first problem, but now, it is embedded in a five-layer medium, as shown in Fig. 2(b). Again, the conducting strip is excited by a 1.0-A current source at one end and terminated by a resistor with  $R = 50 \Omega$  at the other end. For both problems, the desired frequency band is from 400 MHz to 4 GHz.

Figs. 3 and 4 show the imaginary parts of the eigenvalues for all the modes in the two problems. Since these are two-port lossless circuits, the real parts that represent the resistance can be neglected. For the eigenvalues with a positive imaginary part, the inductive component dominates, which has a linear behavior as the function of frequency; hence, the approximation in (17) is applicable. For the eigenvalues whose imaginary



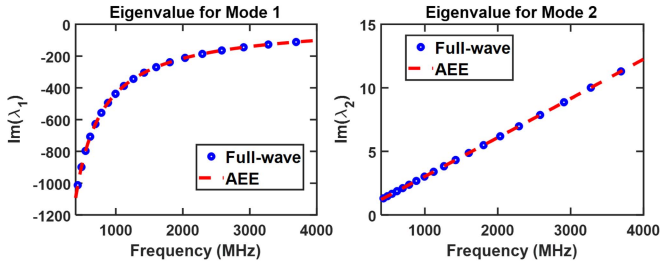


Fig. 3. Imaginary parts of the eigenvalues versus frequency for test problem 1.

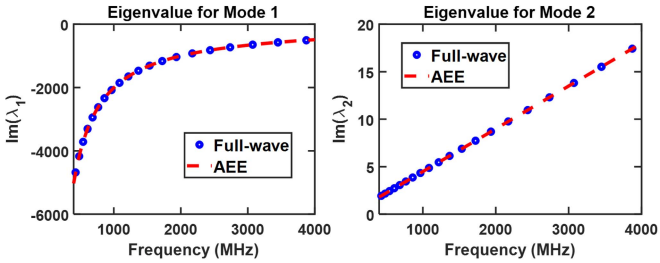


Fig. 4. Imaginary parts of the eigenvalues versus frequency for test problem 2.

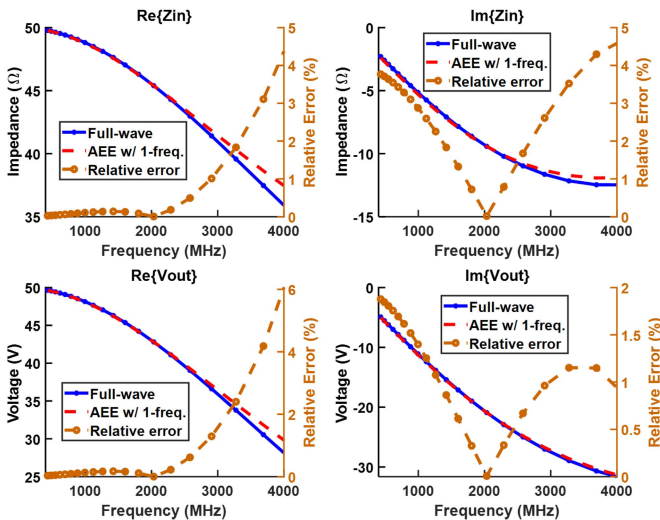


Fig. 5. Quasi-static AEE using a single frequency at 2 GHz versus the full-wave analysis for test problem 1. Top: real and imaginary parts of the input impedance. Bottom: real and imaginary parts of the output voltage.

part is negative, they are capacitively dominant and can be approximated by (18).

