Errata and Corrections

Errata to "Codes and Designs Related to Lifted MRD Codes"

Tuvi Etzion, *Fellow, IEEE*, and Natalia Silberstein

In the above titled paper [1], the following correction is necessary.

The matrix in Example 7 (p. 1014) is incorrect. The correct matrix is as follows.

$$
\begin{array}{c}
\begin{array}{cccc|cccc|cccc}
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\
\mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\
\mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} \\
\mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\
\hline
\end{array} \\
\left(\begin{array}{cccc|cccc|cccc}
1&0&0&0&1&0&0&0&1&0&0&0\\
0&1&0&0&0&0&0&1&0&0&1&0\\
0&0&1&0&0&1&0&0&0&0&0&1\\
0&0&0&1&0&0&1&0&0&1&0&0\\
\hline
1&0&0&0&0&1&0&0&0&1&0&0\\
0&1&0&0&0&0&1&0&0&0&0&1\\
0&0&1&0&1&0&0&0&0&0&1&0\\
0&0&0&1&0&0&0&1&1&0&0&0\\
\hline
1&0&0&0&0&0&1&0&0&0&1&0\\
0&1&0&0&0&1&0&0&1&0&0&0\\
0&0&1&0&0&0&0&1&0&1&0&0\\
0&0&0&1&1&0&0&0&0&0&0&1\\
\hline
1&0&0&0&0&0&0&1&0&0&0&1\\
0&1&0&0&1&0&0&0&0&1&0&0\\
0&0&1&0&0&0&1&0&1&0&0&0\\
0&0&0&1&0&1&0&0&0&0&1&0
\end{array}\right)
\end{array}
$$

References

[1] T. Etzion and N. Silberstein, "Codes and designs related to lifted MRD codes," *IEEE Trans. Inf. Theory*, vol. 59, no. 2, pp. 1004–1017, Feb. 2013.

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Corrections to "ADMiRA: Atomic Decomposition for Minimum Rank Approximation"

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***Abstract*—In this correspondence, a corrected version of the convergence analysis given by Lee and Bresler is presented.**

***Index Terms*—Compressed sensing, matrix completion, performance guarantee, rank minimization, singular value decomposition.**

I. Corrections

First, we present a correction of [1, Th. 6.1]. The proof of [1, Lemma 6.2] had an error, which we correct by replacing [1, Lemma 6.2] by Lemma 2.1. To improve the threshold for the R-RIC, we also provide improved versions of [1, Lemmata 6.4 and 6.5]. The modified convergence results due to these replacements, presented in Theorem 1.1, are identical to the original version [1, Th. 6.1], except minor improvements on the constant in the noise term, from 13ϵ to 11ϵ, and on the required R-RIP condition, from $\delta_{4r}(\mathcal{A}) < 0.04$ to $\delta_{4r}(\mathcal{A}) < 0.065$. In fact, to guarantee just linear convergence (that is, replacing the error decay constant 0.5 in Theorem 1.1 by a number $0 < a < 1$), it suffices to satisfy $\delta_{4r}(\mathcal{A}) < 0.26$. These corrections apply straightforwardly also to [1, Th. 4.1] for the compressible matrix case, with similar improvement to $\delta_{4r}(\mathcal{A}) < 0.065$, and slight reduction of the constant factor on ϵ from 16 to 14.

Theorem 1.1 (Correction of [1, Th. 6.1]): Let $\widehat{X}_k$ denote the estimate of X_0 in the kth iteration of ADMiRA. Assume that $\operatorname{rank}(X_0) \leq r$. If $\delta_{4r}(\mathcal{A}) \leq 0.065$, then for each $k \geq 0$, $\widehat{X}_k$ satisfies the following recursion:

$$\|X_0 - \widehat{X}_{k+1}\|_F \leq 0.5\|X_0 - \widehat{X}_k\|_F + 5.5\epsilon$$

where ϵ is the unrecoverable energy. From the aforementioned relation, it follows that

$$\|X_0 - \widehat{X}_k\|_F \leq 2^{-k}\|X_0\|_F + 11\epsilon, \quad \forall k \geq 0.$$

Second, we withdraw the claim [1, Th. 4.2] that ADMiRA converges within finitely many steps. The finite convergence property of CoSaMP in the vector case does not extend to that of ADMiRA in the matrix case. However, the linear convergence of ADMiRA implied by Theorem 1.1 is still valid.

II. Proof of Theorem 1.1

The proof of Theorem 1.1 is almost identical to that of the original version [1, Th. 6.1]. Only a few lemmata are replaced as presented below. We will present only the differences from the corresponding original versions.

First, we replace [1, Lemma 6.2], which had an error in its proof, by the following result, proved in the Appendix.

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Lemma 2.1 (Replaces [1, Lemma 6.2]): Under the same conditions as in Theorem 1.1

$$\|\mathcal{P}_{\Psi'}^{\perp}(X_0 - \widehat{X})\|_F \le 0.493\|X_0 - \widehat{X}\|_F + 3.642\|\nu\|_{\ell_2^p}.$$

Lemma 2.1 is the most significant change in the guarantee of ADMiRA from that of CoSaMP.

Next, similarly to the recent improvements on the analysis of CoSaMP [2], we improve the constants in [1, Lemmata 6.4 and 6.5] as follows. The proofs of Lemmata 2.2 and 2.3 only require replacing vector notations and RIP in [2] by the corresponding matrix notations using the analogy and correspondences between the sparse vector and low-rank matrix case introduced in [1]. Therefore, we do not provide the details in this correspondence.

