Unipolar Induction Revisited: New Experiments and the "Edge Effect" Theory

Francisco J. Müller

Physics Department, Florida International University, Miami, FL 33144 USA

A brief historical review is made of the 180-year-old debate on Faraday's unipolar inductor. By introducing two convenient modifications of Faraday's original experiment of 1832, pertinent answers are experimentally found to the most debated problems: 1) Can Faraday's law be used? Yes; 2) Do the magnetic field lines rotate when the magnet rotates? No. 3) Can the seat of induction be unambiguously determined? Yes. 4) Is there a fundamental difference between rotational and translational motional induction? Yes: the "edge effect", whereby a negative $\mathrm{v} \times \mathrm{B}$ field appears whenever a magnetic edge moves perpendicularly to itself. An additional experiment is presented to verify the theory. Finally, 5) Can Relativity Theory be applied? The Special Theory, no; the General one, yes.

***Index Terms*—Edge effect, electromagnetic motional induction, relativity theory, seat of *emf*, unipolar induction.**

I. Introduction

IN 1962, E. W. Bewley, a General Electric engineer, wrote that "*to this date Faraday's disk*" (the first DC generator ever created) "*remains the least understood of all electric generators*" [1]. Has the situation improved in the last half century? It does not seem so. Writing 20 years after Bewley, A. I. Miller, an expert in relativity theory and its historical emergence, again said that the problem "*still awaits a complete solution*" [2].

Perusal of the literature after Miller's book still shows a great variety and even opposition between authors concerning Faraday's generator of 1832 [3], sometimes labeled as "Faraday's paradox", others as Faraday's "paradoxical generator" [4].

In 180 years theories have gone to such extremes as: 1) Denying that Faraday's Law can be applied to Faraday's disk. Notably, Feynman [5] had this position; also Cohn [6] and Kaempffer [7]. 2) Others, on the contrary think that, if properly integrated, Faraday's law can be applied, [8]–[10]. 3) A favorite "solution" to the *emf* generated by Faraday's disk is to use Lorentz's force, $\mathrm{F} = \mathrm{q}(\mathbf{v} \times \mathbf{B})$, as applied to the charges rotating with the disk. But a problem arises when the magnet also rotates with the disk as done by Faraday in 1832. Then the meaning of the velocity term, $\mathbf{v}$, becomes ambiguous, to say the least. 4) The latter ambiguity is connected to the problem of deciding if the magnetic field (lines) rotate or do not rotate when the magnet rotates. 5) Finally several authors disagree concerning the applicability, or not, of Special Relativity theory. Kennard [11] will say that the induction depends only upon the absolute rotation of the system and, hence, that it is a "*stumbling block in the way of* [the] *ultra-relativists*". But many authors like Trocheris [12], Cullwick [13], Guala-Valverde [14][15], Berg and Alley [16], forcefully defend the applicability of Special Relativity, to the point that the latter authors consider unipolar induction to be "*the only table-top demonstration of Special Relativity*". Panofsky and Phillips, [8], in a more nuanced fashion, say that strictly speaking Special Relativity cannot be applied to the rotating system. Instead, they indicate that General Relativity must be used, and refer their readers to a paper written by Schiff in 1939 [17]. In the latter paper Schiff attributes unipolar induction ultimately to the "*counter-rotation of the distant galaxies of the Universe",* which warps a space/time term in the matrix, and then the unipolar voltage appears.

The purpose of this paper is to find a more "down-to-earth" solution, not only by introducing two convenient modifications of Faraday's original set up but also by using a comparative experimental strategy, *ie.*, by contrasting the case of *rotational* unipolar induction with a regular case of *rectilinear* motional induction. It will be seen that in rotation there is no need of relative motion between magnet and conductor for induction to occur, whereas in translation relativity of motion is crucially required, as stated by Einstein in 1905 [18]. Rarely this comparison is found in the literature although Crooks *et al.* [4] came closest to it. They, however, do not present realistic experiments to test, and explain, what will be called in this paper the "edge effect".

II. Detailed Historical Development

Faraday's original experiment of 1832 can be described in three basic steps. Fig. 1A shows the familiar copper disk rotating over a cylindrical permanent magnet. Through sliding contacts at the center and rim of the disk Faraday could collect and measure the induced current in the external circuit, which remained fixed to the Lab. In step B Faraday glued the disk to the magnet, rotating both together. He obtained the same current "*as when only the copper rotated above the magnet*". That being the case and since iron is a good conductor, Faraday removed the disk altogether in step C and still obtained the same induced current in circuit ECRI as before. This is the configuration later called "unipolar induction" by Weber (probably because only one pole of the magnet is involved).

After the remarkable result of Step C, Faraday concluded that the field lines *did not* rotate with the magnet, so the latter could "cut" them and produce a voltage along the internal radius of the magnet, IR, (dotted line in the figure).

But around 1854 Faraday changed his mind, thinking that the lines did rotate with the magnet. The "cutting action", then, was not along IR but along the External Connector, (ECR).

Manuscript received February 07, 2012; revised August 20, 2013; accepted September 03, 2013. Date of publication September 16, 2013; date of current version December 23, 2013. Corresponding author: F. J. Müller (e-mail: fjmuller@bellsouth.net).

Digital Object Identifier 10.1109/TMAG.2013.2282133

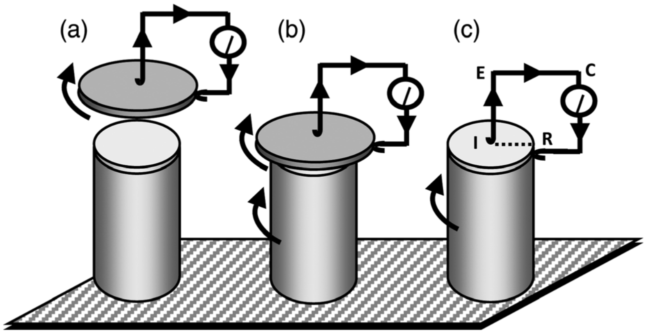

Fig. 1. Faraday's 3-step experiment: (A) Copper disk rotating above magnet; (B) magnet and disk rotating together; (C) magnet rotating alone. In all three cases the induced current was the same in the external circuit.

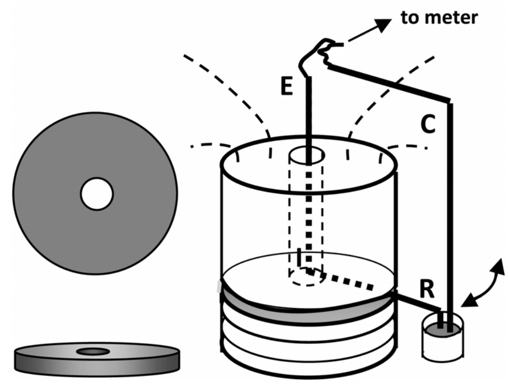

Fig. 2. Ring ceramic magnets, (left), are stacked together (right) to mimic a cylindrical magnet with central hole and an equatorial gap through which a rectangular circuit **ECRI** is inserted. A mercury cup at R allows slight angular motions as indicated by the double arrow. The wire is totally insulated except at the tips dipping into the mercury.

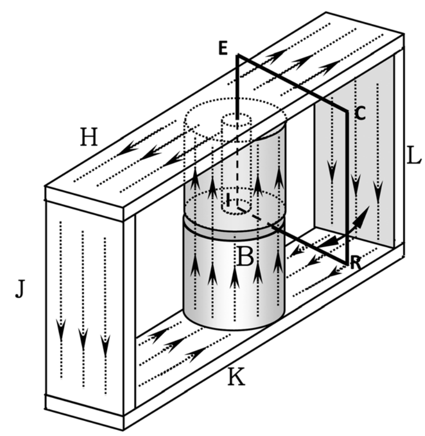

Fig. 3. The system of Fig. 2 is totally enclosed within a ferromagnetic frame HJKL which essentially shields the ECR branch from the B field. Size of circuit ECRI is exaggerated for clarity's sake. At R the mercury cup of Fig. 2 can still be inserted and also the connections to the meter at E.