Fig. 5 shows the input impedance and output voltage of the quasi-static AEE for test problem 1 using a single frequency at 2 GHz. The results are compared with the full-wave analysis. The relative errors in the quasi-static analysis calculated using the full-wave solution as the reference are also plotted. The maximum error is around 6%. Fig. 6 shows the results in which two frequency points at 400 MHz and 4 GHz are used to include the effect of both inductance and capacitance via (16). A much better agreement is achieved where the maximum error is reduced to about 0.8%. The error remains to be less than 1% for frequencies up to 5 GHz, at which the device's electrical size is about one tenth of the wavelength, and the

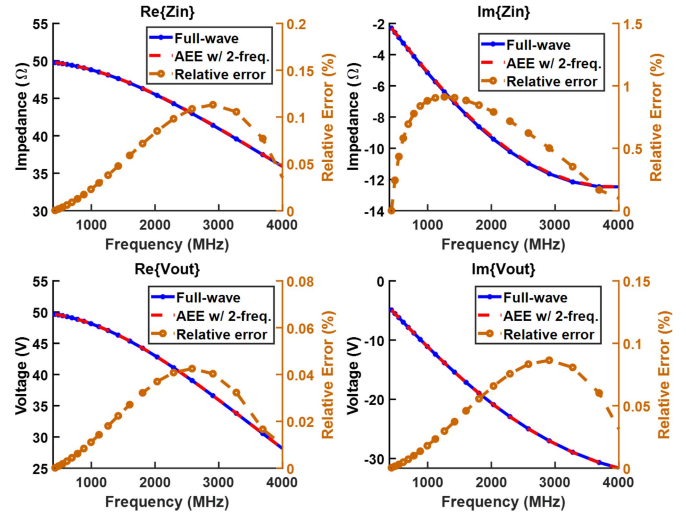


Fig. 6. Quasi-static AEE using two frequencies (400 MHz and 4 GHz) versus the full-wave analysis for test problem 1. Top: real and imaginary parts of the input impedance. Bottom: real and imaginary parts of the output voltage.

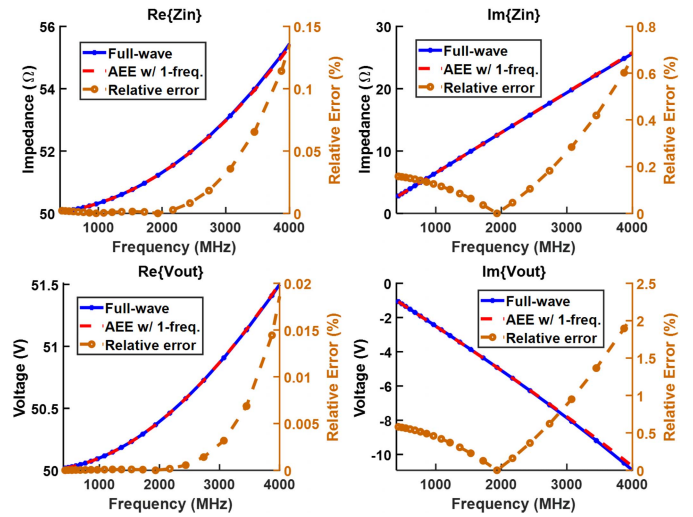


Fig. 7. Quasi-static AEE using a single frequency at 2 GHz versus the full-wave analysis for test problem 2. Top: real and imaginary parts of the input impedance. Bottom: real and imaginary parts of the output voltage.

error gradually increases to 6% at 10 GHz, at which the device's electrical size is about one fifth of the wavelength. The choice of the sampling frequencies is found to be noncritical in the AEE as long as the frequencies are sufficiently apart. For the one-frequency case, the obvious choice is the central frequency. For the case with two sampling frequencies, they can be chosen either at the beginning and end of the frequency band or 25% away from the two ends.

Figs. 7 and 8 show the input impedance and output voltage of the quasi-static AEE versus the full-wave analysis for test problem 2, respectively. As can be seen, the maximum error using the single 2-GHz frequency point is around 2.1%, which is reduced to 0.22% by using two frequency points at 400 MHz and 4 GHz.

