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Integral Formulation for Losses Computation in Tripolar Submarine Cables with Shield Wires, Moisture Barriers and Armor

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Method–of–Moments (MoM) is coupled with PEEC in order to compute Joule and hystheresis losses in the armor, shields and moisture barriers of a submarine tripolar cable. The large size and complex structure of the linear algebraic system resulting from the problem discretization demands for a suitably tailored solver. We describe a block–Gauß–Seidel preconditioner for Krylov subspace methods that allows for a matrix–free solver implementation, resulting in a dramatic reduction of computation time and memory requirements w.r.t. other brute–force modeling approaches.

Index Terms—**PEEC, Method-of-Moments, eddy current losses, submarine cables.**

I. Introduction

In offshore wind farms, connections between wind turbines and substation(s) are realized with medium voltage three-phase submarine cables, in which each phase conductor is surrounded by a shield composed of thin conductive wires and by a metallic moisture barrier (MB) ([2]). The whole submarine cable is also protected by an armour made of (magnetic and conductive) steel wires. The geometry of the cable is depicted in Fig.1. In previous work [2], [3], [4], [5] it was shown that, by applying suitable integral formulations and exploiting the existing symmetries and periodicity, the study of each sub-component of such a cable can be performed at a very small fraction of the computational cost of competing methods.

When different sub-components with non compatible symmetry groups (*e.g.* the double–helix geometry of shielding wires and the helical winding in the armor) are to be coupled, though, the resulting linear algebraic system is very large and complex and, as a whole, no longer enjoys the convenient structural properties of the decoupled sub-systems (*e.g.* the circulant matrix property descending from helical geometry [3]).

In this work we detail the construction of a block–Gauß–Seidel preconditioner used for the matrix–free implementation of a Krylov–Subspace iterative solvers.

By using this approach we manage to fully exploit the convenient structural properties of diagonal blocks and to obtain a cheap fast converging iterative solution of our system.

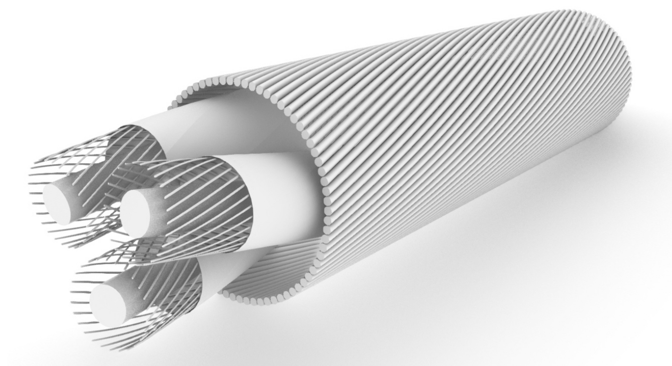

Fig. 1. Representation of the tripolar submarine cable.

II. Numerical Models for the Cable Sub–components

The efficient numerical methods for the simulation of each of the components of the cable have been developed in [2], [3], [4], [5] are briefly recalled below for sake of completeness.

A. Armor

The numerical model for the ferromagnetic armor was presented in [1], [3] and was based on the MoM formulation described in [9]. According to this approach, the armor wires are subdivided into a set of straight cylinders and the *equivalent magnetization* in the i–th cylinder is expressed as

$$\mathbf{M}_i = \frac{1}{\mu_0}\left(I - K_i^{-1}\right)\left(\mathbf{B}_i^{ext} + \sum_{j=1}^{N}\mathbf{B}_{i,j}^{M}\right)$$

where $\mathbf{B}_i^{ext}$ is the magnetic flux density due to the phase conductors and to the other cable components, while K_i is a tensor defined along the armor wires accounting for the equivalent permeabilities in the longitudinal and transverse directions (cfr. [4, Sec. IV]); $B_{i,j}^{M}$ denotes the magnetic flux density due to the equivalent magnetization in the j–th cylinder.This latter can be expressed in terms of the equivalent magnetization $\mathbf{M}_j$ as

$$\mathbf{B}_{i,j}^{M} = -\frac{\mu_0}{4\pi}\int_{\partial\Omega_j}(\mathbf{n}\times\mathbf{M}_j)\times\frac{\mathbf{x}_i-\mathbf{x}}{|\mathbf{x}_i-\mathbf{x}|^3}\,d\sigma$$

so as to reduce the problem to a linear algebraic system of the form

$$C_{arm}\ M_{arm} = B_{arm} \tag{1}$$

where the vector of unknowns M_{arm} is formed by the equivalent magnetizations of the cylinders and the forcing term B_{arm} by the magnetic flux densities computed at the barycenters of the cylinders.