Hence, the debate started: Is the seat of induction along IR or along ECR? Do the lines rotate or not? Is relative motion needed between the magnet and the conductor where the seat of *emf* is located?

No experiment could discriminate between these alternatives, since the *emf* is the same in all cases, both mathematically and experimentally. The closest to a solution was provided by E. H. Kennard, who in 1917 [11] removed the external connector ECR and measured an electrostatic voltage between I and R when rotating the whole system. The effect, then, appeared to be *absolute*, not relativistic. But Kennard used a solenoid as source of the field and a cylindrical capacitor to increase the charge separation between I and R. In a critical review of the experiment in 1922, Tate [19] limited Kennard's results only to solenoids, not to rotating permanent magnets.

It was at this juncture that I planned to repeat Kennard's experiment, but using permanent magnets. The enterprise, however, proved impossible in spite of thirty years of repeated attempts. Electrostatic noises appear, which being hundred times bigger than the expected induction obliterate the results. In addition, other theoretical conflicts had already appeared on the scene when in 1962 Feynman [5] asserted that Faraday's Law, $emf = \text{-}\mathrm{d}\Phi/\mathrm{d}t$, could not be applied to Faraday's disk. It was clear for him that in Fig. 1, the flux through the rectangular circuit is constant, (even zero, since the B field is parallel to the area of the circuit). Yet, an *emf* exists, contrary to the zero prediction of the Law.

In contrast, Panofsky and Phillips [8] stipulated that the current path is not IR in Fig. 1A and 1B, but IR', where the point R' at the rim should move with the disk. In reality what is needed here is a microscopic analysis of the actual path of the electrons. But this will take us too far off from the main problem. To circumvent this adventitious debate it is better to introduce a completely filamentary circuit, realizing that the source of confusion is due to the extended nature of the disk. Hence, the following method was used.

III. Methodology

A. *Modifications of Faraday's Original Experiment*

Two basic modifications were introduced.

The legend of Fig. 2 describes how, using ring ceramic magnets, the circuit ECRI of Fig. 1 can be turned into a completely filamentary circuit that is separated from the magnet, yet is immersed in the same magnetic field as before. The internal radius, IR, "sees", basically a homogeneous B field in the gap, parallel to the cylindrical axis. Wires are twisted at E, leading to a voltmeter capable of measuring sub-millivolt quantities.

A second modification to Faraday's original experiment was also introduced by inserting four ferromagnetic mild steel plates, HJKL, as shown in Fig. 3.

With this arrangement, instead of eliminating the external connector ECR as done by Kennard, it is *shielded* from all magnetic fields, (see B field arrows in Fig. 3). Thus, ambiguities about the seat of *emf* are avoided as will be seen below. By mechanically controlling the motion of each part of the system, namely: IR, ECR and the central magnet, so that each one can be moved independently of the other, there will be eight possible combinations of motion to be tested, as shown in Table II. Motions are confined to slight oscillations as indicated by the double arrow in Fig. 3 and they were controlled by means of inter-connecting rods, as shown in Fig. 5.

For comparative purposes, as explained in the Introduction, eight additional tests were performed, but moving IR, ECR and the magnets in rectilinear fashion, with a system similarly enclosed in a ferromagnetic frame as illustrated in Fig. 4. Instead

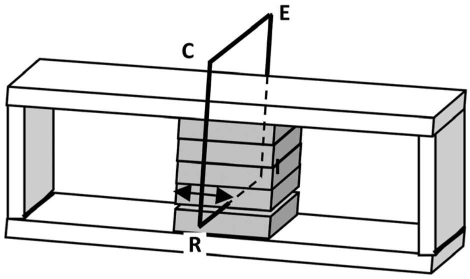

Fig. 4. Rectangular ceramic magnets (48 × 22 × 10 mm) are stacked together at the center, leaving a gap between two of them so that the circuit ECRI can be inserted as shown. The rectangular iron bars confine the B field exactly as in Fig. 3. Motions are rectilinear oscillations as indicated by the double arrow.

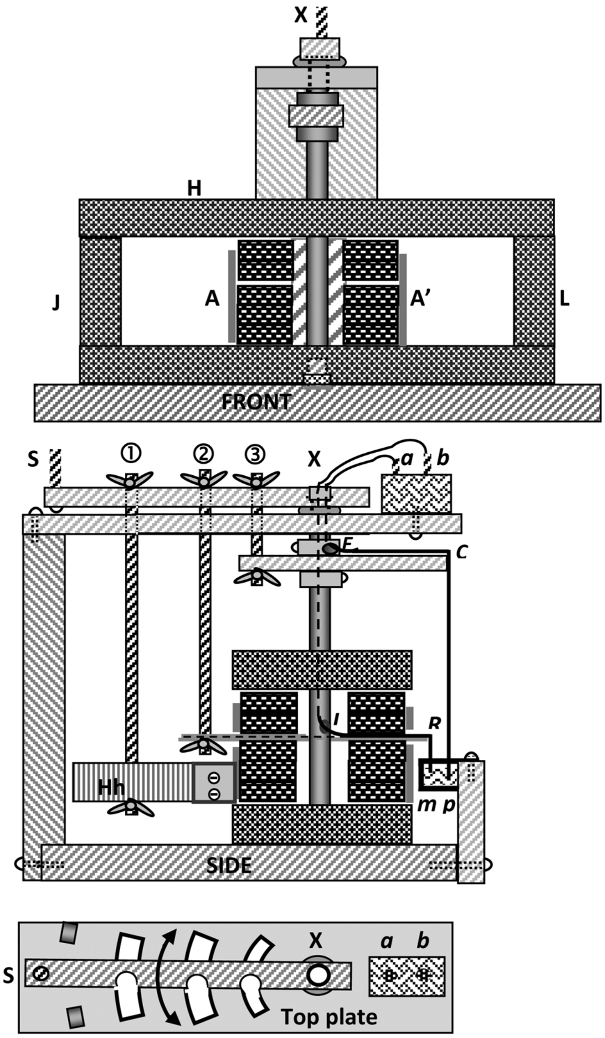

Fig. 5. Front, side and top view of rotational inductor of Fig. 3. All lightly shaded areas are aluminum. The iron frame HJKL has dimensions 10″ × 4″ × 4″ and 3/4″ thickness. Motion of IR, ECR and central magnets is effected via threaded rods 1, 2, 3, which can be independently attached to the top lever SX. Oscillatory motion is applied by hand at screw S synchronically with an acoustic signal. The axis XX is an aluminum tube through which wire XI enters at X and exits at I through a lateral hole to form branch IR. The latter is attached to a copper disk (dashed horizontal line) which is connected to rod #2 through a thin handle. Wire ECR enters the tube through a collar ring at E and exits at the top X together with wire XI for electrical connections at posts a, b. ECR oscillates when its supporting plate is connected to rod #3. Both wires, IR and ECR, dip into mercury pool *mp* at R. An aluminum jacket AA' embraces the central magnets and has a handle Hh connected to rod #1 to move the magnets. To facilitate motion plastic "washers" are inserted between the magnets and the bottom iron plate. At the top a small gap is left between the magnets and the iron plate H.

of cylindrical ceramic magnets, rectangular ones are used, to be displaced as shown by the double arrow in Fig. 4.

In all experiments, rotational or translational, the frame HJKL always remained fixed to the laboratory.

B. *Detailed Operational and Measurement Techniques*

Figs. 5 and 6 and their legends describe the practical way to assemble and operate the rotational (Fig. 5) and the translational (Fig. 6) inductors.

In rotation, (Figs. 3 and 5) when the central magnets rotate the magnetic flux through the iron plates remains constant thanks to the axial symmetry of the B field. In translation, however, (Figs. 4 and 6) when the magnets move rectilinearly the length of the magnetic circuit along the plates decreases on one side and increases on the other. This produces a change of B flux and hence an *emf* around ECRI due to transformer induction. If a similar loop L is inserted as in Fig. 6, so that it links the changing flux inside the top plate, and if properly connected, the net transformer *emf* can be eliminated.

Given the moderate strength of the magnets and the small velocities used the *emf's* obtained were in the submillivolt range. Hence, some electronic amplification was needed before feeding the signals into the final recording device. Fig. 7 is a flow diagram of the components used. The legend describes some operational details.