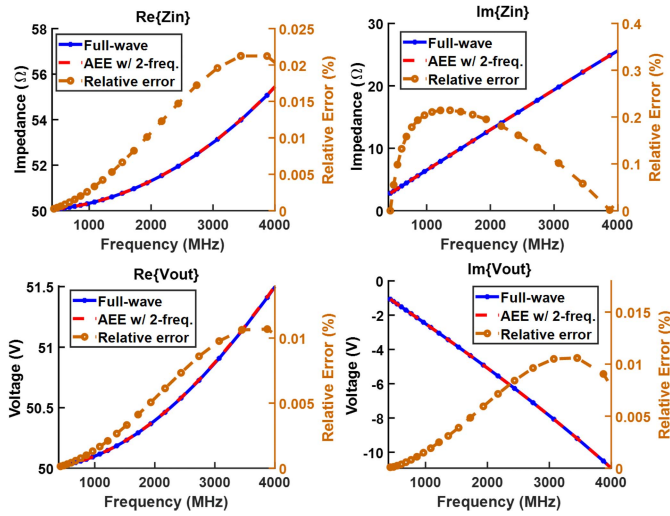


Fig. 8. Quasi-static AEE using two frequencies (400 MHz and 4 GHz) versus the full-wave analysis for test problem 2. Top: real and imaginary parts of the input impedance. Bottom: real and imaginary parts of the output voltage.

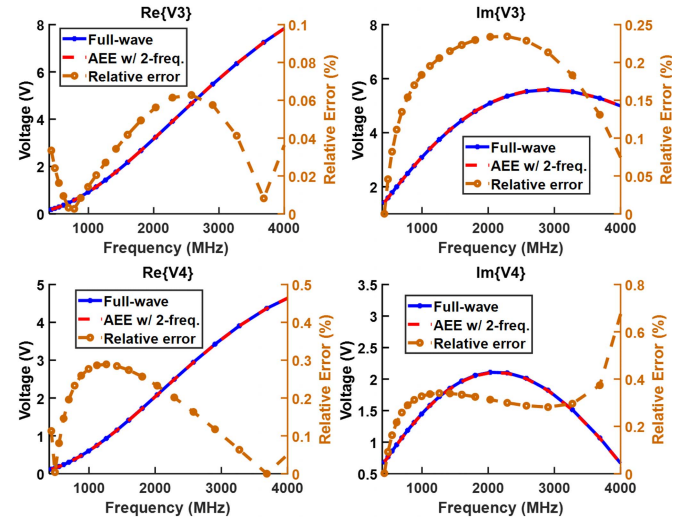


Fig. 10. Quasi-static AEE using two frequencies (400 MHz and 4 GHz) versus the full-wave analysis for test problem 3. Top: voltage at port 3. Bottom: voltage at port 4.

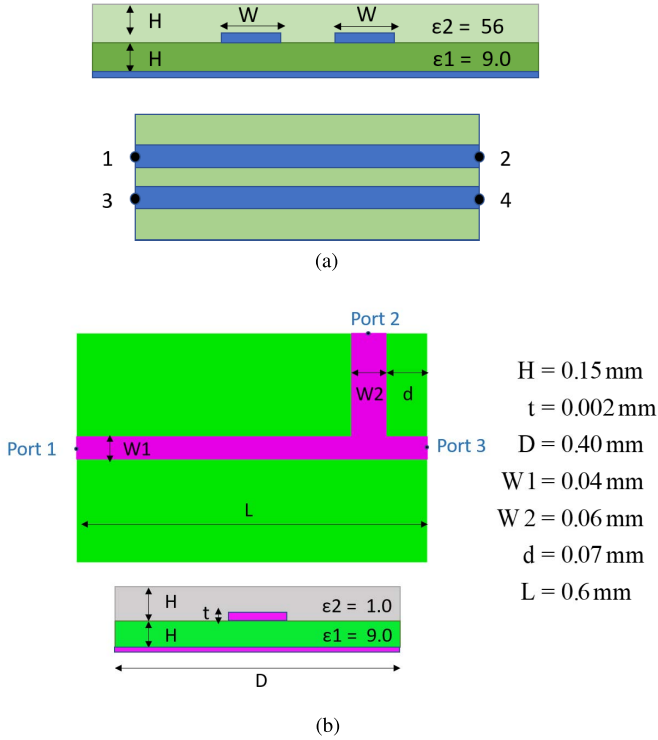


Fig. 9. Test problems for multiport circuits. (a) Test problem 3 ( $W = 0.04$  mm,  $H = 0.15$  mm,  $T = 0.002$  mm,  $S = 0.04$  mm, total width = 0.52 mm, and total length = 1.00 mm). (b) Test problem 4 consisting of one piece of conductor and three ports.

