Classics in Magnetics

On the Magnetic Aftereffect

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1) Discussion of the past experience regarding the time dependence of magnetization. Hypothesis of a formal analogy between the Jordan loss component and the dielectric aftereffect loss. 2) Fundamental experiments that examine this hypothesis based on the classical theory. 3) Measurement of the frequency response of the permeability. Order-of-magnitude confirmation of the theory. 4) Execution of the switching experiment using a ribbon and solid core of a Fe–Ni alloy. Invalidity of the superposition principle. 5) Interpretation of experimental results.

I. Introduction

A FREQUENTLY asked question is whether the magnetization of ferromagnetic materials follows the field strength instantaneously or whether a finite time is necessary for the emergence of a certain magnetization. In search for an answer, numerous different experiments were employed without a final solution of the problem.

In one experimental approach, the magnetization evolution following the sudden application of a dc field is recorded ("switching experiment"). The other important experimental method is the observation of the magnetic behavior as a function of the frequency of an alternating field ("alternating current experiment"). With both methods, the reaction of the eddy currents must be considered in order to answer the physically interesting question of the delay of the magnetization relative to the field actually present at a certain location.

A. High-Frequency Limit

The consideration of eddy current effects is most comfortably possible for the investigation of the inductance and the energy loss of iron wires at high frequency. Such investigations revealed[1] that with small field strengths, the ferromagnetic permeability disappears at a frequency $\sim 10^{10}$ Hz. It is not yet decided whether this disappearance of the ferromagnetic behavior is bound to a critical *time* or to a critical length (*penetration depth*). (Due to eddy currents, only a thin layer of $\sim 10^{-5}$ cm switches magnetization for the samples in question.)

From these experiments, one can conclude that the bulk of the reversible magnetic processes will occur virtually instantaneously for observations with time constants much larger than 10^{-10} s.

B. Switching Experiments

However, a number of experimental processes indicate that with the switching scheme, delay features are observed that cannot be explained by eddy current effects in a simple way. In particular, this already follows from the classical observations of Ewing who observed a small but measurable increase of the magnetization after 60 min by the use of a magnetometer; although computationally, the time constant of the eddy current losses is of the order of magnitude of 0.02 s [4].

From the data concerning "switching experiments" in the literature, one can generally conclude that the observed delay of the *largest part* of the change of induction can be attributed to the retarded field evolution due to *eddy currents. Only a small fraction of the magnetization is afflicted with an "aftereffect."*

In particular, using a rough calculation, Bozorth [5] showed that the experimental results of Lapp [6]—who investigated the temporal emergence of the induction of iron at different places of the hysteresis loop in great detail—can be satisfactorily explained by eddy current effects. Something similar should apply to most works concerned with the gradual emergence of the induction in switching experiments (for example, [7]). Due to the curvature of the magnetization curve, the situation is more complicated when switching in strong fields. However, the computational estimate usually shows that only a small fraction of the change of induction exhibits time delays that are too long to be attributed to the effect of eddy currents in a simple way.

We note that there is another kind of magnetic delay that can occur at certain temperatures, which is, however, not related to the physical question asked here. Some magnetic materials change their hysteresis loops due to a treatment in a magnetic field at temperatures above 400° due to the relaxation of internal material stresses of magnetostrictive origin [8]. The

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[1]For newer measurements with undamped waves, see Hoag and Jones [1] and Sänger [2]. See also Arkadiew [3].

temporal change of the magnetizability as a consequence of the effect of the measurement field then mimics a retarded induction. This feature was observed by Kühlewein [9] and interpreted also in a similar sense. We would like to exclude these features from the concept of the "magnetic aftereffect."

Careful alternating current experiments at intermediate frequencies also indicate the presence of a genuine aftereffect which becomes effective already far below the mentioned high-frequency limit ($\sim 10^{10}$ Hz). Through approximate consideration of the eddy current field, it was shown [11] that the hysteresis loop measured with alternating current has a larger area than the ballistically determined loop.

C. Jordan Aftereffect Loss

These experimental facts might have supported Jordan [12] in deriving his hypothesis of a magnetic aftereffect for the interpretation of an observation of precise loss measurements on iron-based inductors with alternating current. The phase shift ε between the fundamental frequencies of the magnetizing current and the magnetic induction (and thus the loss angle of the measured inductance) according to Jordan can be represented, for small field strengths and small frequencies, empirically as

$$\varepsilon = \varepsilon_n + \varepsilon_h \cdot \frac{H}{H_0} + \varepsilon_w \cdot \frac{\omega}{\omega_0} \tag{1a}$$

$$= \frac{n}{5000} + \frac{h}{5000} \cdot H + \frac{w}{5000} \cdot \frac{\omega}{5000}. \tag{1b}$$

The term ascending with the angular frequency ω is quantitatively explained by eddy current losses and can easily be computed for sheet metals and wires. The frequency-independent terms could be attributed to hysteresis losses in principle. Now, the classical investigations of Rayleigh regarding the hysteresis at small field strengths showed that the loops can be represented by similar and similarly situated parabolic curves. This implies that the loss of energy has to increase with the third, and the phase shift thus with the first, power of the field strength. According to Jordan, therefore, only the second term of (1) would be interpreted as "hysteresis," whereas the first term, which represents a frequency- and field strength-independent phase angle, would be attributed to a magnetic aftereffect.

Jordan suggested that the interpretation of such a frequency-independent phase shift is a consequence of a temporal delay of the induction in analogy to findings from the elastic and dielectric aftereffects. These findings showed that such a phase shift, which is based on internal delays, is frequently found to be constant over a large range of frequencies, even though, in principle, a reduction for extremely low and extremely high frequencies must be expected.

The explanation of the *dielectric* aftereffect is based on the spatial distribution of electrically charged particles in the dielectric. To understand the *elastic* aftereffect, the existence of rigid regions that can flow in an elastic environment is assumed. The fact that there is no magnetic analog to electrical conductivity or to plastic flow was also a reason that the

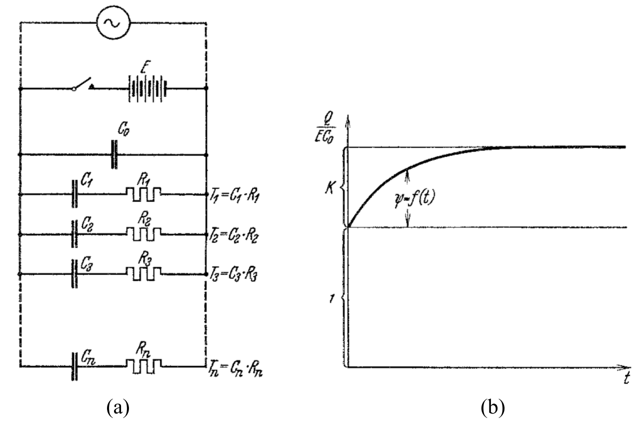


Fig. 1. (a) Equivalent-circuit scheme for a capacitor with dielectric aftereffect. (b) Delay function.

acceptance of the analogy in the magnetic case in the sense of Jordan was doubted[2].

In particular, a doubt about the correctness of this interpretation was justified as long as a simultaneous quasi-static determination of the hysteresis loops had not been accomplished with sufficiently low amplitudes as a check of the Rayleigh law. Recently, the sensitive ballistic measurements of Wittke [15] and Ellwood [16] showed the approximate correctness of the Rayleigh law in the sense that the Jordan loss energy prevailing at small field strengths is not found in ballistic measurements. In addition, further findings regarding the "refresh effect" (see below) as well as the absence of the harmonics at small amplitudes support the hypothesis of Jordan (see Neumann [17]).