Fig. 8 gives the details of the ring ceramic magnets used and the positioning of the wire IR in the radial direction in Fig. 5. Table I collects the intensity of the magnetic field measured inside the gap at various radial positions along IR using a teslameter from Tel-Atomic Inc.

IV. Results

A. *Overall Qualitative Results*

The basic results of the 16 experiments performed are collected in Table II. Further experimental details and quantitative treatment of the data are relegated to the next section, not to darken the logic of the results which behave almost like in a truth table, having 0, + and − values only.

<u>Case 1</u> is trivial. Nothing moves, nothing happens.

<u>Case 2</u>, both in rotation and translation yields a positive *emf* result, clearly showing that when IR moves it "cuts" (to use Faraday's language) the main B field lines within the gap.

<u>Case 3:</u> the zero emf's here demonstrate that branch ECR has been effectively isolated from the B field, as intended. (Some leakage, however, was also present, due to the gap between the magnets and the upper and lower plates).

<u>Case 4:</u> now a dramatic difference is seen between rotation and translation. In rotation, *no* induction results when the central magnet slightly rotates in oscillatory fashion. In translation the opposite is true: an emf results which has the same intensity, but opposite polarity, as when only IR moved back and forth. Thus, rectilinear induction is perfectly symmetric and relativistic: motion of IR with respect to the magnet (and Lab ob-

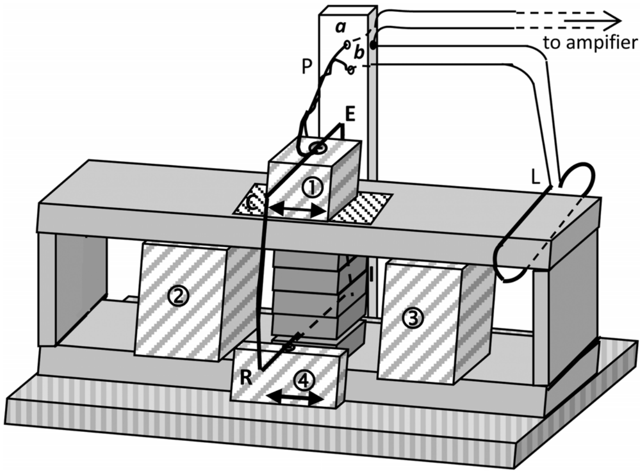

Fig. 6. Practical details of the rectilinear inductor of Fig. 4. In addition to the four iron plates, the central stack of rectangular magnets and the circuit ECRI the system has four wooden blocks,1, 2, 3, 4, and a compensating loop L. Blocks 1 and 4 can slide left and right (see double arrows) to move the wires IR and CE respectively (by hand). The wires are attached to the blocks by means of a central screw. Blocks 2 and 4 are stuck between the iron plates and serve as limiters for the magnets' motion. Vertical wires CR (front) and EI (back) cannot have independent motion (no mercury pool was used). Circuit ECRI is flexible although rigid enough to keep its general shape. A wooden post P receives the exiting wires which pass through small holes *a, b.* and then exit to the amplifier. The compensation loop L is connected in series with the loop ECRI to cancel the variable reluctance effect (see text).

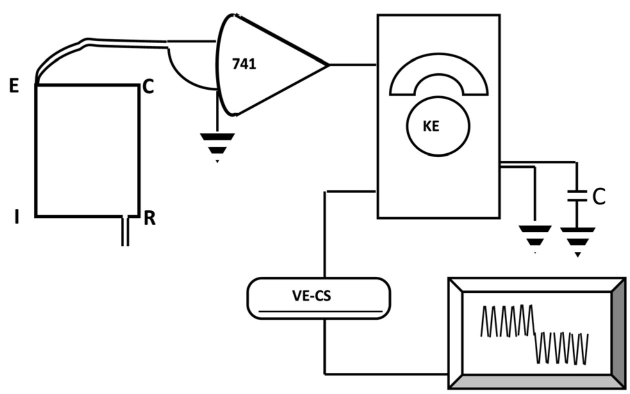

Fig. 7. The circuit ECRI is connected from E to an Operational Amplifier 741, then to a Keithley Electrometer, KE. The output of the electrometer goes to an interfacer VE-CS, (a Velleman CompuScope-164) that delivers the signal to a Laptop for digital recording and analysis. The capacitor C after the electrometer is essential to minimize electronic noises.

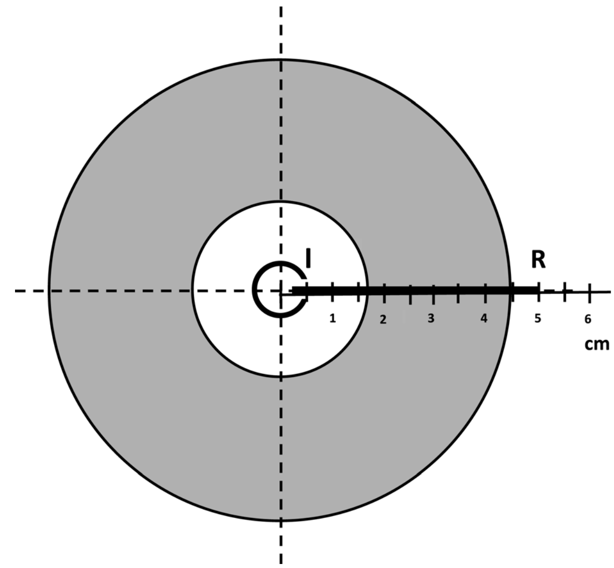

Fig. 8. The ring ceramic magnets had ID $= 1.7$ cm and OD $= 4.5$ cm. Wire IR extended from center to about 5 cm (or 0.5 cm beyond the edge of the magnet). The oscillations were made through an angle of 23.6° at a frequency of 1 Hz ($\omega_{\rm ave} = 0.824$ rad/sec). The magnetic intensities along IR (perpendicular to the paper) were measured in milliTesla and the values are given in Table I.

TABLE I
INTENSITY OF B FIELD ALONG RADIUS IR IN FIG. 8

r *cm*	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
B *mT*	45	69	147	204	216	207	207	184	149	77

TABLE II
INDUCED *EMF'S* RESULTING FROM MOTIONS (m) OF THE COMPONENTS OF FIGS. 3 AND 5 (ROTATION) AND FIGS. 4 AND 6 (TRANSLATION)

Case	IR	ECR or (I)ECR	Mag-net	*Emf* results Rotation	Translation
1	0	0	0	0	0
2	m	0	0	+ε	+ ε
3	0	m	0	0	0
4	0	0	m	0	− ε
5	m	m	0	+ε	+ε
6	m	0	m	+ε	0
7	0	m	m	0	− ε
8	m	m	m	+ε	0

server) produces the same (but opposite) induction as motion of the magnet with respect to the IR and Lab observer. In contrast, rotational induction is asymmetric and non-relativistic: rotation of the wire IR with respect to magnet and Lab observer produces a definite induction, but identical rotation of the magnet with respect to IR and the Lab observer, produces absolutely no induction. In the language of Faraday we can say that the B field lines *do not* rotate when the magnet rotates, but they do translate when the magnet translates rectilinearly. All this being so, cases 5 to 8 are easily explained, as follows:

Case 5: the result is identical to that of Case 2, when only the IR moved, both in rotation and translation. Adding the motion of ECR changes nothing, given that ECR is isolated from the B field as shown in Case 3.

Case 6: again the result is identical to that of Case 2 in rotation, since the rotating magnet is irrelevant (Case 4). But in translation a net zero *emf* results, by cancellation of the $+emf$ of Case 2 and the $-emf$ of Case 5. This case shows that without relative motion no *emf* results in the rectilinear case. In contrast, the *emf* "survives" in rotation, even without relative motion between magnet and wire.

Case 7: In rotation this case combines the zero result of Cases 3 and 4. Hence the result is identically zero. In translation we are adding the zero result of Case 3 with the negative result of Case 4. Hence a negative *emf* is observed.