### B. Quasi-Static AEE for Multiport Circuits

Next, we examine the quasi-static AEE for multiport circuits. For this, two test problems are designed. The first is similar to test problem 1, but with two conducting strips separated at a distance of  $S = 0.04$  mm, as shown in Fig. 9(a). With a 1.0-A current source applied at port 1 and a 50- $\Omega$  resistor applied at the other three ports, the voltages observed at ports 3 and 4 due to mutual coupling are plotted in Fig. 10.

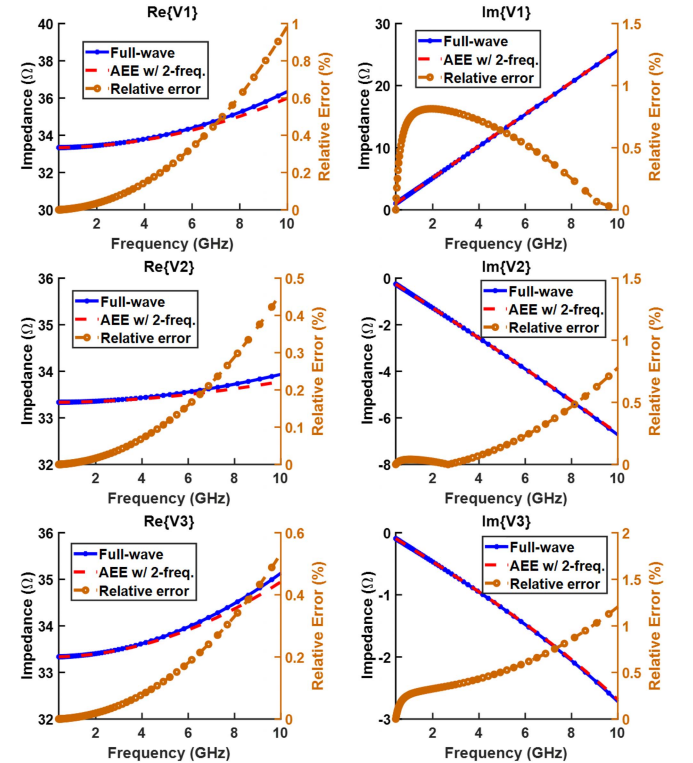


Fig. 11. Quasi-static AEE using two frequencies (400 MHz and 10 GHz) versus the full-wave analysis for test problem 4. From top to bottom: real and imaginary parts of the voltage at ports 1–3.

Two frequencies at 400 MHz and 4 GHz are selected to perform the quasi-static AEE. The agreement between the quasi-static and full-wave results is excellent over the interested frequency range.

The second test problem consists only one piece of conductor but has three ports, as shown in Fig. 9(b). Port 1 is excited by a 1.0-A current source, and ports 2 and 3 are loaded with a 50- and 100- $\Omega$  resistor, respectively. The entire

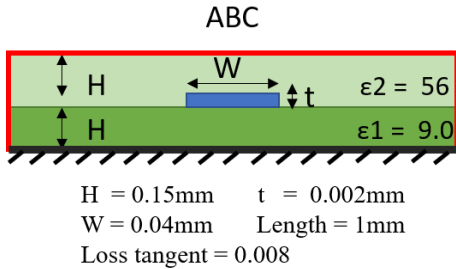


Fig. 12. Test example with a dielectric loss and ABC applied.

device is enclosed in a perfectly conducting box with a dimension of  $0.8\text{ mm} \times 0.6\text{ mm} \times 0.3\text{ mm}$ , and the desired frequency range for the analysis is from 400 MHz to 10 GHz. The voltages at the three ports are compared between the quasi-static and full-wave results in Fig. 11, which again shows an excellent agreement with a maximum error of 1.3% at 10 GHz. The accuracy of the two test problems indicates that the quasi-static AEE can correctly and accurately model the multiport RF circuits.