Thus, it seems appropriate to accept the formal analogy to the elastic and dielectric aftereffects as a working hypothesis in order to verify the validity of this assumption via ensuing further experiments.

II. Theory and Fundamental Experiments of the Dielectric and the Elastic Aftereffect

Let us briefly discuss the basics of the classical theories of the dielectric and elastic aftereffect with respect to the most important verification experiments in the spirit of the works of Wiechert [18] and Becker [19] (elastic aftereffect) and Wagner [20] (dielectric aftereffect). The description is possible in a simple way by example of the dielectric aftereffect, because it can be treated in a simple equivalent-circuit scheme [Fig. 1(a)].

It is assumed that only a small fraction K of the material exhibits aftereffect features. With the switching experiment, the largest part of the charge follows the applied potential instantaneously; only the small fraction K is delayed. The delay function $\psi(t)$ shown in Fig. 1(b) can be generally thought of as a superposition of exponential functions with different time constants (the aftereffect part K is drawn exaggeratedly large). In the equivalent-circuit scheme in Fig. 1(a), this is represented by a number of parallel current circuit branches consisting of the series connection of different resistors and capacitors, all of which are in parallel with the main

[2]See the discussion remarks in [13] and also the contribution of Gans [14].

capacitor C_0. The apparent capacity of such a network for alternating current must monotonically decrease with increasing frequency from $C_0(1+K)$ to C_0. At the same time, the phase shift (the apparent loss angle) exhibits a maximum at the frequency whose reciprocal is some average time constant of the parallel circuits.

For suitably chosen parallel circuits in Fig. 1(a), all aftereffect procedures observed through switching and alternating current experiments in practice can be represented. Wagner, specifically via the assumption of a Gaussian distribution around a most probable time constant T_0, could attribute the empirically known switching curves to three constants (the total amount of the aftereffect fraction K, the most frequent time constant T_0, and the measure for the sharpness of the grouping b).

There is now a very specific simple type of distribution of the time constants, which is often fulfilled, to a large extent, for the dielectric and, in particular, for the elastic aftereffect. It results in a logarithmic evolution of the response function ("load function") in switching experiments and, in the case of the alternating current experiments, a frequency-independent phase shift over a large range of times or frequencies. This limiting case is apparently of interest, as it represents an analogy to the findings of Jordan with regard to the behavior of magnetic materials with alternating currents. For this limiting case, the circuit in Fig. 1(a) is simplified by the condition of the equality of all capacitances

$$C_1 = C_2 = C_3 = \cdots = C_n = C, \quad \frac{C}{C_0} = \beta.$$

Furthermore, the time constants of the parallel circuits should form a geometric series

$$T_K = T_{K-1} \cdot e.$$

This corresponds to a limiting case of the Wagner representation ($b = 0$), for which a uniform distribution of the logarithms of the time constants is assumed over a large range of times.

Following Becker [19], we find a simple mathematical description by assuming that the parallel circuits for small and large time constants are limited by a minimum or maximum time constant $T_{\min}$ or $T_{\max}$, respectively.

It follows that:

$$\frac{T_{\max}}{T_{\min}} = e^n$$

and

$$K = \frac{n \cdot C}{C_0} = \beta \ln \frac{T_{\max}}{T_{\min}}. \tag{2}$$

The calculation results in the response function ("load function")[3]

$$\psi(t) = K + \beta \left\{ Ei\left(\frac{-t}{T_{\max}}\right) - Ei\left(\frac{-t}{T_{\min}}\right) \right\}. \tag{3}$$

The transcendental "*Ei* function" serves only for the description of the transients for $t \approx T_{\min}$ and $t \approx T_{\max}$ and is not usually experimentally controllable. In the intermediate time region, $t \gg T_{\min}$ and $t \ll T_{\max}$

$$\psi(t) = \text{const} + \beta \ln t \tag{3a}$$

is valid. The solution for alternating current results in the phase shift ε :

$$\text{tg } \varepsilon = \beta \left[\frac{\pi}{2} - \text{arctg} \frac{1}{\omega T_{\max}} - \text{arctg } \omega T_{\min} \right]. \tag{4}$$

For the intermediate frequency range, $\omega \cdot T_{\max} \gg 1$ and $\omega \cdot T_{\min} \ll 1$ particularly

$$\text{tg } \varepsilon = \beta \frac{\pi}{2}. \tag{4a}$$

The frequency dependence of the capacity is represented by

$$C = C_0 \left[1 + \frac{\beta}{2} \right] \ln \left[\left(\frac{T_{\max}}{T_{\min}} \right)^2 \frac{1 + (\omega T_{\min})^2}{1 + (\omega T_{\max})^2} \right]. \tag{5}$$

In the intermediate frequency range

$$\frac{\Delta C}{C} = \text{const} - \beta \ln \omega. \tag{5a}$$

If the aftereffect obeys the model stated here, three different observations can be traced back to the same constant β according to (3a), (5), and (6): 1) the temporal delay in the switching experiment; 2) the magnitude of the phase shift; and 3) the change of the capacitance (or the inductance for magnetic materials) with frequency for the alternating current experiment.

A direct examination of the superposition principle, which is fundamental for the entire theory, is possible by an "ON/OFF switching experiment." If switched ON during time τ, then the subsequent observable temporal change can be represented as

$$\psi_{EA}(t) = \beta \ln \left(1 + \frac{\tau}{t} \right). \tag{6}$$

This experiment yields a stricter and more conclusive examination of the foundations of the theory than a comparison of the constant β inferred from the different observations. In the case of the magnetic aftereffect, so far only Tobusch [21] was able to observe a temporal evolution, which qualitatively corresponds to the Wiechert theory. However, he does not give absolute values. A comparison with the results from the alternating current experiment—since then frequently conducted—is not even possible within an order of magnitude.

In one case, Hermann [22] observed a frequency dependence of the inductance at extremely low frequencies with a special kind of iron. At the same time, however, an abnormally large loss angle arose in a narrow frequency range. Thus, the characteristic frequency-independent loss angle of Jordan was not present here.

As far as I know, the simultaneous determination of the Jordan loss angle ε_n, either from exact inductance measurements with alternating current or from switching experiments, and the fundamental value β of the aftereffect theory by another way has not been attempted. Some experiments regarding this determination, which can have no claim to completeness,[4] are here communicated.

[3]In R. Becker [19], it is $T_{\max} = 1/r$, $T_{\min} = 1/R$. Equations (3)–(5) are, hence, more general than those explicitly communicated in the paper by R. Becker as they also apply for long times, i.e., small frequencies.

[4]The author concluded the experiments prematurely for external reasons.

III. Measurement of the Frequency Dependence of the Inductance Due to the Magnetic Aftereffect

The frequency response of the inductance of iron-loaded coils is usually caused by eddy current flux displacement and the self-inductance that is always parallel to the coil capacity in practice.

The effect of the eddy currents, which is a reduction of the inductance, can be characterized by the limiting frequency f_g defined by Wolman [23] and the effect of the dielectric charging currents, which causes an apparent increase of the inductance due to the resonant frequency f_r of the coil. It then follows (for $f \ll f_g$ and $f \ll f_r$):

$$\frac{L_f}{L_0} = 1 - \frac{8}{15}\left(\frac{f}{f_g}\right)^2 + \left(\frac{f}{f_r}\right)^2 = 1 + \left(\frac{f}{f_r}\right)^2\left[1 - \frac{8}{15}\left(\frac{f_r}{f_g}\right)^2\right]. \tag{7}$$

The effect of the eddy currents can usually be neglected, since $f_r \ll f_g$ for coils that are used for precision measurements in the audio frequency range.