Case 8: all three things moving: IR, ECR and the magnet. Spectacularly, a positive *emf* results in rotation but a net zero *emf* in translation. In both situations the result is simply the addition

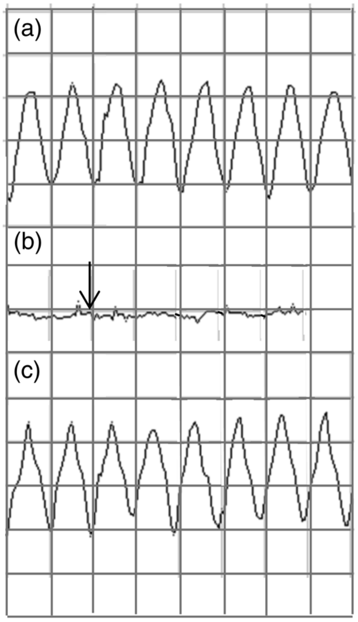

Fig. 9. Rotational experiments. (A) IR moving alone (Case 2 of Table II); (B) magnet moving alone (Case 4); the arrow indicates the start of magnet oscillations; (C) all components moving together: $\mathrm{IR} + \mathrm{ECR}$ + magnet (Case 8) Vertical scale = 50 mV/division; horizontal scale = 1 sec/division.

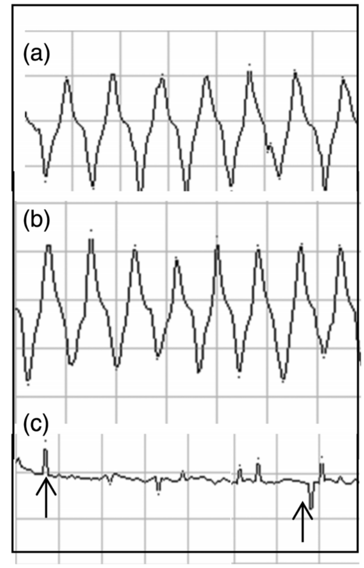

Fig. 10. Rectilinear experiments. (A) IR moving alone; (B) magnet moving alone (C) all components, $\mathrm{IR} + \mathrm{ECR}$ + magnet, moving together. The arrows indicate the start and stop moments of measurement. The scale is the same as in Fig. 9.

of Cases 2, 3 and 4, which means $+\varepsilon + 0 + 0 = +\varepsilon$ for rotation and $+\varepsilon + 0 - \varepsilon = 0$ for translation.

B. Some Quantitative Results

Fig. 9 shows the graphs obtained for three selected cases of the rotational unipolar induction experiments.

It is remarkable that graph (C) when all components are rotating together (Case 8 of Table II) is practically identical to graph (A) for the motion of IR alone. This means that motions of the magnet and of ECR are irrelevant. The magnet alone yields graph (B) which is practically zero, almost identical to the background noise seen at the left of the start arrow.

For comparison, the curves obtained with the rectilinear induction experiments are shown in Fig. 10.

These results show the expected relativistic symmetry between curves (A) and (B). Moving the IR conductor relative to the magnet gives identical results as moving the magnet relative to the conductor, but with opposite signs.

Predictably, when magnet and conductor move together (Curve C) the result should be zero. And it is zero, except for the electronic noises, especially the low frequency blips which cannot be filtered by the capacitor. The arrows in Fig. 10C indicate the “start” and “stop” instants of the measurement interval.

The numerical values of the peak-to-valley voltages corresponding to Figs. 9 and 10 are reported in Table III, including their standard deviations, (except when the results are zero or close to zero). The values include the overall amplification of the system which was 434 X for rotation and 323 X for translation.

The overall amplification of the system was not linear and had to be re-calibrated for each specific intensity range of the expected signal.

To judge if the observed results were in agreement with some expected “theoretical” value an attempt was made to estimate the induced *emf* using the values of the B field along IR in the magnetic gap reported in Table I. If these values average to some value $\mathrm{B_{gap}}$, the usual integration of the Lorentz field $\mathrm{v \times B}$ along IR would yield, for a rotating system ($\mathrm{v} = \omega r$),

$$emf = B_{gap}\omega \int_{R1}^{R2} rdr = B_{gap}\omega({R_2}^2 - {R_1}^2)/2. \tag{1}$$

The average $\mathrm{B_{gap}}$ was 150 mT between radial positions $\mathrm{R_1} = 0.5$ and $\mathrm{R_2} = 5.0$ cm from Table I. With $\omega = 0.824$ rad/sec and amplification of 434 X the predicted *emf* was 133 mV, slightly higher than the observed 123 mV for IR rotating alone.

For the rectilinear case with a $\mathrm{B_{gap}}$ of 232 mT, $\mathrm{v} = 0.042$ m/s, wire IR length of $\mathrm{L} = 2.2$ cm and amplification of 323 X the calculated emf was $\mathrm{BvL} \times 323 = 138$ mV, very close to the observed 131 mV for IR moving alone.

There are two probable reasons why an observed value could be smaller than the calculated one: (1) the influence of the capacitor in parallel with the output signal (Fig. 7). In fact, all peaks of Figs. 9 and 10 showed two “shoulders” due to the exponential behavior of the capacitor, which retards the signal going up and coming down. (2) The other decreasing factor is the unavoidable B field leakage at all gaps between the magnets and the iron plates, (more of this in the Discussion).

V. Theoretical Explanation of the Results: The “Edge Effect” Theory

Looking at the plots of Figs. 9 and 10 the remarkable difference between the rotational and the rectilinear experiments comes out in a striking way. Curves (A) in both figures are perfectly in agreement with all expectations: moving the wire IR inside the magnetic gap produces a definite motional induction, easily predicted by the Lorentz voltage, BvL, or even by the

TABLE III
QUANTITATIVE INDUCTION RESULTS

Case	IR	ECR or ECR(I)	Magnet	*emf* results (milivolts) Rotation	Translation
1	0	0	0	0	0
2	m	0	0	+123 ± 7.2	+131 ± 0.73
3	0	m	0	3	2
4	0	0	m	3	− 129 ± 5.4
5	m	m	0	+121 ± 4.0	(129)
6	m	0	m	+121 ± 7.2	2
7	0	m	m	4	(-127)
8	m	m	m	+122 ± 9.2	3

* Single digit results were obtained after subtracting a back-ground noise of 2 mV; numbers in parentheses were calculated by adding previous cases, for ex. $\mathrm{Case5} = \mathrm{Case2} + \mathrm{Case3}$.

magnetic flux "swept" by the moving wire, (Feynman not withstanding).

But then, the results of plots (B) and (C) are exactly reversed for both experiments. In rotation the moving magnet induces no *emf*; in translation it does, (curves B). This entails the reversion of Curves (C): in rotation a positive *emf* is obtained moving magnet and wire, together, whereas in translation, no *emf* results without relative motion between magnet and wire. The "down-to-earth" theory that is proposed in this paper is simply this:

When an axi-symmetric magnet rotates about its axis of symmetry, absolutely *no change* of magnetism occurs anywhere in the universe. As far as magnetic intensities are concerned one could say that the magnet, or the B field, is not rotating at all. Hence the moving wire, IR, can produce its BvL effect regardless of whether the magnet rotates or not, (Curves A and C)

In contrast, when the rectangular magnets of Fig. 4 and 6 are displaced, huge magnetic "storms" start to happen at points near the leading and trailing edges of the magnet. Panofsky and Phillips [8] have expressed these changes by means of the convective operator, $(\mathbf{v}\cdot\nabla)$, so they write Maxwell's third law in a generalized form as:

$$\nabla\times\mathbf{E} = -\frac{d\mathbf{B}}{dt} = -\frac{\partial\mathbf{B}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{B} = 0 + (\mathbf{v}\cdot\nabla)\mathbf{B} \quad (3)$$

where the zero term comes from the obvious fact that the magnets are permanent.

