Corrections to "Weight Distribution of Cosets of Small Codes With Good Dual Properties"

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WE provide two corrections to [1] which do not affect the validity of any of the the reported results. First, we note that Conjecture 9 on page 6497 is not correct; a counter example follows from Cohen's theorem [2] which asserts the existence of linear codes with covering radius up to the sphere-covering bound. The second correction is related to the "Proof of Theorem 2 using Theorem 5" on page 6496. In that proof, the n-point Discrete Fourier Transform (DFT) should be on $n+1$ points. The other steps of the proof hold without modification. We reproduce below the corrected proof with the needed modifications in bold. The issue with the n-point DFT is that it makes Identity (1) below incorrect for $b = n$.

Proof of Theorem 2 using Theorem 5 (corrected): If $w \in [0:n]$, define the indicator function $I_w : \{0,1\}^n \to \{0,1\}$ by $I_w(x) = 1$ iff $|x| = w$. Thus, $B_n(w) = E_{U_n} I_w$ and $\overline{\mu_{Q+u}}(w) = E_{\mu_{Q+u}} I_w$. For each $b \in [0:n]$, we have the character sum identity

$$\sum_{a=0}^{\mathbf{n}} e^{\frac{2\pi i a b}{\mathbf{n+1}}} = \begin{cases} \mathbf{n+1} & \text{if } b = 0 \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

It follows that for each $x \in \{0,1\}^n$,

$$I_w(x) = \frac{1}{\mathbf{n+1}} \sum_{a=0}^{\mathbf{n}} e^{\frac{2\pi i a(|x|-w)}{\mathbf{n+1}}} = \sum_{a=0}^{\mathbf{n}} \alpha_{a,w} e_{\theta_a}(x),$$

where $\alpha_{a,w} = \frac{1}{\mathbf{n+1}} e^{\frac{-2\pi i w a}{\mathbf{n+1}}}$ and $\theta_a = \frac{2\pi a}{\mathbf{n+1}}$. Thus, for all $w \in [0:n]$ and $u \in \{0,1\}^n$, we have

$$\begin{aligned} |\overline{\mu_{Q+u}}(w) - B_n(w)| &= |E_{\mu_{Q+u}} I_w - E_{U_n} I_w| \\ &= \Big|\sum_{a=0}^{\mathbf{n}} \alpha_{a,w}(E_{\mu_{Q+u}} e_{\theta_a} - E_{U_n} e_{\theta_a})\Big| \\ &\le \sum_{a=0}^{\mathbf{n}} |\alpha_{a,w}| |E_{\mu_{Q+u}} e_{\theta_a} - E_{U_n} e_{\theta_a}| \\ &= \frac{1}{\mathbf{n+1}} \sum_{a=0}^{\mathbf{n}} |E_{\mu_{Q+u}} e_{\theta_a} - E_{U_n} e_{\theta_a}|. \end{aligned}$$

By Jensen's inequality, $\left(E_{u\sim U_n} |E_{\mu_{Q+u}} e_\theta - E_{U_n} e_\theta|\right)^2 \le E_{u\sim U_n} |E_{\mu_{Q+u}} e_\theta - E_{U_n} e_\theta|^2$, or any $0 \le \theta < 2\pi$. It follows that:

$$\begin{aligned} &E_{u\sim U_n} \|\overline{\mu_{Q+u}} - B_n\|_\infty \\ &\quad = E_{u\sim U_n} \max_w |\overline{\mu_{Q+u}}(w) - B_n(w)| \\ &\quad \le E_{u\sim U_n} \frac{1}{\mathbf{n+1}} \sum_{a=0}^{\mathbf{n}} |E_{\mu_{Q+u}} e_{\theta_a} - E_{U_n} e_{\theta_a}| \\ &\quad = \frac{1}{\mathbf{n+1}} \sum_{a=0}^{\mathbf{n}} E_{u\sim U_n} |E_{\mu_{Q+u}} e_{\theta_a} - E_{U_n} e_{\theta_a}| \\ &\quad \le \max_\theta |E_{\mu_{Q+u}} e_\theta - E_{U_n} e_\theta| \\ &\quad \le \max_\theta \sqrt{E_{u\sim U_n} |E_{\mu_{Q+u}} e_\theta - E_{U_n} e_\theta|^2}. \end{aligned}$$

Theorem 2 then follows from Theorem 5.

References

[1] L. Bazzi, "Weight distribution of cosets of small codes with good dual properties," *IEEE Trans. Inf. Theory*, vol. 61, no. 12, pp. 6493–6504, Dec. 2015.

[2] G. D. Cohen, "A nonconstructive upper bound on covering radius," *IEEE Trans. Inf. Theory*, vol. IT-29, no. 3, pp. 352–353, May 1983.

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