Then, with the simultaneous presence of an aftereffect according to (6) and (7), it is to be expected that

$$\frac{L_f}{L_0} = \text{const} - \beta\, \ln f + \left(\frac{f}{f_r}\right)^2. \tag{8}$$

For a graphic evaluation, this function is more appropriately represented as

$$\frac{f}{L_0}\frac{dL_f}{df} = -\beta + 2\left(\frac{f}{f_r}\right)^2. \tag{8a}$$

It immediately follows that the frequency-dependent inductance must possess a minimum at:

$$f_{\min} = f_r\sqrt{\frac{\beta}{2}} = f_r\sqrt{\frac{\operatorname{tg}\varepsilon_n}{\pi}}.$$

For the usual low-loss magnetic cores, ε_n is of the order of magnitude $2 - 20 \times 10^{-4}$. The minimum of inductance is thus expected to be at $1 - 2\%$ of the resonant frequency. From this, and from the expected small magnitude of the effect, the necessary experimental strategy for its detection follows.

The fundamental difficulty for the execution of the experiments lies in the current dependence of the inductance. The experimentally necessary measuring currents affect the inductance much more strongly than the variation of the measuring frequency.

After careful extrapolation to zero measurement current, the inductance curve of Fig. 2(a) resulted for a bulk core from an iron–nickel alloy. The representation as a straight line according to (8a) is shown in Fig. 2(b). We find $\beta_L = 0.9 \times 10^{-3}$. The simultaneous determination of the loss angle yielded $\varepsilon_n = 2.4 \times 10^{-3}$, and thus, according to (5), $\beta_\varepsilon = 1.5 \times 10^{-3}$. The experiment, therefore, yields the theoretically expected order of magnitude for the change of inductance. A more exact numerical agreement could not be found even for further measurements of ring and bulk cores. Whether the deviation can be explained by insufficient measurement accuracy would have to be verified by further refinement of the alternating current methods at lower frequencies. The described measurement result certainly supports—even if only qualitatively—the interpretation of the Jordan loss as a temporal aftereffect.

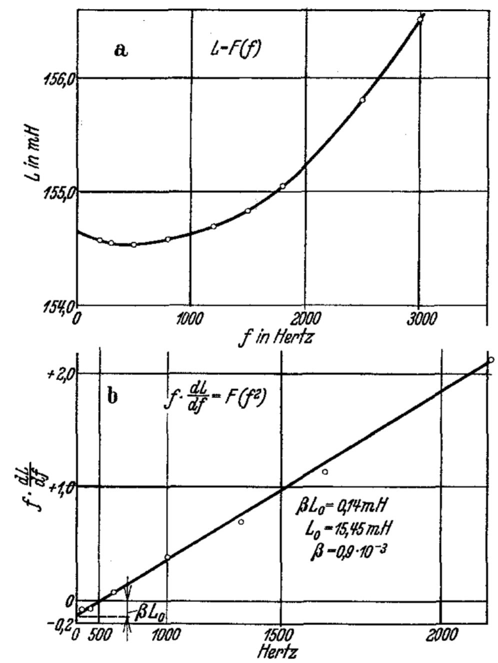


Fig. 2. (a) Frequency response of the inductance of a bulk core extrapolated to current = 0. (b) Decomposition in aftereffect contribution and contribution of the charging currents of the coil capacity.

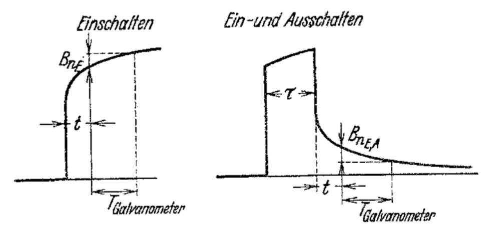


Fig. 3. Schematic of the switching experiments.

IV. Execution of the Switching Experiment

The temporal emergence of the induction can be measured ballistically by connecting the ballistic galvanometer to the secondary circuit only after a certain time t. Due to the finite oscillation time of the galvanometer (the excursion time amounted to ≈10 s), one does not measure the entire part of the induction after time t, but only a fraction, which evolves within a time on the order of magnitude of the excursion time. The experimentally determined aftereffect curves thus mean $B_{n_E} = B_{T_G} - B_t$ (see Fig. 3), if T_G is a time (≈ oscillation duration) characteristic of the galvanometer. Since, however, as the figures show, all the observed curves are logarithmic, the only observable quantity [according to (3a)] will still be determined correctly.

To achieve sufficient sensitivity, toroidal coils with a large iron volume were used and a large number of secondary turns were attached. (For the alternating current attempt, the windings had to be dimensioned differently.) To generate different switching times, a camshaft with six cams was driven by an engine with a reduction gear with a large variation range for the speed of rotation. For the *switching-ON* experiment, the time t (see Fig. 3) is determined by the angle between the cam that operates the relay for the magnetizing current and the cam that operates the relay of the galvanometer circuit. The ON-time τ of the "*ON/OFF switching experiment*" is due to the width of the cam that closes the magnetizing electric circuit. The calibration of the times was accomplished with the help of the oscillograph and checked occasionally. (Because the response times of the relays were separately measured, calibration at a single engine speed was usually sufficient.) With a continuously running camshaft, the action of the other circuit elements was made possible by switching ON a separate switch for each test point.

A. Sources of Error

By the use of a large number of secondary turns, the fundamental frequency of the secondary coil is relatively small due to the coil capacity. The consequence is a transient effect in the capacitively loaded secondary circuit when switching. This effect has to be sufficiently decayed at the observation time t, so that it does not mask the small aftereffects. The smallest decay time was achieved by aperiodic damping of the secondary circuit with the help of a parallel resistor (of several hundred thousand ohms).

The lower limit of the experimentally accessible observation times t was due to the mentioned transients. (The time constant of the primary circuit could always be kept sufficiently small.) Errors were easy to avoid, since the spurious excursion depended exponentially on time in contrast to the logarithmic character of the aftereffect. The error due to a transient effect could be easily controlled experimentally by the reduction of the secondary resistance, whereby the time constant of the secondary circuit is artificially increased. The resonance frequency varied between 200 and 1200 Hz for the different devices.

The characteristic time delay for the eddy currents for the examined cores was several orders of magnitude smaller and could not at all become effective in this observation (relevant eddy current time constant $\approx 10^{-6}$, see [24]). We here discuss the results obtained from an alloy of 40% Ni and 60% Fe in the form of a ring core and a bulk core.

B. Measurements Using the Ring Core

The ring core was wound from a ribbon of 0.035 mm thick and 15 mm wide. The cross section was 2.8 cm^2. The material was annealed in the rolled up state and then carefully wrapped in paper insulation. This prevented the influences of eddy currents to a large extent. The magnetic measurements on the finished core yielded: initial permeability $\mu_0 = 1000$, coercivity $H_c = 1.5$ Oe, and remanence = 7000 G. The alternating current loss values according to the Jordan decomposition [see (1b)] were $h = 5200$, $w = 6$ (corresponds to the Wolman critical frequency $f_g = 0.45 \times 10^6$), and $n = 16$ (i.e., $\varepsilon_n = 3.2 \times 10^{-3}$ and $\beta_\sim = 2 \times 10^{-3}$). The field strength,

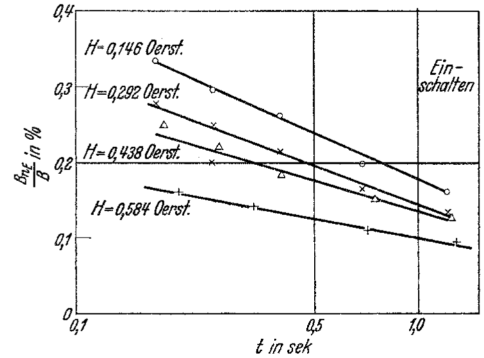


Fig. 4. Switching-ON experiment for small field strengths. Iron–nickel tape-wound core.