Applying now a well-known vectorial theorem

$$\nabla\times(\mathbf{v}\times\mathbf{B}) = \mathbf{v}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{v}) + (\mathbf{B}\cdot\nabla)\mathbf{v} - (\mathbf{v}\cdot\nabla)\mathbf{B} \quad (4)$$

and recalling that $\mathbf{B}$ has no divergence and that for constant linear velocity the second and third term of the right hand become zero, the previous theorem reduces to

$$\nabla\times(\mathbf{v}\times\mathbf{B}) = -(\mathbf{v}\cdot\nabla)\mathbf{B}. \quad (5)$$

Applying (5) to the last term of (3), finally leads to

$$\nabla\times\mathbf{E} = -\nabla\times(\mathbf{v}\times\mathbf{B}) \quad (6a)$$

or simply,

$$\mathbf{E} = -(\mathbf{v}\times\mathbf{B}). \quad (6b)$$

This negative -$(\mathbf{v}\times\mathbf{B})$ field is what will be called in this paper the "edge effect". Its function is to combine with the positive $\mathbf{v}\times\mathbf{B}$ that *still is operative* along the wire IR when it moves with the magnet, so that the net result is *zero* as seen by experience in the rectilinear case 8.

Special Relativity also predicts this negative -$\mathbf{v}\times\mathbf{B}$ effect, by the Lorentz transformation of the fields, and it explains why there is no net induction when wire and magnet co-move in a straight line. The $\mathbf{E}'$ field "seen" by a laboratory observer when the magnets move with velocity $\mathbf{v}$, say to the right, is

$$\mathbf{E}' = \gamma(\mathbf{E} - \mathbf{v}\times\mathbf{B}) \quad (7)$$

where γ is the Lorentz factor $(1\text{-}\mathrm{v}^2/\mathrm{c}^2)^{-1/2}$. Since the magnets have no intrinsic $\mathbf{E}$ field to begin with ($\mathbf{E} = 0$) and since the speeds are very small ($\gamma = 1$), we get:

$$\mathbf{E}' = -\mathbf{v}\times\mathbf{B}. \quad (8)$$

In addition, a charge q in the moving wire IR "feels" a Lorentz field,

$$\mathbf{E} = \mathrm{F}/q = +\mathbf{v}\times\mathbf{B} \quad (9)$$

So the combination of (8) and (9) yield the zero result predicted by Special Relativity theory as experimentally observed.

But then Special Relativity cannot explain why the same transformation leaves intact the positive $\mathbf{v}\times\mathbf{B}$ effect in rotation, even when magnet and wire move together. It is here where Panofsky and Phillips [8] and also A. I. Miller [2] note that the persistence of $\mathbf{E} = \mathbf{v}\times\mathbf{B}$ in rotation invalidates the use of Special Relativity. The resulting field, $\mathrm{E} = \mathrm{B}\omega\mathrm{r}$, has a non-vanishing divergence,

$$\nabla\cdot\mathrm{E} = 2\mathrm{B}\omega = \rho/\varepsilon \quad (10)$$

and leads to the appearance of a volume charge ρ. Through this charge a co-moving observer could detect (and measure) the absolute rotation of the system. This is contrary to the special relativistic principle.

Hence, the previous authors [2], [8] say that the General Theory of Relativity must be used for the rotational inductor, not the Special Theory.

But continuing with our "down-to-earth" explanation, it is also possible to use Faraday's Law (Maxwell's "flux rule") to arrive at the same conclusion about the edge effect. Recalling that Faraday's law can be applied to any closed region of space, *even if its boundary does not coincide* with an actual material boundary [20], [21], then Fig. 11 can be used to explain the dramatic difference between rotating a magnet and translating it.

In the figure the $\mathbf{B}$ field is perpendicular to the paper in all cases and is confined to the region of the magnets and the iron

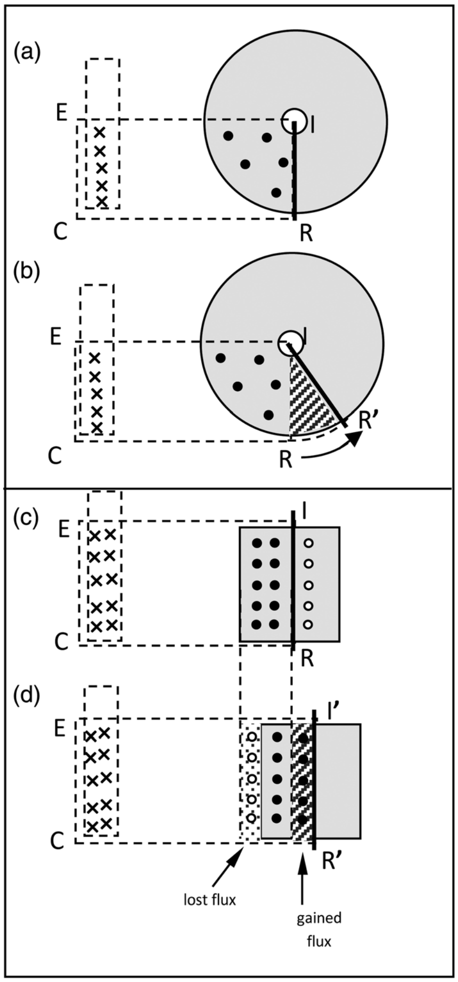

Fig. 11. The edge effect and Faraday's law. All B fields are perpendicular to the paper and confined to the magnets and iron plates. Panels A, B, rotational experiment; panels C, D, translational experiment.

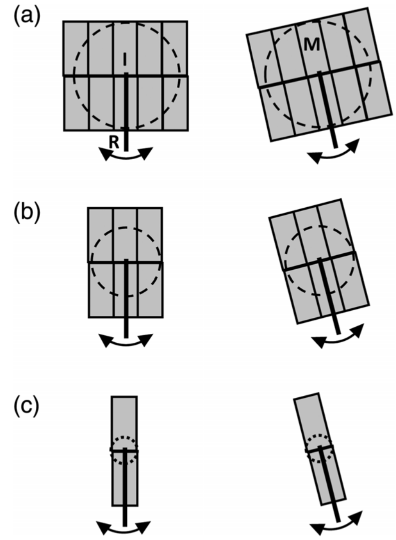

Fig. 12. Arrays of rectangular ceramic magnets of decreasing widths. On the left side figures only the radial wire IR oscillates. On the right side, both, the wire and the magnets M oscillate together. The inscribed circle is indicated with dashed lines in each case. The B fields are perpendicular to the paper and confined to the magnets in all cases. Each individual magnet was 22.4×48.5 mm (surface) and 10 mm thick. Average $\mathbf{B}$ was about 200 mT.

plates thanks to the confining enclosure used in this paper. Focusing on the area ECRI, the dots represent B field lines "coming out" of the magnets and the crosses an equal number of field lines "going down" through the left vertical plate of the system. Hence, the net B flux is zero, exactly as in the original vertical circuit ECRI, to which the horizontal ECRI is electromagnetically equivalent.

Then, when wire IR co-rotates with the magnet (Fig. 11B) there is a "swept flux" represented by the shaded sector in the figure which, indeed, increases the total flux traversing the area ECRI as it becomes ECRR'I. Hence, a positive *emf* is observed.

In contrast, when wire IR co-moves with the magnet in Fig. 11D, the flux swept by the wire as it moves to I'R' (shaded rectangle) is equal to the flux "lost" at the trailing edge of the rectangle (dotted area). Elementary as it sounds one can imagine that of fifteen lines coming out of the magnet in Fig. 11C, ten (solid circles) belong to the flux through ECRI (compensated by ten crosses going down through the left plate) while five open circles are "waiting" to be enclosed into the area when the magnet moves to I'R' But then when this happens, five (open circles) are "lost" at the trailing edge of the magnet. The result is no net change of flux. Hence, no emf is observed.

Admittedly it is difficult to visualize how the vertical circuit ECRI with real wires is equivalent to its horizontal version ECRI. But as long as the flux sweeping element, IR, is the same in both, the rest of the connecting circuit, real or imaginary, can have any shape whatsoever, provided it does not cross any additional field lines.

The advantage of the horizontal projection of the ECRI circuit is that it graphically shows how the trailing edge of the rectangular magnet (the "edge effect") looses the flux gained by the sweeping wire IR. In contrast, in the cylindrical magnet the fourth quadrant of the circle has no edge, and keeps the flux constant as the magnet rotates. (In rotation no edges move perpendicularly to themselves). One can even define the phenomenon of unipolar induction by saying that it is a type of motional induction that can take place even when magnet and conductor(s) move together. The mathematical treatment of the "edge effect" can be done via (3)–(6) above as done by Panofsky and Phillips [8], or using Dirac's delta function as will be shown below.