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B. Screen Wires

The numerical model for the screen wires is developed in [2] and consists of a circuital model of the form

$$R_n I_n + j\omega \sum_{m=1}^{N} L_{n,m} I_m = -j\omega A_n$$

where R_n and $L_{n,m}$ are the wire resistances and (mutual) inductances, respectively, and the forcing term is the magnetic vector potential along the wires due to to the phase conductors and to the presence of the other cable components, and I_n denotes the current in the n–th wire. The above system of equations may be expressed in matrix form as

$$C_{sw}\ I_{sw} = A_{sw} \tag{2}$$

where I_{sw} is the unknown vector of wire currents and A_{sw} accounts for the total magnetic vector potential along each wire.

C. Moisture Barriers

The numerical model for the moisture barriers is developed in [5] based on the PEEC formulation for thin media presented in [10].

Denoting the vector electric potential on the thin conductive sheats (which is assumed to have only one component, normal to the sheats surface) one may write a system of integral equations of the form

$$\begin{aligned} &\frac{1}{\sigma}\int \mathbf{curl}_S T \cdot \mathbf{curl}_S T' \mathrm{d}\Omega + \\ &\frac{j\omega d\mu_0}{4\pi} \times \sum_{k\in\mathbb{Z}} \int\int \frac{(R^k \mathbf{curl}_S T)\cdot \mathbf{curl}_S T'}{|\mathbf{x} - F^k \mathbf{x}'|} \mathrm{d}\Omega \mathrm{d}\Omega' = \\ &\qquad\qquad - j\omega \int \mathbf{A}_0 \cdot \mathbf{curl}_S T' \mathrm{d}\Omega. \end{aligned}$$

The above equation is discretized by using piecewise linear basis functions on a triangular surface mesh and leads to an algebraic system which reads

$$\tilde{C}_{mb}\ \tilde{T}_{mb} = \tilde{A}_{mb} \tag{3}$$

where the vector $\tilde{T}_{mb}$ consists of the values of the potential T at the mesh nodes, while $\tilde{A}_{mb}$ represents the nodal values of the magnetic vector potential due to other cable components. Notice that, due to the use of T as an unknown, system (3) does not have a unique solution (indeed, the surface current density is the curl of T, therefore the latter is defined modulo a gradient field). Unlike the approach used in [10] where the system is solved in a least squares sense, we impose that the electric vector potential have zero mean value on each sheath by means of the Lagrange multipliers method, which results in an *augmented* system that we write as

$$C_{mb}\ T_{mb} = A_{mb} \tag{4}$$

III. Coupled Formulation

In order to couple the three subsystems (1), (2) and (4), the first step is that of separating the different contributions to the forcing terms in each equation, so that we can represent, with obvious notation, the magnetic flux density on the armor as

$$B_{arm} = B_{0,arm} - C_{sw-arm} I_{sw} - C_{mb-arm} T_{mb}, \tag{5}$$

the magnetic vector potential on the screen wires as

$$A_{sw} = A_{0,sw} - C_{arm-sw} M_{arm} - C_{mb-sw} T_{mb}, \tag{6}$$

and, finally, the magnetic vector potential on the moisture barriers

$$A_{mb} = A_{0,mb} - C_{arm-mb} M_{arm} - C_{sw-mb} I_{sw} \tag{7}$$

The resulting coupled linear algebraic system has the form

$$\begin{bmatrix} C_{arm} & C_{sw-arm} & C_{mb-arm} \\ C_{arm-sw} & C_{sw} & C_{mb-sw} \\ C_{arm-mb} & C_{sw-mb} & C_{mb} \end{bmatrix} \begin{bmatrix} M_{arm} \\ I_{sw} \\ T_{mb} \end{bmatrix} = \begin{bmatrix} B_{0,arm} \\ A_{0,sw} \\ A_{0,mb} \end{bmatrix} \tag{8}$$