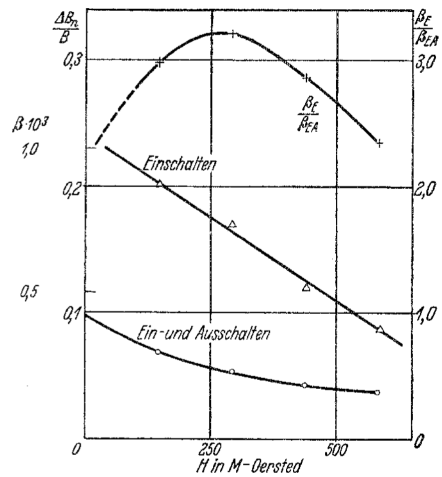


Fig. 5. Field strength dependence of the curve slope from Figs. 4 and/ or 7 for switching-ON, and ON and OFF experiments (tape-wound core).

for which the eddy current losses equal the aftereffect losses, amounts to only $H_{\text{ch}} = 0.005$ Oe.

The *switching-ON experiment* was carried out in such a way that the field was switched periodically 1 min to $+H$ then after a 1 min break to $-H$. Observation was done with both current polarities, and the average value was computed. For experiments with small field strengths, the secondary winding contained 80 000 turns with a resonance frequency of 200 Hz. For the experiments with large field strengths, only 8000 turns were used, and the resonance frequency increased to 1200 Hz.

For the smallest employed field strengths, the results are shown in Fig. 4. For sensitivity reasons, it was not possible to reduce the field strength to the order of magnitude of the characteristic field strength of $H_{\text{ch}} = 0.005$ Oe. The fact that this was possible for the observations of the bulk core (see below; there the measurements ranged from $H = H_{\text{ch}}$ to $H = 20H_{\text{ch}}$) without remarkable amplitude dependence of the results justified the assumption that an extrapolation to still smaller field strengths is permissible also here.

The aftereffect induction B_{n_E} relative to the switching-ON induction is shown in Fig. 4 as the ordinate (the value of the switching-ON induction was determined in a separate measurement). The measurement points consistently fall on straight

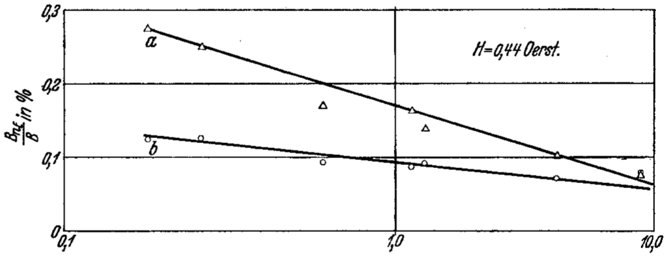


Fig. 6. Curve a: switching-ON experiment, for alternating magnetization current polarity. Curve b: switching-ON experiment for single-sided excitation without polarity reversal (tape-wound core).

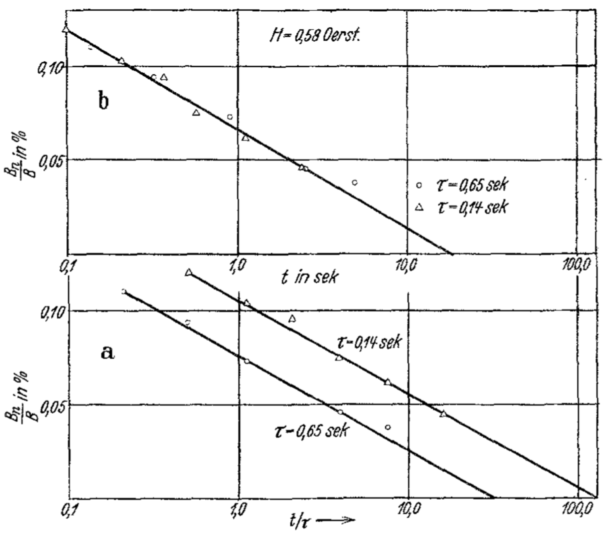


Fig. 7. ON and OFF switching of the tape-wound core. (a) Representation as a function of the ratio of observation time to switch-ON time. (b) Representation as a function of the observation time t.

lines when using a logarithmic abscissa scale. In Fig. 5, the dependence of the slope of these lines on the field strength is plotted. In addition to the easily extracted parameter

$$\frac{\Delta B_n}{B} = \frac{B_{n_{10t}} - B_{n_t}}{B_{\mathrm{Ein}}}$$

(percent change with tenfold increase in time); the β-scale is indicated, that is, the percent change with e-times increase in the observation time. The obtained β_E value is 1×10^{-3} and *thus amounts to half of the* $\beta_\sim$ *value determined from the alternating current experiment.*

We again obtain—as from the change of inductance—the order of magnitude expected from the aftereffect theory, but no numerical agreement. In Fig. 6, curve b shows a case deviating from the usual test method (curve a) for which the current was always switched ON with identical polarity and not—as usual—reversed polarity between two observations. (Naturally, the associated induction reference value was determined separately.) The slope of the curve is nearly halved.

The switching-ON experiments yielded the same results if the ON and OFF times (which were always large compared with time t) were changed within wide limits.

The *ON and OFF switching* yielded a surprising result, which is not to be classified in the context of the superposition theory

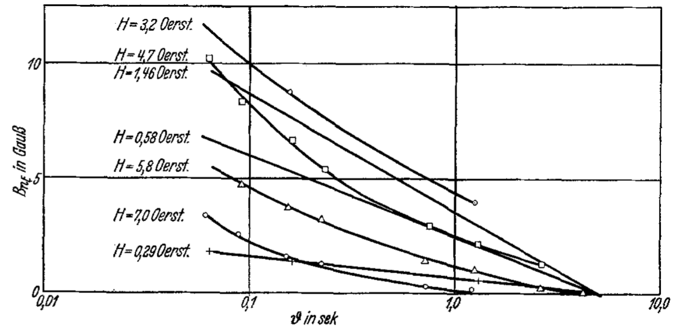


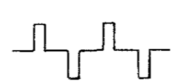
Fig. 8. Switching-ON experiment of the tape-wound core with large field strengths.

of the aftereffect. In Fig. 7(a), the aftereffect function $B_{n_{EA}}/B$ is plotted as a function of the relationship t/τ (see Fig. 3). From (6), a hyperbolic decrease would have to arise according to the theory for $t/\tau > 1$, and the curves with different τ values should coincide. In contrast to this, the parallel straight lines are observed. If one, however, plots as a function of the observation time t [Fig. 7(b)], then the data points for different switch-ON times τ are strictly on one and the same straight line.