### C. Quasi-Static AEE With Dielectric Losses and Absorbing Boundary Condition

All the previous examples use the PEC boundary conditions to truncate the computational domain, and the dielectrics are assumed to be lossless. To extend the proposed AEE to handle lossy cases, we now consider a problem where the dielectric is lossy. We still use the configuration in test problem 1 described in Section III-A, except that the dielectrics have a loss tangent of  $\tan \delta = \varepsilon''/\varepsilon' = 0.008$  in both layers. In this case, the real part of the resulting Z-matrix is no longer negligible, and after eigenvalue decomposition, the eigenvalues contain the characteristic resistance  $R_n$ .

Different from the characteristic inductance  $L_n$  and capacitance  $C_n$  whose values remain constant over the frequency band, the resistance  $R_n$  is frequency dependent. However, the behavior of  $R_n$  versus frequency can be determined based on *a priori* knowledge. For dielectric losses, the loss tangent can be represented by an equivalent conductivity  $\sigma_{\text{eq}} = \omega\varepsilon'\tan\delta$ , which is a linear function of frequency. Thus, the resistance varies as  $R_n \sim R_{\text{const}}/f$ . For the radiation loss due to the application of an absorbing boundary condition (ABC) or an impedance boundary condition (IBC) to truncate the computational domain, a simple analysis of the radiated power indicates that the corresponding characteristic resistance  $R_n$  is independent of frequency.

Now, we consider test example 1 with both dielectric loss and ABC applied, as shown in Fig. 12. The values of  $R_n$  for the two modes are shown in Fig. 13. The resistance  $R_n$  in the first mode behaves as  $R_n \sim K_1/f + K_2$ , where the  $K_1/f$  term comes from the dielectric loss and the constant term  $K_2$  is contributed by the ABC. The loss for mode 2 is very small and has a negligible contribution to the total response. By selecting two frequencies at 400 MHz and 4 GHz,  $L_n$  and  $C_n$  can be calculated from the imaginary part of the  $n$ th eigenvalue, and

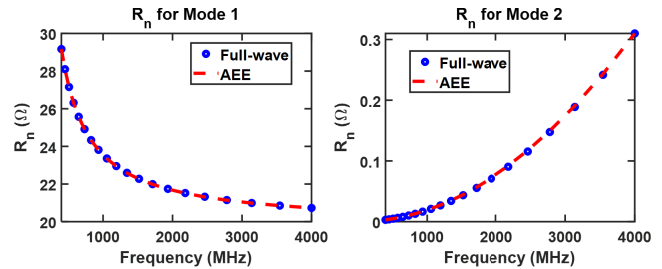


Fig. 13. Characteristic resistance versus frequency for lossy test problem 1 where an ABC is applied and a loss tangent of 0.008 is set in both layers.