which we express in short form as

$$Cs = b$$

The construction of the non–diagonal matrix blocks C_{xx-yy} in (8) can be extremely time and memory consuming, but the evaluation of their action on the unknowns amount to nothing less but computing the fields generated by the currents and dipoles in the different parts of the cable. The latter observation suggests the possibility of solving (8) by means of a staggered iterative procedure such as, for example, the Jacobi iteration which consists of letting the forcing terms at the k–th iteration be given by

$$\begin{aligned} B_{arm}^{(k)} &= B_{0,arm} - C_{sw-arm} I_{sw}^{(k)} - C_{mb-arm} T_{mb}^{(k)} \\ A_{sw}^{(k)} &= A_{0,sw} - C_{arm-sw} M_{arm}^{(k)} - C_{mb-sw} T_{mb}^{(k)} \\ A_{mb}^{(k)} &= A_{0,mb} - C_{arm-mb} M_{arm}^{(k)} - C_{sw-mb} I_{sw}^{(k)} \end{aligned}$$

so that the system to be solved becomes

$$\begin{aligned} &\begin{bmatrix} C_{arm} & 0 & 0 \\ 0 & C_{sw} & 0 \\ 0 & 0 & C_{mb} \end{bmatrix} \begin{bmatrix} M_{arm}^{(k+1)} \\ I_{sw}^{(k+1)} \\ T_{mb}^{(k+1)} \end{bmatrix} + \\ &\begin{bmatrix} 0 & C_{sw-arm} & C_{mb-arm} \\ C_{arm-sw} & 0 & C_{mb-sw} \\ C_{arm-mb} & C_{sw-mb} & 0 \end{bmatrix} \begin{bmatrix} M_{arm}^{(k)} \\ I_{sw}^{(k)} \\ T_{mb}^{(k)} \end{bmatrix} = \begin{bmatrix} B_{0,arm} \\ A_{0,sw} \\ A_{0,mb} \end{bmatrix}. \end{aligned}$$

or the Gauß–Seidel iteration which consists of letting at the k–th iteration, the forcing terms be given by

$$\begin{aligned} B_{arm}^{(k)} &= B_{0,arm} - C_{sw-arm} I_{sw}^{(k)} - C_{mb-arm} T_{mb}^{(k)} \\ A_{sw}^{(k)} &= A_{0,sw} - C_{arm-sw} M_{arm}^{(k+1)} - C_{mb-sw} T_{mb}^{(k)} \\ A_{mb}^{(k)} &= A_{0,mb} - C_{arm-mb} M_{arm}^{(k+1)} - C_{sw-mb} I_{sw}^{(k+1)} \end{aligned}$$

so that the system to be solved becomes

$$\begin{bmatrix} C_{arm} & 0 & 0 \\ C_{arm-sw} & C_{sw} & 0 \\ C_{arm-mb} & C_{sw-mb} & C_{mb} \end{bmatrix} \begin{bmatrix} M_{arm}^{(k+1)} \\ I_{sw}^{(k+1)} \\ T_{mb}^{(k+1)} \end{bmatrix} + \begin{bmatrix} 0 & C_{sw-arm} & C_{mb-arm} \\ 0 & 0 & C_{mb-sw} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} M_{arm}^{(k)} \\ I_{sw}^{(k)} \\ T_{mb}^{(k)} \end{bmatrix} = \begin{bmatrix} B_{0,arm} \\ A_{0,sw} \\ A_{0,mb} \end{bmatrix}.$$