Decay of the induction after switching OFF is, therefore, independent of the switch-ON time. This empirical result from the ON and OFF switching can, thus, be characterized by *a single* numerical constant, namely, by the slope of the logarithmic straight lines. (The intersection of our experimental straight lines with the x-axis is not physically interesting; it is given by the oscillation duration of the ballistic galvanometer.) The amplitude dependence of these constants, β_{EA}, is shown in Fig. 5, analogous to the result of the switching-ON experiment. The ratio of the two slopes, β_E/β_{EA}, is also shown in Fig. 5. Extrapolation for small field strengths is not completely certain. The ratio seems to approach the number 2.

In agreement with the mentioned findings for the switching-ON experiment (curves a and b in Fig. 6), this result can be formulated as follows. With a current–time curve of the form

the aftereffect curves are identical for ON and OFF switching and independent of the switching time. With a current–time curve of the form



the independence from the switching time is also fulfilled, but the parameter β of the logarithmic decay curve is twice as large for ON switching as for OFF switching. Whether this finding is still correct for extrapolation to amplitudes smaller than H_{ch} could not be verified experimentally.

The extension of the switching-ON experiment to large field strengths is shown in Fig. 8. Here, the absolute value of the aftereffect induction is plotted as the ordinate. For field strengths larger than the coercivity, a curvature of the straight lines sets in. The aftereffect decreases practically to zero for

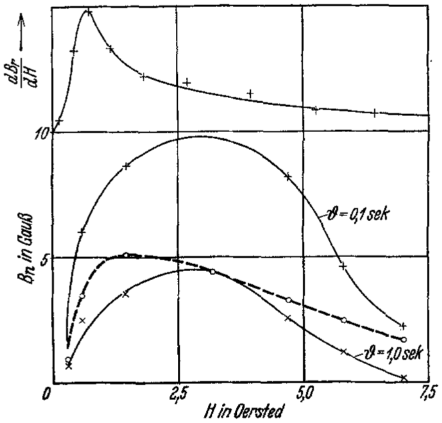


Fig. 9. Representation of the result of Fig. 8 as a function of the field strength.

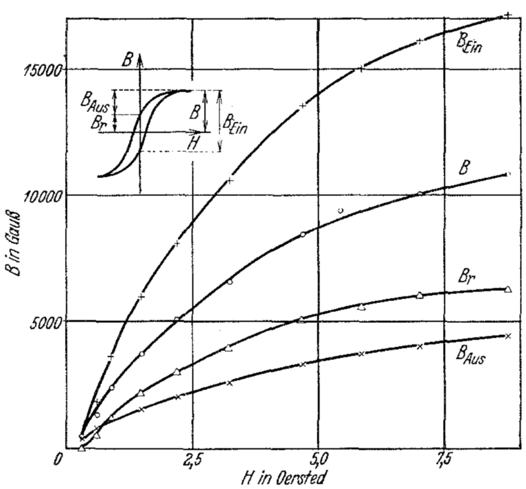


Fig. 10. Field strength dependence of different induction values that are of interest for the switching experiment.

$H > 7$ Oe. In Fig. 9, the value of the aftereffect induction is shown for 0.1 and 1 s. In addition, the slope of the curves for the time 1 s is shown in dashed line. It is remarkable that the aftereffect curves decay at higher field strengths than the curve dB_r/dH (B_r is the remanence of the loop found for sweeping up to field strength H in each case) shown for comparison. Furthermore, Fig. 10 shows the usual ballistic measurements for identification of the magnetic characteristics of the core.

C. The Bulk Core

From the powder of the same alloy, 40% Ni and 60% Fe, a bulk core was pressed. The apparent permeability drops strongly due to the isolation of the individual metal grains. We measured $\mu_0 = 56$, $h = 46$, and $w = 1.6$. The aftereffect parameters were $n = 9.6$, $\varepsilon_n = 1.9 \times 10^{-3}$, and $\beta_\sim = 1.2 \times 10^{-3}$. The field strength, for which hysteresis and aftereffect losses become identical, thus computes to $H_{\text{ch.}} = 0.3$ Oe. The cross section of the core was 9 cm^2; 30 000 secondary turns were used.

The measurements between 0.2 and 6 Oe were in qualitative agreement and also yielded similar quantitative results in the representation of Fig. 11. With the ON- and OFF-switching experiment, a noticeable curvature is determined compared with the tape-wound core. The independence of τ in the representation as $f(t/\tau)$ demanded by the superposition theory is also not fulfilled here, although the switching-ON curve [Fig. 11(b)] is clearly logarithmic. Toward long times, the curves are parallel straight lines similar to the case of the tape-wound core. At least, the deviation from the behavior of the tape-wound core is very distinct in the representation of the curves [as shown in Fig. 11(b)] as a function of t. While the curves of the tape-wound core coincide, we here obtain a deviation as expected from the superposition theory. For large times, however, the curves seem to coincide here also.

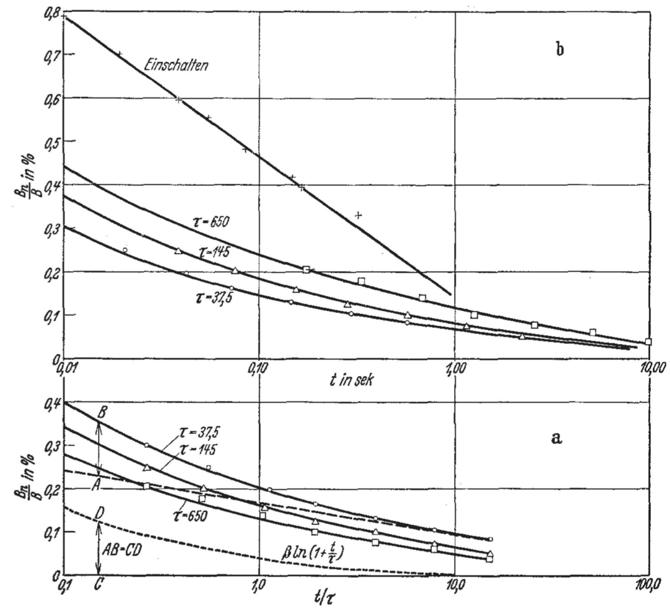


Fig. 11. Switching experiments with the bulk core. (a) ON and OFF switching as a function of t/τ. (b) ON switching, and ON and OFF switching as the functions of t.

It is possible to split the curves into a part that corresponds to the classical superposition theory and a part that corresponds to the findings for the tape-wound core. The second part (drawn dashed for $\tau = 37.5$) results in a straight line in our representation; the remainder (drawn dotted) can be shown to follow the function $\beta \ln(1 + t/\tau)$ [according to (6)].

From the value $\beta_E = 10.2 \times 10^{-3}$, the part $\beta_S = 3.9 \times 10^{-3}$ is found to be in accordance with superposition. The remainder $\beta_m = \beta_E - \beta_S = 6.3 \times 10^{-3}$ would be the characteristic magnetic aftereffect. The logarithmically evolving part of the ON- and OFF-switching experiment yields $\beta_{EA} = 3.1 \times 10^{-3}$ and, thus, half of β_m, analogous to the findings with the tape-wound core. The obvious explanation is that the aftereffect of the bulk core has two different origins. One is magnetic in nature and obeys the same laws as the aftereffect observed with the tape-wound core. The other one is an additional feature and obeys the superposition principle. Perhaps, it is caused by an elastic aftereffect of the isolation material and becomes apparent in the magnetic observation via the magnetoelastic coupling.