VI. Supplementary Experiments Verifying the "Edge Effect" Theory in Hybrid Motions and Geometries

Any good theory should not only be capable of explaining pre-existing phenomena but also new phenomena that could be related to the known ones. Such is the case of induction experiments in which a "hybrid" mixture of geometry and motions is used. For example, instead of rotating a cylindrical magnet it could be simply translated. Similarly a rectangular magnet could be rotated about its geometric center rather than being laterally displaced.

This new case is illustrated in Fig. 12 in which a rectangular array of ceramic magnets can be oscillated around the central axis of the system, (perpendicular to the paper), while the total width of the array can be stepwise decreased by eliminating one row of magnets from each side at a time.

When performing this experiment the idea is to study if the induced *emf* that was previously observed when both wire IR

TABLE IV
INDUCED *EMF'S* IN WIRE IR PLUS RECTANGULAR MAGNETS OSCILLATING TOGETHER AND WITH DECREASING WIDTHS OF THE MAGNETS

Radius of inscribed circle (cm)	Observed *emf* for IR moving alone (mV)	*emf* of IR+magnets moving together (mVolt)		
		observed	calculated via Eq.(18)	calculated with angle correction
4.85	8.04 ±0.95	8.14 ±1.27	8.05	8.26
3.36	7.08 ±0.46	5.95 ±0.88	5.58	5.7
1.12	6.17 ±0.50	1.87 ±0.29	1.86	1.89
0.0	5.56 extrapolated	0 extrapolated	0	0

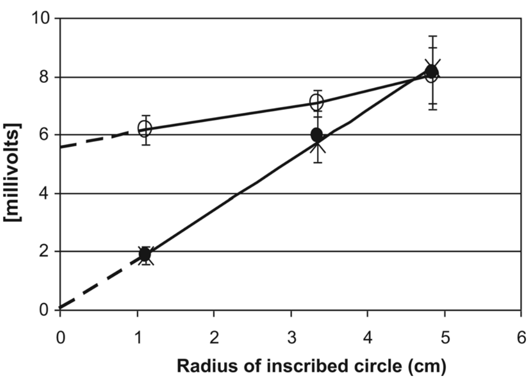

Fig. 13. Plot of the *emf* values from Table IV. Open circles (top curve): *emf* when IR oscillates alone (left diagrams of Fig. 12); Closed circles: *emf* when IR + magnets oscillate together (right diagrams of Fig. 12); solid line (X): calculated values using (18).

and magnet rotated together is now affected (decreased?) by the presence of the edges, especially when the edges come closer to the wire as the magnet's lateral dimension progressively decreases. The experiment was done as usual with the same type of shielding iron frame as in Figs. 3 to 6. (For details see legend of Fig. 12)

The observed data for three different widths of the array are collected in Table IV and are also plotted in Fig. 13. As mentioned in the legend of Fig. 12 the wire IR oscillates alone in the left side figures. The corresponding *emf*'s observed are reported in the second column of Table IV and plotted in the upper curve of Fig. 13. The values tend to decrease slightly, although the frequency and length of the oscillating wire (5.0 cm) was always the same.

On the right side of Fig. 12 the magnets oscillate *together* with the IR wire. Now there is a more dramatic decrease of the induced *emf* as indicated by the third column of Table IV and the lower plot of Fig. 13. In fact, the *emf* values extrapolate to zero, when the width of the magnet becomes zero. Not so with the IR wire oscillating alone, in which the values extrapolate to some 5.6 millivolts at $r = 0$

Interestingly the magnet + IR *emf's* are proportional to the radius of the circle that can be inscribed in the rectangular array of magnets in each case (dashed circles in Fig. 12). A quick explanation of this decrease is that only that portion of the wire IR which is contained within the inscribed circle seems to fully receive the induced *emf*. Or to put it in different words: only the cylindrical portion of the magnets behave as the rotational unipolar inductor of Fig. 3.

Strictly speaking, however, if only the "inscribed" portion of the magnet produced the observed *emf's* then their values should decrease as the square of the inscribed radius, according to (1), $\mathrm{emf} = {}^1\!/_2 \mathrm{B}\omega \mathrm{R}^2{}_{\mathrm{inscribed}}$.

But this is not observed. How can the linear decrease of the *emf's* with the inscribed radii be explained?

It appears that the vectorial theorem given above, (4), must be re-interpreted for the rotating rectangular magnets. The $-\mathrm{v} \times \mathrm{B}$ effect taking place when a magnet is linearly displaced might not be fully operative when the same magnet is rotated about its center of figure. To quantify the "edge effect" in the latter case the velocity $\mathbf{v}$ must be interpreted in terms of an angular velocity ω. Thus $\mathbf{v} = \omega \mathrm{r}$, and (4) will become

$$\nabla \times (\mathbf{v} \times \mathbf{B}) = 0 - 0 + \mathbf{B}\omega - (\mathbf{v} \cdot \nabla)\mathbf{B} = \mathbf{B}\omega - (\mathbf{v} \cdot \nabla)\mathbf{B} \quad (11)$$

The new term, $\mathrm{B}\omega$, in (11) which was not present in (5) is due to the fact that $(\mathbf{B} \cdot \nabla)\mathbf{v} = \omega(\mathbf{B} \cdot \nabla)\mathbf{r} = \omega\mathbf{B}$ and will play a crucial role here. Thus, solving for the last term of (11) and inserting the result in Maxwell's third law, (3), results in

$$\nabla \times \mathbf{E} = -\nabla \times (\mathbf{v} \times \mathbf{B}) + \mathbf{B}\omega. \quad (12)$$

Taking the surface integral of each term and applying Stoke's theorem to the curl integrals, yields

$$\int_0^R \mathbf{E} \cdot d\mathbf{r} = -\int_0^R \omega r B dr + \oint\!\!\oint \mathbf{B}\omega \cdot d\mathbf{S} \quad (13)$$

where R is the length of IR in Fig. 12. The last term of (13) has to be integrated with care. The relevant area is the lower left quarter of the magnet as demarcated by the circuit ECRI in Fig. 11A. In the case of the rotating rectangular magnet, however, the changing area (shaded area in Fig. 14) is more difficult to calculate since the edge introduces an abrupt step function of B in the calculation.

In addition, the B field seems not to be constant but to decrease with the width of the magnet as indicated by the top plot of Fig. 13. So $\partial \mathrm{B}/\partial\varphi$ is not zero. Only the parameter ω is constant and can be taken out of the last integral in (13). A possible way to proceed is to use the delta function as done by Crooks *et al.* [4], but in two dimensions and in polar coordinates r, φ.

Thus, shifting the differential of (13) to the B field and leaving the area as $\Delta\mathrm{S}$ after the magnet has rotated some angle $\Delta\varphi$ the integral becomes

$$Int = \omega \int_0^{\varphi_3} \frac{B\delta(\varphi - \varphi')d\varphi}{\pi} \Delta S(\varphi). \quad (14)$$

To calculate $\Delta\mathrm{S}(\varphi)$ we imagine the magnet rotating as shown in Fig. 14. The relevant area is the trapezoid I0123R composed of a rectangle $\mathrm{R}r_{in}$ and a triangle I01 of area ${}^1\!/_2({r_{in}}^2 \tan \Delta\varphi)$. Since the rotated angles were small (around 4°) the previous area can be expressed as $\Delta\mathrm{S}(\varphi) = {}^1\!/_2({r_{in}}^2 \Delta\varphi) + \mathrm{R}r_{in}$.