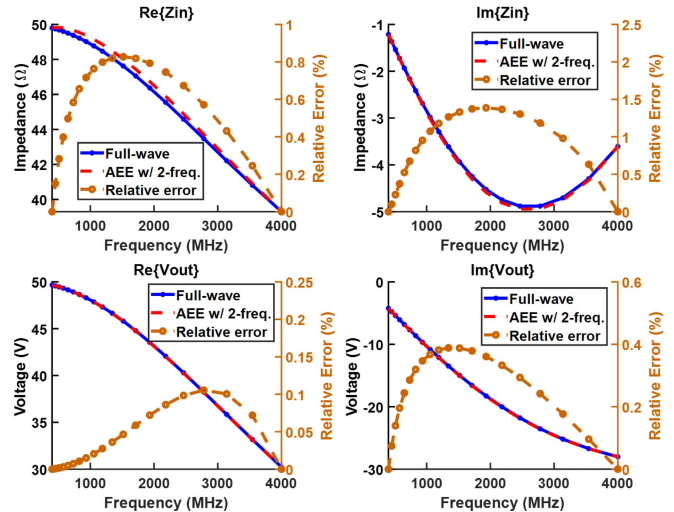


Fig. 14. Quasi-static AEE using two frequencies (400 MHz and 4 GHz) versus the full-wave analysis for the lossy test problem 1 with an ABC and a loss tangent of 0.008. Top: real and imaginary parts of the input voltage. Bottom: real and imaginary parts of the output voltage.

the coefficients  $K_1$  and  $K_2$  for  $R_n$  can be obtained from the real part of the eigenvalue.

Fig. 14 compares the results between the quasi-static AEE and full-wave analysis and the relative error is below 1% except for the imaginary part of the input impedance whose absolute value is small. We note that in the case, there are more than two loss mechanisms and the user has no *a priori* knowledge about these losses; two sampling frequency points are not sufficient to determine the frequency-dependent resistances accurately. In this case, one has to either use three frequencies or model only significant losses. For example, in the absence of the radiation loss for a shielded circuit, one can assume that  $R_n \approx K \cdot f^\beta$  and use the eigenvalues at the two frequencies to determine constants  $K$  and  $\beta$ . This, however, is not a critical issue for practical applications because RF circuits are always designed to minimize undesired losses.

### D. Numerical Modeling of an SAW Filter

In this section, we construct a very simple design of an SAW filter to illustrate the application of the quasi-static AEE. The input and output layouts are chosen to be the same square-shaped, as shown in Fig. 15. As for the SAW part, it consists of two interdigitated transducers (IDTs) with



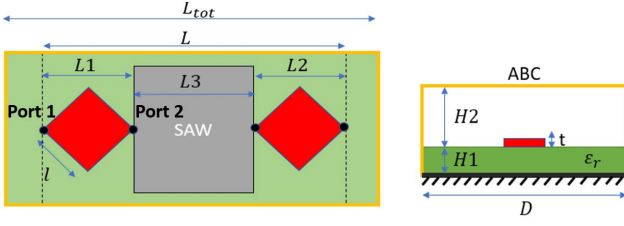


Fig. 15. Simple SAW filter to test the quasi-static and full-wave analyses. (a) Top view of the geometry ( $L1 = L2 = 1.41$  mm,  $L3 = 3.17$  mm,  $l = 1.0$  mm, and  $L_{tot} = 7.6$  mm). (b) Side view ( $H1 = 0.15$  mm,  $H2 = 0.60$  mm,  $D = 5.5$  mm,  $t = 0.008$  mm,  $\epsilon_r = 9.6$ , and  $\tan \delta = 0.008$ ).

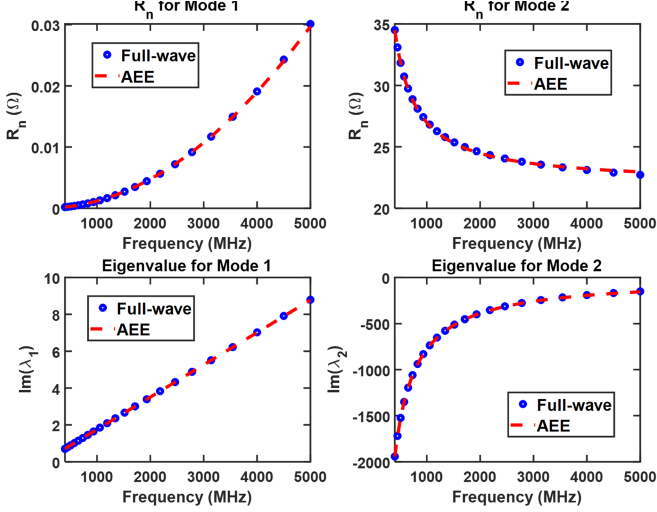


Fig. 16. Real (top) and imaginary (bottom) parts of the eigenvalues of the layout for the SAW filter example.