V. Interpretation of the Switching Experiment Carried Out With the Tape-Wound Core

A. Deviation From the Superposition Theory

From the comparison of the switching-ON experiment with the alternating current experiment, whereby an agreement of the β-values within a factor 2 was reached, the classical aftereffect theory seemed to be approximately correct. The peculiar results of the ON- and OFF-switching experiments cannot be explained, however, within the picture that is used in the aftereffect theory: the hypothesis of superposability has to be given up. Contrary to the characteristics of the scheme for the dielectric aftereffect [Fig. 1(a)], we need to be able to explain *in which way a long-term delay after switching OFF can be excited by the brief power-ON procedure*. This would be possible by the formal introduction of a nonlinear element in the electrical circuit diagram. For example, resistors would have to be thought of as replaced by electric rectifiers. The desired behavior would be found for the kind of electric rectifier that exhibits an increased resistance for the current direction during the switching-OFF process. Physically, this implies that certain particles exhibit vanishing inertia during the switching ON and only show a delay during the switching-OFF process. The test result from the tape-wound core (independence of the time for both ON- and OFF-switching experiments) can be described only by the assumption that individually *different particles are retarded with respect to the field* for ON and OFF switching. This is in stark contradiction to the picture of the classical aftereffect theory where defects, which are located in the material, follow *any force* with certain inertia.

Since the study of the Barkhausen effect, it is logical to search for the delay in the switching experiment in the Barkhausen domains that show delayed switching. Imagine (for small field strengths) individual domains that are included in the reversibly behaving magnetically elastic base material that have individual coercivity and magnetization represented by a rectangular loop. If a particle is magnetized close to coercivity with the switching ON of the measuring field, then, for various reasons (thermal fluctuations and local eddy currents), the Barkhausen jump starts only after a certain time delay. In this picture, it is clear that, in general, a domain will jump *either with ON or with OFF switching* in an ON- and OFF-switching experiment.

Tracing the aftereffect back to Barkhausen jumps automatically explains the characteristic nonsuperposability and the independence of the switching-OFF delay and the switching-ON time observed by us.

B. Description of the Switching Experiment Based on a Hysteresis Model

If one wants to attribute the hysteresis features to a superposition of elementary rectangle-like loops and, thus, explain the Rayleigh relationship, as well as the dependence of the Rayleigh hysteresis constant on the pre-magnetization [25], then the following has to be assumed: 1) the width of the elementary loops has a certain probability distribution whereby loops with zero width also occur and 2) the elementary loops are to be thought of as afflicted with an initial magnetization state ("pre-magnetization"). Pre-magnetizations from $H = 0$ up to $H > H_c$ exist; otherwise, the area of strongly pre-magnetized cycles [25] could not be understood. It is easy to see (see still further below) that for small fields, the distribution curve must have a horizontal initial slope over the coercivity a, as well as over the pre-magnetization b; otherwise, the Rayleigh law cannot be explained. From this first approach for the statistics of the hysteresis-producing domains, quantitative conclusions can be drawn regarding the number of domains that just switch in a switching experiment.[5]

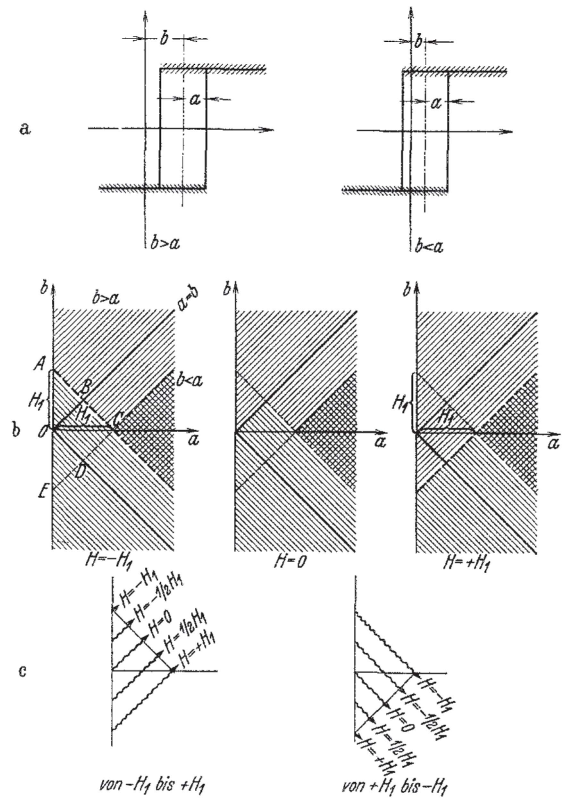


Fig. 12. Schematic for the interpretation of hysteresis and aftereffect for small field strengths by elementary rectangle loops. (a) Two basic types of the loops associated with pre-magnetization. (b) Change of the direction of magnetization of the domains during cyclic remagnetization for representation in the *ab* plane. (c) Shift of the front of switching domains.

The number of equally sized domains can be instructively thought of as plotted as an area depending on the quantities a and b, whereby in the ab plane, each point represents a domain characterized by coercivity a and pre-magnetization b. There are two different kinds of domains [see Fig. 12(a)]; one kind is strongly pre-magnetized, $b > a$, such that they possess a clear magnetization orientation in the field-free condition, and one kind is weakly pre-magnetized, $b < a$, with the orientation at $H = 0$ depending on the history. With a cyclic magnetization, apparently only the second kind contributes to the remanence. The first kind only causes hysteresis. In the representation of the domains in the ab plane [Fig. 12(b)], the direction of the respective magnetization is indicated by the direction of the shading. All domains that are plotted between the ordinate axis and the 45° line belong to the first kind (continuous shading). In the demagnetized condition, the direction is undefined in the

[5]I owe the following description to a discussion with Professor R. Becker.

region $b < a$. In Fig. 12(b), the three conditions for the cyclic switching experiment (as it was also carried out) are indicated with commutated current ($H = -H_1$, $H = 0$, and $H = +H_1$). If (for small field strengths) the distribution is independent of the position within the plane, then the number of switching domains grows with area and thus with H^2. Since the mean loop width is also proportional to H, the cubic dependence of the loss area on the field magnitude follows immediately from the Rayleigh relationship. The number of the domains that contribute to the remanence lies in the rectangle *0BCD* from which the square increase of the remanence with the field strength, also known from the Rayleigh approach, follows.

The predictions of our diagrams are remarkable regarding the number of domains switching at any given time. It is easy to see that different domains become unstable for each field strength. Fig. 12(c) shows the progression of the front of the switching domains in the *ab* plane during a gradual change of field strength between $-H_1$ and $+H_1$. The number of switching domains apparently grows linear with field strength. For $H = 0$ (switching OFF), one obtains half the number (distance $0B$) as if one switches on the field $H = +H_1$ (distance *EC*). This corresponds, however, to our experimental findings (Fig. 5), which seemed initially unexpected and unexplainable in the context of the classical aftereffect conceptions. In addition, based on Fig. 12, it is clear that during the execution of a cyclic magnetization between $H = 0$ and $H = +H_1$ (without polarity reversal), only the triangle *0ED* in the region $a < b$ is effective. The line segment *ED*, then, represents the number of switching domains in the switching experiment. The line segment *ED*, however, amounts to half of the line segment CE, which became effective in the conventional symmetrical switching experiment. Thus, the difference between the curves a and b in Fig. 6 is explained quantitatively.

The number of switching domains and, thus, the aftereffect for higher field strengths will depend on the statistics of the domains in the whole *ab* plane.