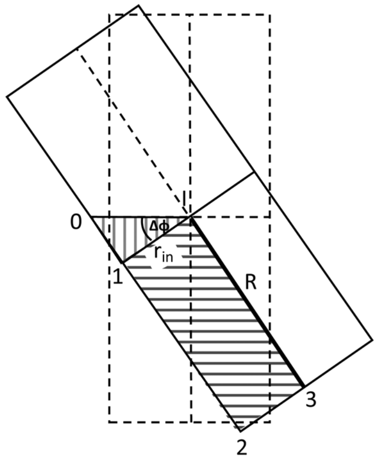

Fig. 14. Magnet rotated through angle $\Delta\varphi$. The values of φ at points 0, 1, 2, 3 are as follows: $\varphi_0 = 0$; $\varphi_1 = \Delta\varphi$; $\varphi_2 = \Delta\varphi + \tan^{-1}(\mathrm{R}/r_{in})$; $\varphi_3 = \Delta\varphi + \pi/2$.

Inserting the latter value in (14) and evaluating the integral between $\varphi = 0$ and $\varphi_3 = \Delta\varphi + \pi/2$ to ensure that the φ angle "sweeps" the whole shaded area, yields, after some algebra,

$$Int = B\omega\left(\frac{\varphi}{\pi} + \frac{1}{2}\right) r_{in}R\left(1 + \frac{\varphi}{2}.\frac{r_{in}}{R}\right) \tag{15}$$

where the difference $\Delta\varphi$ has been simply expressed as φ given that $\varphi_0 = 0$ at the origin of coordinates for the polar angle, which is at point 0 in Fig. 14.

For very small rotation angles as used in this paper, φ can be neglected in the parentheses terms of (15), thus becoming

$$Int = {}^1\!/_2\mathrm{B}\omega R r_{in}. \tag{16}$$

Replacing the last term of (13) by (16) and performing the other two integrations from 0 to R yields, for the *emf* due to the magnet oscillating through small angles (less than 4°),

$$emf_{magnet} = \text{-}{}^1\!/_2 B\omega R^2 + {}^1\!/_2 B\omega R r_{in}. \tag{17}$$

The latter *emf* is the one resulting when the magnet oscillates alone. When both magnet and IR oscillate together then the *emf* due to the wire alone should be added to the previous result. So the net value of the induced *emf* will be

$$\begin{aligned} emf_{net} &= [{}^1\!/_2 B\omega R^2]_{\text{wire}} + [\text{-}{}^1\!/_2 B\omega R^2 + {}^1\!/_2 B\omega R r_{in}]_{\text{magnet}} \\ &= {}^1\!/_2 \mathrm{B}\omega\mathrm{R} r_{in}. \end{aligned} \tag{18}$$

Equation (18) neatly explains the linear dependence of Fig. 13 (lower plot) on the first power of the radius of the inscribed circle. In particular, when $r_{in} = R$, (18) yields ${}^1\!/_2 B\omega R^2$, explaining the coincidence of the biggest values of Fig. 13 for the two plots.

This coincidence is the best indication that the theory developed throughout (11)–(18) is experimentally realistic given the fact that the experiment moving IR alone is independent of the theory. So the coincidence of *emfs* when $r_{ins} = R_{\text{wire}}$ functions as a confirmation of the theory.

In spite of this the parenthetical terms in (15) which were neglected for small angles were considered in the calculation of predicted values appearing in the last column of Table IV. It is seen that the added correction does not improve much the calculations done using (18), *ie*, without the angle correction.

The actual experimental parameters used for this refined calculation were:

$\varphi = 1.4°$; $\omega = 0.59$ rad/s; B $= 200$ mT; R $= 4.85$ cm; the $\mathrm{R_{ins}}$ were taken from the first column of Table IV. The amplifier gain was $58\times$.

VII. Discussion

Several authors cited in the body of this paper have anticipated some of the solutions to the problems connected with Faraday's unipolar inductor in ways similar to the solutions offered here. Others, on the contrary, have given solutions or viewpoints plainly in contradiction with the results of this paper. Above all else, it is to be noted a certain lack of experimental realism in the previous authors and papers when compared to the experiments described in the present one. These three points will be discussed now.

a) Anticipation of Similar Solutions: Crooks *et al.* [4] have certainly described the edge effect as a step function whose surface integration leads to BvL in straight line motion and whose negative sign is predictable by Lenz's law. On the other hand, Panofsky and Phillips [8] are contented with briefly quoting the vectorial theorem ((4) above) and concluding that $-\mathrm{v} \times \mathrm{B}$ is *"also the effective electric field in the moving medium"*. It is not clear if the latter statement means that this $-\mathrm{v} \times \mathrm{B}$ field will oppose (and cancel) the $+\mathrm{v} \times \mathrm{B}$ or not, since $-\mathrm{v} \times \mathrm{B}$ acts at the edge and $+\mathrm{v} \times \mathrm{B}$ acts where the wire is, *ie*, far from the edge (some 24 mm in Fig. 6). It seems, in this respect, that Crooks' *et al.* use of the flux integration better allows to understand how the $-\mathrm{vBL}$ at the edge opposes the $+\mathrm{BvL}$ along the wire, (which in their case is just an imaginary line fixed in space but residing along the boundary of the flux area). The distance between the edge (where the B field changes abruptly) and the wire IR at the middle of the magnet and which is part of the flux contour, has *no* influence on the intensity of the *emf* induced around the circuit. The distance could be small or large, but the net effect (in this case a cancellation effect) would be the same. Such is the virtue of Faraday-Maxwell's Law of induction, as exemplified by the loop around the iron plate in Fig. 6. The *emf* in the loop is the same regardless of the size of the loop and its distance to the plate.

None of the previous authors [2], [8], mention realistic experiments to support their views (as discussed below). In addition, they do not consider the "hybrid" cases of Fig. 12. The advantage of a hybrid case is to show that the difference between rotating and translating a magnet is not so much connected with rotation and translation *per se* (that is, with the difference between accelerated versus inertial motion) as with the *shape* of the magnet in *relationship* to its motion. It is only natural to rotate a cylindrical magnet about its axis of magnetic symmetry; but if it were rotated about another axis, for example, one perpendicular to the symmetry axis, then the unipolar induction *emf* will completely disappear since then the cylindrical edges would be moving perpendicularly to themselves. Similarly, if the cylinder of Fig. 3 were to be translated linearly, instead of rotated, the *emf* would be zero for the same reason as before. (I have performed both experiments only to verify the zero *emf* in both cases).

b) Lack of Experimental Realism of Previous Authors: Both groups mentioned before, ideally use either an infinite magnetized bar moving along its length [8] or a flat plate magnet *"long enough"* [4] so that the *"returning magnetic field through space around the magnet can be made as small as desired"* so that it will not interfere with the positive results of the $\mathrm{v} \times \mathrm{B}$ field in the wire co-moving with the magnet. But there are problems here. First, the magnet, of course, cannot be really infinite, but even if it were "*long enough"* the unavoidable effect of the returning field lines will *always* and *totally* cancel the unipolar $\mathrm{v} \times \mathrm{B}$ induction. The only way to effectively eliminate all effects of the "returning field lines" is by enclosing them, as done in this paper, in a fixed and totally confined ferromagnetic frame, although some small leakage will be always present.

With respect to this leakage and in response to some criticisms in the past, contending that all the effects reported in this paper are 100% due to the unavoidable leakages, I constructed a version of Fig. 5 in which all the gaps (between magnet an upper plate, and between the upper plates and the vertical walls) were greatly exaggerated, by interposing pieces of wood. Then it is seen that motion of ECR (case 3 in Tables II and III) yields a definite non-zero *emf*, while the main IR induction (Case 2) is greatly weakened. As the gaps were stepwise reduced it was beautifully seen how all the results extrapolate back to the values reported in Table III. Thus, the leakages rather than producing the reported *emf*s destroy and blur them. In short, small leakages are unavoidable but they are not the "cause" of the observed results in Table II and Table III.

c) Applicability, or not, of Special Relativity Theory: Concerning this point, I disagree with Berg and Alley's thesis that the rotating unipolar inductor is a "proof", in fact, "*the only table-top demonstration of Special Relativity"* [16]. Their reasoning is a repeat of Panofsky and Phillips' [8] relativistic demonstration that a magnetization $\mathbf{M}$ moving with a velocity $\mathbf{v}$ relative to a non-proper observer (at rest), becomes electrically polarized (vector $\mathbf{P}'$ in the observer's frame) according to

$$\mathbf{P}' = \gamma(\mathbf{P} - \mathbf{v} \times \mathbf{M}/\mathrm{c}^2). \quad (19)$$