uniform finger spacing and constant finger overlap, which forms a coupled-resonator filter [41], [42]. Note that we are not concerned with the performance of the filter, and the objective is simply to compare the quasi-static AEE approach with the full-wave analysis. In the quasi-static analysis, we first perform the AEE to approximate the Z-parameters of the input and output layouts. The SAW resonator is modeled as a lumped network characterized by a Y-matrix using the analytical formula in [43]–[45]. By cascading the three Z-matrices, the characteristics of the entire device can be modeled. In the full-wave analysis, the entire filter is modeled using the FEM and the SAW resonator is still modeled as a lumped network and incorporated into the FEM analysis to perform a wave-circuit cosimulation [33]. The substrate material is assumed to be lossy with a dielectric loss tangent of 0.008. The entire structure is enclosed by a truncation boundary where the first-order ABC is applied.

The real and imaginary parts of the eigenvalues for the input and output layouts are plotted in Fig. 16 for the two decomposed eigenmodes. As discussed in Section II, the characteristic resistance  $R_n$  consists of two frequency-dependent terms:  $R_n \sim K_1/f + K_2$ , where the constant comes from the ABC and the term that is inversely proportional to the frequency is introduced by the dielectric loss. After extracting  $R_n$ ,  $L_n$ , and  $C_n$  of the layout from sampling frequencies at

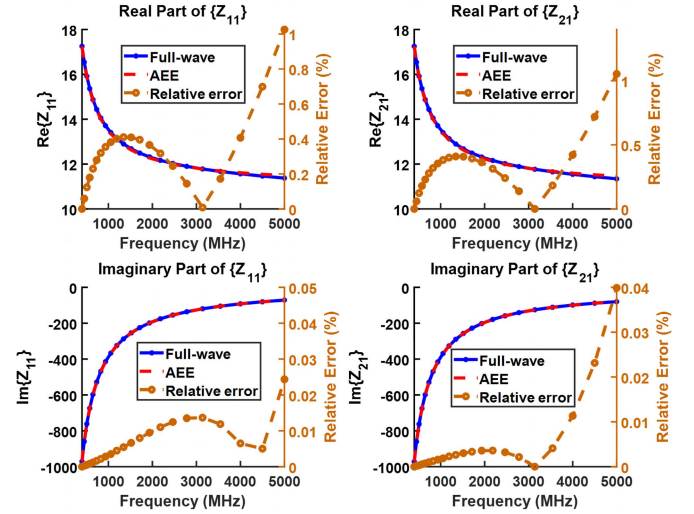


Fig. 17. Z-parameters of the layout for the SAW filter example. Left:  $Z_{11}$ . Right:  $Z_{21}$ . Because of geometrical symmetry,  $Z_{22} = Z_{11}$ , and because of reciprocity,  $Z_{12} = Z_{21}$ .