Just as the Rayleigh law determined the frequency surface for small a and small b (to a horizontal plane), the frequency surface for larger field strengths can be determined by sufficient hysteresis observations. From this, deriving the dependence of the aftereffect switching experiment on the field strength should be possible. For our sample, only the dependence of the remanence of symmetrical loops on the field strength was determined. In Fig. 12, this corresponds to the volume of the prism over the rectangle *0BCD* that is limited by the frequency surface. It is easier to determine the area of the sectional plane over the line segment *BC* from the size of dB_r/dH for a given field strength H_1. The aftereffect experiment, however, gives a measure for the area over the line segment *AC*. The main difference between the evolution of the aftereffect curve and the curve dB_r/dH in Fig. 9 (slower decay of the aftereffect) can thus be explained by the fact that for large H, fewer domains lie on *BC* than on *AB*. This means that there are a few domains with large coercivity (and small pre-magnetization), but relatively many domains with small coercivity and large pre-magnetization.

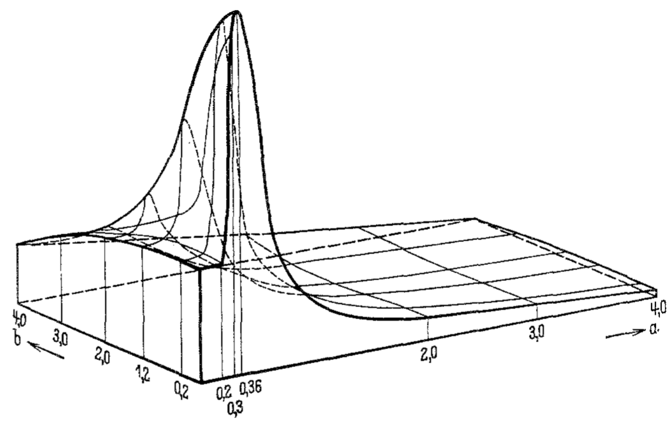


Fig. 13. Distribution function of the domains for the explanation of hysteresis and the aftereffect of the tape-wound core.

In Fig. 13, the distribution landscape (only the part $b > 0$) is shown. The function was drawn in such a way that both the aftereffect and the dB_r/dH curve of Fig. 9 are represented correctly. The experimental curves do not sufficiently specify the sought-after function quantitatively; however, the process can be drawn correctly to large extent for continuity reasons. The surface is presented with curves intersecting with planes $a = \text{const}$ and $b = \text{const}$.

The already mentioned feature of the function is the slow decay of the plateau (which represents the Rayleigh area at $a \approx 0$, $b \approx 0$) for increasing b with small a. By contrast, a rapid decay occurs with increasing a for small b after exceeding a high maximum. This feature of the distribution function is also expected qualitatively alone from *hysteresis observations*.

Unpublished measurements of Dr. H. Kühlewein (performed in the research laboratory of the Siemens Company) show that in a plot of the hysteresis surface and the remanence as a function of the field strength, the hysteresis surface increases noticeably even after the remanence value is already saturated. Kühlewein already explained this further increase of the loss surface by the switching of strongly pre-magnetized areas. This feature is obviously the direct result of the fact that our landscape in Fig. 13 drops more slowly toward the b-axis than toward the a-axis.[6]

C. Cause of the Delay (Switching Experiment and Alternating Current Experiment)

To explain the qualitative characteristics and, also, some quantitative results of the switching experiment, only a few assumptions about the physical nature of the delay were required.

We regarded the number of those domains that are pre-magnetized close to switching as critical to the size of the effect. The domains that need a finite and observable time for switching apparently lie on a strip with a certain width on the front line (Fig. 12, for example AC).

[6]We only formally introduced the suggested model for the treatment of hysteresis and the aftereffect by particles, which are enclosed in the material and characterized by pre-magnetized rectangle loops. The pre-magnetization b does not need to be a true internal scattering field. The wall displacement processes discussed by R. Becker [26] also yields apparently pre-magnetized loops if the movement of the wall is afflicted with hysteresis.

Now one asks: 1) what determines the delay of a single domain and 2) how is this delay related to the distance to the front line. The knowledge of the single mechanism must, then, be sufficient to derive both the transient law with the switching process as well as the phase shift with alternating current.

Here, we only attempt to discuss the possible hypotheses of a future theory in order to receive a plausible qualitative picture for an interpretation of the observations. There are two procedures that might play a role for the occurrence of the lag for the excitation of the Barkhausen jumps. First, the elementary coercivity and also the pre-magnetization are both subject to thermal fluctuations Consequently, with limited likelihood, re-magnetizations still appear here and there with a switched-ON field after a certain given waiting period. Second, the evolution of the triggered Barkhausen jump is retarded by the delaying effect of local eddy currents. The switching experiment can be interpreted solely on the basis of the *fluctuation hypothesis* (without consideration of the eddy currents). The domains that cause the lag are all beyond the front line. If the sum of the external field strength H and the pre-magnetization b reaches the value of the coercivity a of a domain, the jump occurs. The necessary waiting time increases (according to the diminishing probability of excitation) with the distance to the front line. The assumption of the fluctuation hypothesis as the only cause of the aftereffect, however, fails to explain the alternating current experiment. Due to the fluctuation features, a frequency dependence of the eddy current loss is to be expected, but in the sense that with decreasing frequency, the loss increases monotonically. For quasi-static magnetic switching, the largest loss is to be expected, because even the most unfavorably located domains are also excited.

Experimentally, however (see above), one observes a loss per period that is frequency-independent to a large extent, which does not occur for quasi-static observations. For this reason, the treatment of the eddy current effects with the excitation of the Barkhausen jumps seems to be important for the aftereffect. It is qualitatively clear that this kind of eddy current loss means a new additional loss component for the alternating current experiment, which is not included in the eddy current loss that can be computed from the overall permeability.

A computational treatment of the problem would be necessary in order to decide in which way fluctuation features and eddy current procedures are simultaneously responsible for the experimental facts. Based on qualitative considerations, it, however, seems *unlikely* to us that the consideration of *eddy current effects alone* is sufficient for the explanation of the logarithmic time law. In particular, the occurrence of the very slow changes will be difficult to explain other than by fluctuation features. The link between the β-value of the switching experiment and the alternating current phase shift ε here does not appear to be as simple and clear as it was with the model of the dielectric linear aftereffect. Our test results, at least, let us assume that a relationship between the two numbers that does not deviate substantially from expectations of the classical theory is also present here. Perhaps one can see the link between β and ε as follows. The number β is a measure of the number of domains that switch for a certain order of magnitude of the time T during the e-times increase of the observation time. These domains represent a strip in our *ab* plane. During the execution of the alternating current experiment with field strength amplitude = switching field strength and a period duration which correspond to our order of magnitude T of the waiting period, mainly the *same domains* are just barely excited by the alternating current and thus experience a strong (loss producing) eddy current braking. (Domains located substantially less favorably do not become excited at all; those located favorably follow readily and contribute only insignificantly to the delay.) Thus, it appears plausible that one and the same characteristic number, the number of domains with a certain time constant, is critical to both the switching experiment and the alternating current experiment.

This analogy with the theory of the dielectric aftereffect might be critical as well for the explanation of the *frequency response of the permeability* (Section III). This phenomenon shows that a part of the permeability is caused by the amount of induction of the Barkhausen jumps. While individually different domains are relevant with a change of the frequency, the sum of all excited domains determines the permeability. The higher the frequency, the fewer domains are stimulated; therefore, the permeability decreases with increasing frequency.

Note that the so-called *refreshing effects* (temporary increase of the reversible permeability with switching processes, see for example [27]) are presumably also linked to the procedures treated here. If one assumes a certain reversible permeability for the Barkhausen domains, which is higher directly before the jump than afterward, then an increase of the permeability after each switching process, which fades away gradually, easily follows. During the change of induction, a depletion of switch-ready domains takes place, and the increased permeability approaches the stable equilibrium value again.