Both methods above are stationary iteration methods of the form

$$Ps^{(k+1)} = -(C-P)s^{(k)} + b$$

with

$$P = \begin{bmatrix} C_{arm} & 0 & 0 \\ 0 & C_{sw} & 0 \\ 0 & 0 & C_{mb} \end{bmatrix} =: P_J$$

for Jacobi and

$$P = \begin{bmatrix} C_{arm} & 0 & 0 \\ C_{arm-sw} & C_{sw} & 0 \\ C_{arm-mb} & C_{sw-mb} & C_{mb} \end{bmatrix} =: P_{GS}$$

for Gauß–Seidel. It is a well known condition that methods of the above form converge if and only if the spectral radius r of the iteration matrix $-P^{-1}(C-P)$ is less than one, that is

$$r_J := \lambda_{max}\left[-P_J^{-1}(C-P_J)\right]$$

for Jacobi and

$$r_{GS} := \lambda_{max}\left[-P_{GS}^{-1}(C-P_{GS})\right]$$

for Gauß–Seidel, where $\lambda_{max}[\cdot]$ indicatates the maximum absolute value of the eigenvalues of a matrix. Both such iterations have successfully tested for the computation of losses in the cable system at 60Hz or below but, unfortunately, the convergence conditions appear to be verified in practice only for very low frequency, and often become larger than the threshold at frequencies as low as 100Hz. An alternative approach consists of using either of the above preconditioners for the implementation of a Krylov–subspace iterative solver. Such method can be implemented in a *matrix free* form, *e.g.* without the need of assembling the system matrix, as long as one can solve a system where the coefficient matrix is given by the preconditioner P. We implemented and tested, in particular, the Bi-Conjugate Gradient Stabilized (BiCGStab) [11] method and report successful results below. Gauß–Seidel and Jacobi preconditioners have been found to both perform well in numerical experiments, Gauß–Seidel consistently requiring a (slightly) lower number of iterations to reach convergence at a very similar cost per iteration. For this reason we only implemented the Gauß-Seidel preconditioner in the final version of our simulator.

IV. Numerical Results

Fig.4 shows a comparison between the results obtained with the proposed formulation and FEM results, the latter obtained with the help of commercial software [7], in terms of losses in armor, shields and moisture barriers (MB). In the FEM simulations a Transition Boundary Condition is used to model the geometrically thin moisture barriers. Results are presented in the frequency range 50 Hz - 1 kHz. Operating frequency for cables of this type is 50Hz for installation in Europe, and 60Hz in USA. Higher frequencies correspond to the harmonic of the carried currents, and are of interest for network studies performed by the owners of the offshore systems. The geometric parameters for the simulated cable are shown in Fig.IV, while the material parameters are given in TABLEI.A good agreement can be noticed between the proposed formulation and FEM in the frequency range of interest for the application.

TABLE I
Material Properties for Cable Components

Armor wires	
resistivity	1.38×10^{-7} Ω m
relative permeability	300
hysteresis loss angle	60°
Screen wires	
resistivity	1.7241×10^{-8} Ω m
Moisture barriers	
resistivity	1.5×10^{-8} Ω m

FEM simulations require about 10^7 degrees-of-freedom and 512 GByte of RAM and are run on a 4 processor Intel Xeon E7 v2 multi-core with 1.5 TB of RAM. Thin sheet boundary conditions are applied to model MBs.

Fig.3 shows the convergence performance of preconditioned BiCGStab iterations for the solution of the problem at hand which, while it does display a slight degradation at higher frequencies, always reaches convergence (within 10 times the realtive truncation error of double precision floating point) in about 10 iterations.

The coupled formulation (implemented in Octave [8]) requires only less than 10^4 degrees-of-freedom and less than 1 GByte of RAM.

Computational times reflect the different complexity of the models, with each FEM simulation requiring 4 hours of computation and each simulation with the proposed method less than 10 minutes on the same computer.

V. Conclusions

Method–of–Moments (MoM) is coupled with PEEC in order to compute Joule and hystheresis losses in the armor, shields and moisture barriers of a submarine tripolar cable. The large size and complex structure of the linear algebraic system resulting from the problem discretization demands for a suitably tailored solver. We implemented a matrix–free Krylov subspace iterative solver with a block–Gauß–Seidel preconditioner to solve the system. Numerical experiments show very good agreement with more brute–force computation and a computational cost that is dramatically reduced both in terms of time and memory requirements. A slight degradation of the accuracy of results appears at the higher end of the considered frequency range but the error is still acceptable for engineering applications. This degradation may be explaine by noting that, while at low frequency the ratio of sheath thickness