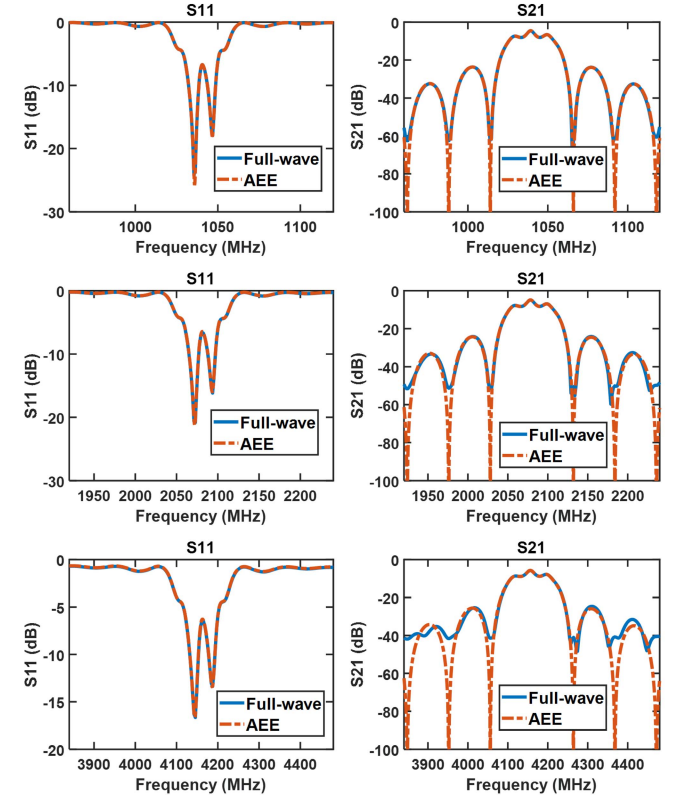


Fig. 18. Quasi-static and full-wave simulated S-parameters centered at three different frequencies for the SAW filter example. From top to bottom, the center frequency is 1040, 2080, and 4160 MHz.

400 and 3140 MHz, the Z-parameters can be approximated over the frequency range of interest, which are given in Fig. 17. By cascading the three segments, the characteristics of the entire device can be calculated, and the comparison with the full-wave analysis centered at three different frequencies is shown in Fig. 18. It can be seen that the agreement between the quasi-static and full-wave results is excellent at the first



three cases, and in the last case, there is a 1-dB discrepancy in  $S_{21}$  around the  $-35$ -dB level, which is due to the wave effect at the high frequency. For these three simulations, the quasi-static AEE is 50 times faster than the direct full-wave analysis because it extracts the Z-parameters of the input and output layouts only at two selected frequencies and the circuit analysis time is negligible, whereas the full-wave analysis has to be carried out at every frequency point and the frequency sampling points have to be dense enough to capture drastic variations in the frequency responses of the filter.

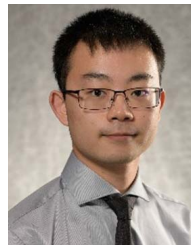
#### IV. CONCLUSION

A new fast frequency sweep method based on AEE has been proposed an efficient analysis of miniature passive RF circuits. In this approach, the Z-parameters are first extracted at one or two sampling frequencies using a full-wave simulation. The eigenvalue decomposition is then performed to diagonalize the impedance matrix. The analytic extension is applied to extend the eigenvalues from the sampling frequencies to all other frequencies via a functional equation. The Z-parameters or Y-parameters can finally be evaluated from their expansion using eigenvectors. For miniature RF circuits, the characteristic capacitance and inductance in each eigenvalue and the corresponding eigenvector are frequency independent, whereas the frequency dependence in the characteristic resistance can be predetermined. The accuracy of this quasi-static AEE has been investigated by comparing its solution with that of the full-wave analysis based on the solution of Maxwell's equations. In the numerical examples, a comparison between the one- and two-frequency approximations was first presented. It has been shown that for some applications where the device is electrically very small, one-frequency approximation can provide acceptable accuracy. A second frequency can be added to further improve accuracy. The modeling of losses introduced by dielectric losses and ABCs has also been discussed, and the frequency behaviors of the corresponding characteristic resistances have been studied. It has been found that the quasi-static AEE is very accurate (with a relative error less than 1%) within the quasi-static range where the circuit components are smaller than one tenth of the effective wavelength and it remains reasonably accurate with a relative error less than 6% when the sizes of the circuit components increase to one fifth of the effective wavelength. In conclusion, the proposed AEE provides a fast frequency sweep for the analysis of RF circuits, and it yields both a mathematical expansion of the Z- or Y-parameters and a physical LEC model. The method can be further extended to high frequencies by considering the frequency dependence of the modal capacitances and inductances and eigenvectors and thus removing its limitation to quasi-static applications.

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