VI. Summary

To clarify the hypothesis that the alternating current loss component introduced by Jordan is based on the magnetic aftereffect, a formal analogy with the dielectric aftereffect is assumed by way of trial. To check this assumption, three experiments are conducted: 1) measurement of the frequency dependence of the permeability; 2) observation of the delay of the induction with the switching-ON experiment; and 3) the same observation with ON- and OFF-switching experiment. The first and second experiments confirm the order of magnitude expectations of the classical theory. The ON- and OFF-switching experiment, however, is in sharp contrast to the formal theory. The finding (obtained from a tape-wound core of a Fe–Ni alloy) that the delay with switching-OFF is independent of the switching-ON time proves rather that the superposition principle is not applicable to the magnetic aftereffect.

For the interpretation of the switching experiment, it is accepted that the aftereffect is due to gradual switching of Barkhausen domains. The statistics of the switching domains can be chosen, such that the field strength dependence of both the hysteresis and the aftereffect features of the switching experiment can easily be explained.

The quantitative link of the delay function of our switching experiment and the Jordan loss component is not clarified yet. For this, thermal fluctuations and local eddy currents, which arise with Barkhausen jumps, would have to be considered computationally.

REFERENCES

[1] J. B. Hoag and H. Jones, "Permeability of iron at ultra-radio frequencies," *Phys. Rev.*, vol. 42, pp. 571–576, 1934, doi: 10.1103/PhysRev.42.571.

[2] R. Sänger, "Frequency dependence of the permeability of iron, nickel and cobalt," *Helvetica Phys. Acta*, vol. 7, pp. 478–480, 1934. [Online]. Available (see "Issue V, Association News," pp. 465 ff.) http://www.e-periodica.ch/digbib/view?pid=hpa-001:1934:7#3

[3] W. Arkadiew, "On the permeability of iron at ultra-radio frequencies," *Phys. Rev.*, vol. 43, pp. 671–672, 1933, doi: 10.1103/PhysRev.43.671.2.

[4] J. A. Ewing, *Magnetic Induction in Iron and Other Metals*. London, U.K.: The Electrician Printing and Publishing Company, 1892. [Online]. Available: https://archive.org/details/magneticinducti03ewingoog

[5] R. M. Bozorth, "Time-lag in magnetization," *Phys. Rev.*, vol. 32, pp. 124–132, 1928, doi: 10.1103/PhysRev.32.124.

[6] C. Lapp, "Research on magnetic viscosity," *Ann. de Physique (Paris), 10th Series*, vol. 8, pp. 278–395, 1927.

[7] H. Kühlewein, "Magnetic aftereffect," *Phys. Zeitschrift*, vol. 32, pp. 472–480, 1931.

[8] R. M. Bozorth, "Theory of the heat treatment of magnetic materials," *Phys. Rev.*, vol. 46, pp. 232–233, 1934, doi: 10.1103/PhysRev.46.232.

[9] H. Kühlewein, "Aftereffect phenomena of hysteresis at high temperatures," *Phys. Zeitschrift*, vol. 32, pp. 860–864, 1931.

[10] W. Neumann, "Magnetic hysteresis at a high frequency," *Zeitschrift Phys.*, vol. 51, pp. 355–373, 1928, doi: 10.1007/BF01338318.

[11] E.-A. Neumann, "Magnetic hysteresis with alternating magnetization," *Zeitschrift Phys.*, vol. 83, nos. 9–10, pp. 619–631, 1933, doi: 10.1007/BF01330864.

[12] H. Jordan, "Ferromagnetic constants for weak alternating fields," *Elektrische Nachrichten-Technik*, vol. 1, pp. 7–29, 1924.

[13] Technical Reports, *Verbandes Deutscher Elektrotechniker (VDE)*, 34th Annual Meeting, Aachen, Germany, 1929.

[14] R. Gans, "The energetics of ferromagnetic materials," in *Magnetismus*. Leipziger Vorträge 1933. Leipzig, Germany: S. Hirzel, 1933, pp. 91–110.

[15] H. Wittke, "Quasistatic magnetic cycles in weak fields," *Ann. Phys.*, 5th series, vol. 20, pp. 106–112, 1934; *Ann. Phys. (Berlin)*, vol. 412, pp. 106–112, 1934, doi: 10.1002/andp.19344120107.

[16] W. B. Ellwood, "Magnetic hysteresis at low flux densities (abstract)," *Phys. Rev.*, vol. 45, p. 743, 1934, doi: 10.1103/PhysRev.45.739; *J. Appl. Phys.*, vol. 6, pp. 215–226, 1935, doi: 10.1063/1.1745323.

[17] E.-A. Neumann, "On the issue of reversible magnetic state changes and the magnetic aftereffect," *Zeitschrift Phys.*, vol. 89, pp. 308–316, 1934, doi: 10.1007/BF01342025.

[18] E. Wiechert, "Laws of elastic after-effect for constant temperature," *Ann. Phys. Chemie*, 3rd series, vol. 50, pp. 335–348, 1893; *Annal. Phys. (Berlin)*, vol. 286, pp. 335–348, 1893, doi: 10.1002/andp.18932861011; *Ann. Phys. Chemie*, 3rd series, vol. 50, pp. 546–570, 1893; *Ann. Phys. (Berlin)*, vol. 286, pp. 546–570, 1893, doi: 10.1002/andp.18932861110.

[19] R. Becker, "Elastic aftereffect and plasticity," *Zeitschrift Phys.*, vol. 33, pp. 185–213, 1925, doi: 10.1007/BF01328304.

[20] K. W. Wagner, "On the theory of imperfect dielectrics," *Ann. Phys.*, 4th series, vol. 40, pp. 817–855, 1913; *Ann. Phys. (Berlin)*, vol. 345, pp. 817–855, 1913, doi: 10.1002/andp.19133450502.

[21] H. Tobusch, "On elastic and magnetic aftereffects," Ph.D. dissertation, Georg-August-Univ., Göttingen, Germany, 1908. [Online]. Available: https://books.google.com/books?id=QJ4UAQAAIAAJ; "On elastic and magnetic aftereffect (hysteresis)," *Ann. Phys.*, 4th series, vol. 26, pp. 439–482, 1908; *Ann. Phys. (Berlin)*, vol. 331, pp. 439–482, 1908), doi: 10.1002/andp.19083310803.

[22] P. C. Hermann, "About magnetic aftereffect," *Zeitschrift Phys.*, vol. 84, pp. 565–570, 1933, doi: 10.1007/BF01330402.

[23] W. Wolman, "The frequency response of the eddy current influence in transformer sheets," *Zeitschrift Technische Phys.*, vol. 10, pp. 595–598, 1929.

[24] W. Wolman and H. Kaden, "On the eddy current delay of magnetic switching," *Zeitschrift Technische Phys.*, vol. 13, pp. 330–345, 1932.

[25] R. Goldschmidt, "Ferromagnetic materials in weak alternating fields," *Zeitschrift Technische Phys.*, vol. 11, pp. 452–455, 1930.

[26] R. Becker, "Elastic tension and magnetic properties," *Phys. Zeitschrift*, vol. 33, pp. 905–913, 1932.

[27] H. Atorf, "Temporal disaccommodation of small symmetrical and asymmetrical hysteresis loops," *Zeitschrift Phys.*, vol. 76, pp. 513–526, 1932, doi: 10.1007/BF01336733.