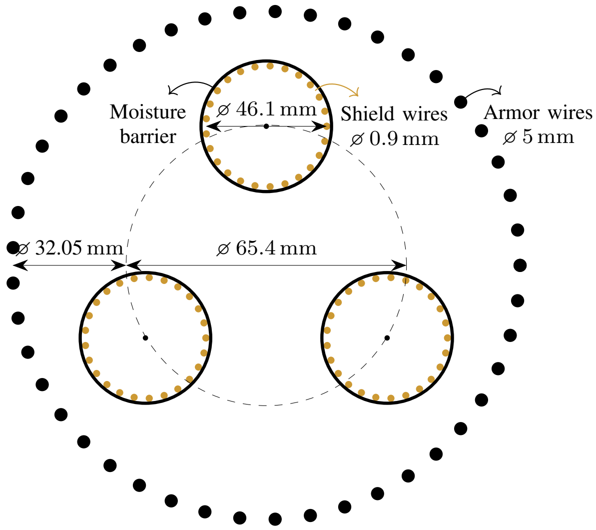

Fig. 2. Representation of the cross-section of the cable with shield wires, moisture barriers, and armor wires.

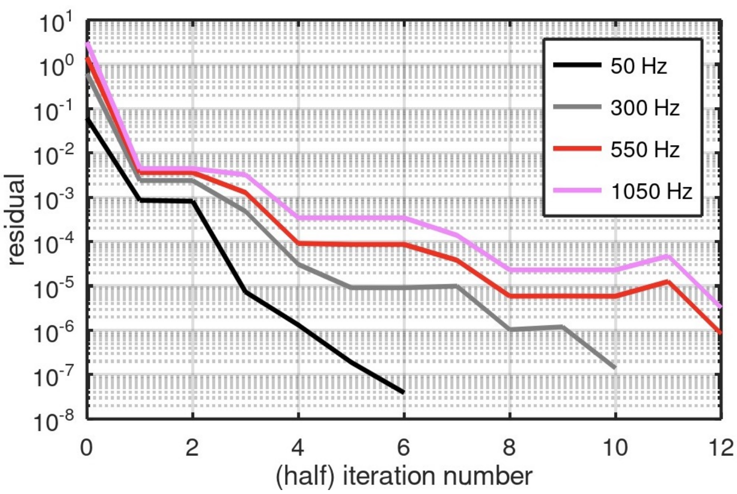

Fig. 3. Convergence performance of preconditioned BiCGStab iterations.

to skin depth is more than one order of magnitude, this ratio is reduced to a few units in the KHz frequency range, so that the assumption of uniform current density in the sheath thickness becomes less valid in that regime; further research is warranted for relaxing the uniform current density assumption and thus improve the model accuracy.

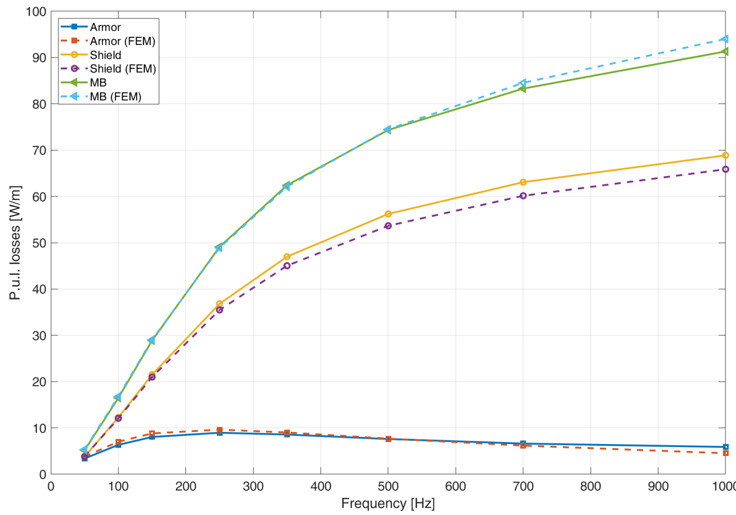

Fig. 4. Comparison with FEM results.

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