Berg and Alley write the simpler version of this equation, which is (7) above. But then instead of using it, with $\mathbf{E} = \mathbf{0}$ and $\gamma = 1$ to obtain the $-\mathbf{v} \times \mathbf{B}$ effect, which cancels the net induction in a co-moving wire, they simply write that

$$\mathrm{E}' = \mathrm{vB} = \omega\mathrm{rB} \quad (20)$$

ignoring the negative sign of (19) as if (20) were the positive unipolar induction predicted by (1) above. This interpretation is at variance with their referenced source authors, Panofsky and Phillips, [8], who clearly saw that the $\omega\mathrm{rB}$ field of (20) has a "*non-vanishing divergence and thus a volume charge is developed"*, ((10) above), and that, therefore, "*the absolute rotational motion of the disk... can in principle be determined"*. This is at variance with Special Relativity, prompting Panofsky and Phillips to say that General Relativity must be used. In fact, A. I. Miller, another author referenced by Berg and Alley, also stated in [2] that "*Minkowski's equations* [*ie*, special relativity] *are not applicable in non-inertial reference systems; rather, general relativity…is required"*, which is exactly what Panofsky and Phillips concluded about the rotating unipolar inductor.

VIII. Conclusions

Concerning the problem-questions mentioned in the Abstract it can simply be reiterated that:

1) With the filamentary ECRI circuit used here, Faraday-Maxwell's Law can be clearly applied, as done in Fig. 11.
2) That the magnetic field of an axi-symmetric magnet, or its subsidiary "B field lines", do not rotate when the magnet rotates about its axis of symmetry, as evidenced by the zero results of Cases 4 of Tables II and III and by Curve B in Fig. 9.
3) That the "seat of *emf"* can be clearly located along the length of wire IR in all cases yielding positive *emf's*, as opposed to location along the external branch ECR. The proof is that motion of ECR alone, never results in any *emf*, (Cases 2 of Tables II and III). Thus, by "default" any induction must reside along IR.
4) That the essential difference between the rotational and the translational motional inductors resides not so much in the nature of the motion (inertial vs. non-inertial) but in the fact that the magnet has, or does not have, "edges" that move perpendicularly or longitudinally to themselves. If the edges move perpendicularly (either in rotation or translation) an "edge effect" appears through $-\mathbf{v} \times \mathbf{B}$ or through $-\mathrm{B}\omega\mathrm{r}$ that tends to oppose the absolute $\mathbf{v} \times \mathbf{B}$ field.
5) Finally, Special Relativity is applicable to the rectilinearly translating magnet, where the $-\mathbf{v} \times \mathbf{B}$ resulting from the Lorentz transformation of the fields, cancels the $+\mathbf{v} \times \mathbf{B}$ of the Lorentz force, yielding a net zero result as expected when magnet and wire co-move together. In rotation, however, the Special Theory cannot be used because it would yield always a zero *emf* contrary to observation. Being a local field theory, (as all field theories are), Special Relativity cannot account for the difference between a translating magnet with edges moving perpendicularly to themselves, and a rotating cylindrical magnet in which all edges move tangentially to themselves. The edge effect, being "far away" from the location where the conductor IR is, does not enter at all in any of the (local) field transformation equations of Special Relativity.

For rotation, therefore, the General Theory must be used. But this approach is out of the scope of this paper, and it can be evaded by using, instead, the more "down-to-earth" edge effect theory.

The absolute nature of the rotational experiments, clearly showing that a conductor can receive a motionally induced voltage, even when the source of the field (the magnet) co-rotates with the conductor, might have novel future applications in the field of geomagnetism and even in planetary and astrophysical magnetic field theories. Current "dynamo" theorists purporting to explain the origin of the Earth's magnetic field do not seem to be aware of Faraday’s “old fact” of 1832. This is somehow intriguing and one can only speculate how much these “dynamo” theories might have to be modified if that amazing experiment, as further developed and interpreted in this paper, is recognized as having full scientific value.

Acknowledgment

The author thanks the Varela Academy of Science for patronizing this research and the Hazelett Strip Casting Corporation for financial support. Also the equipment and laboratory space from the Miami-Dade College, North Campus and the Physics Dept. of Florida International University were very useful.

References

[1] L. V. Bewley, *Flux linkages and Electromagnetic Induction*. New York: Dover, 1963, preface.
[2] A. I. Miller, *Albert Einstein’s Special Theory of Relativity*. Reading, MA: Addison-Wesley, 1981.
[3] M. Faraday, *Electrical Researches in Electricity*. London, U.K.: Great Books, Britannica, 1982, vol. 45, Art. 218-220, 1832.
[4] M. J. Crooks, D. B. Litvin, P. W. Matthews, MaCaulay, and J. Shaw, “One-piece Faraday generator: A paradoxical experiment from 1851,” *Amer. J. Phys.*, vol. 46, pp. 729–731, Jul. 1978.
[5] R. P. Feynman, *Lectures in Physics*. Reading, MA: Addison-Wesley, 1964, vol. 2, sec. 17-1, pp. 17–2.
[6] G. I. Cohn, “Electromagnetic induction,” *Elect. Eng.*, vol. 68, pp. 441–448, 1949.
[7] F. A. Kaempffer, *Elements of Physics*. Waltham, MA: Blaisdell, 1967, p. 164.
[8] W. Panofsky and M. Phillips, *Classical Electricity and Magnetism*. Reading, MA: Addison-Wesley, 1962, p. 167, 297, 339.
[9] P. J. Scanlon, R. N. Henriksen, and J. R. Allen, “Approaches to electromagnetic induction,” *Amer. J. Phys.*, vol. 37, pp. 698–708, Jul. 1969.
[10] N. Savage, “Electromagnetic induction,” *Elect. Eng.*, vol. 68, p. 645, 1949.
[11] E. H. Kennard, “On unipolar induction,” *Philos. Mag.*, vol. 33, pp. 179–190, 1917.
[12] M. G. Trocheris, “Electrodynamics in a rotating frame of reference,” *Philos. Mag.*, vol. 40, pp. 1143–1154, Nov. 1949.
[13] E. G. Cullwick, *Electromagnetism and Relativity*. New York: Longmans, 1959.
[14] J. Guala-Valverde and P. Mazzoni, “The principle of relativity as applied to electromagnetic induction,” *Amer. J. Phys.*, vol. 63, 1995.
[15] J. Guala-Valverde, P. Mazzoni, and R. Achilles, “The homopolar motor: A true relativistic engine,” *Amer. J. Phys.*, vol. 70, p. 1052, 2002.
[16] R. E. Berg and C. O. Alley, “The Unipolar Generator: A Demonstration of Special Relativity,” [Online]. Available: http://physics.umd.edu/lecdem/outreach/QOTW/arch11/q218unipolar.pdf in the University of Maryland website
[17] L. I. Schiff, “A question in general relativity,” in *Proc. Nat. Academy of Science*, 1939, vol. 25, pp. 391–395.
[18] A. Einstein, “Electrodynamics of moving bodies,” *Annalen der Physik*, vol. 17, p. 89.
[19] J. Tate, “Unipolar induction,” *Bull. Nat. Res. Council*, vol. 4, no. 24, pp. 75–95, 1922.
[20] Panofsky and Phillips, op.cit p. 159.
[21] A. Matveyev, *Principles of Electrodynamics*. New York: Dover, 1966, p. 21.

Francisco J. Müller was born in Cuba in 1939, where he studied chemical engineering. He came to the US in 1962 where he finished a Master of Science in Physics at the University of Miami in 1993 after working as Research Associate at the Internal Medicine Department of the same institution for 28 years.

He is director of the Varela Academy of Science (a subsidiary of the Varela Foundation, Inc.) in Miami, since 1990, and in 2002 became an adjunct professor of Physics and Astronomy at Florida International University until present. He has shared his medical research and academic duties with a 40-year private research in electromagnetic unipolar induction, having presented conferences on this subject at various cities in the United States and abroad, including St. Petersburg, Russia, in 1996.