Lemma 2.2 (Replaces [1, Lemma 6.4]): Under the same conditions as in Theorem 1.1

$$\|\widetilde{X} - X_0\|_F \le 1.002\|\mathcal{P}_{\widetilde{\Psi}}^{\perp} X_0\|_F + 1.104\|\nu\|_{\ell_2^p}.$$

Lemma 2.3 (Replaces [1, Lemma 6.5]): Under the same conditions as in Theorem 1.1

$$\|\Pi_r(\widetilde{X}) - X_0\|_F \le 1.007\|\widetilde{X} - X_0\|_F + 1.788\|\nu\|_{\ell_2^p}.$$

Combining the aforementioned lemmata and [1, Lemma 6.3] just as in the proof of [1, Th. 6.1] provides the proof of Theorem 1.1.

APPENDIX

1) Proof of Lemma 2.1: Let $\Phi = \mathrm{atoms}(X_0 - \widehat{X})$. Since $|\Phi| \le \mathrm{rank}(X_0) + \mathrm{rank}(\widehat{X}) \le 2r$, it follows by the selection rule of Ψ' that

$$\|\mathcal{P}_{\Phi}\mathcal{A}^*(b - \mathcal{A}\widehat{X})\|_F \le \|\mathcal{P}_{\Psi'}\mathcal{A}^*(b - \mathcal{A}\widehat{X})\|_F. \tag{1.1}$$

First, we derive a lower bound on the left-hand side of (1.1) as

$$\begin{aligned}
&\|\mathcal{P}_{\Phi}\mathcal{A}^*(b - \mathcal{A}\widehat{X})\|_F \\
&= \|\mathcal{P}_{\Phi}\mathcal{A}^*\mathcal{A}(X_0 - \widehat{X}) + \mathcal{P}_{\Phi}\mathcal{A}^*\nu\|_F \\
&\ge \|\mathcal{P}_{\Phi}\mathcal{A}^*\mathcal{A}(X_0 - \widehat{X})\|_F - \|\mathcal{P}_{\Phi}\mathcal{A}^*\nu\|_F \\
&= \|\mathcal{P}_{\Phi}\mathcal{A}^*\mathcal{A}\mathcal{P}_{\Phi}(X_0 - \widehat{X})\|_F - \|\mathcal{P}_{\Phi}\mathcal{A}^*\nu\|_F \\
&\ge (1 - \delta_{2r}(\mathcal{A}))\|X_0 - \widehat{X}\|_F - \sqrt{1+\delta_{2r}(\mathcal{A})}\|\nu\|_{\ell_2^p}
\end{aligned} \tag{1.2}$$

where the last inequality follows from the R-RIP of $\mathcal{A}$ and Proposition 5.6 and Proposition 5.4 of [1].

Then, we derive an upper bound on the right-hand side of (1.1) as

$$\begin{aligned}
&\|\mathcal{P}_{\Psi'}\mathcal{A}^*(b - \mathcal{A}\widehat{X})\|_F \\
&= \|\mathcal{P}_{\Psi'}\mathcal{A}^*\mathcal{A}(X_0 - \widehat{X}) + \mathcal{P}_{\Psi'}\mathcal{A}^*\nu\|_F \\
&\le \|\mathcal{P}_{\Psi'}\mathcal{A}^*\mathcal{A}(X_0 - \widehat{X})\|_F + \|\mathcal{P}_{\Psi'}\mathcal{A}^*\nu\|_F \\
&\le \|\mathcal{P}_{\Psi'}(\mathcal{A}^*\mathcal{A} - \mathcal{I})(X_0 - \widehat{X})\|_F \\
&\quad + \|\mathcal{P}_{\Psi'}(X_0 - \widehat{X})\|_F + \|\mathcal{P}_{\Psi'}\mathcal{A}^*\nu\|_F \\
&= \|\mathcal{P}_{\Psi'}(\mathcal{A}^*\mathcal{A} - \mathcal{I})\mathcal{P}_{\Phi}(X_0 - \widehat{X})\|_F \\
&\quad + \|\mathcal{P}_{\Psi'}(X_0 - \widehat{X})\|_F + \|\mathcal{P}_{\Psi'}\mathcal{A}^*\nu\|_F \\
&\le \delta_{4r}(\mathcal{A})\|X_0 - \widehat{X}\|_F \\
&\quad + \|\mathcal{P}_{\Psi'}(X_0 - \widehat{X})\|_F + \sqrt{1+\delta_{2r}(\mathcal{A})}\|\nu\|_{\ell_2^p}
\end{aligned} \tag{1.3}$$

where the last inequality follows from Proposition 5.4 of [1] and the R-RIP of $\mathcal{A}$ since

$$\begin{aligned}
&\|\mathcal{P}_{\Psi'}(\mathcal{A}^*\mathcal{A} - \mathcal{I})\mathcal{P}_{\Phi}\| \\
&\le \|\mathcal{P}_{\Psi'\cup\Phi}(\mathcal{A}^*\mathcal{A} - \mathcal{I})\mathcal{P}_{\Psi'\cup\Phi}\| \\
&\le \sup_{\substack{\Psi\in\mathbb{O} \\ |\Psi|\le 4r}} \|\mathcal{P}_{\Psi}(\mathcal{A}^*\mathcal{A} - \mathcal{I})\mathcal{P}_{\Psi}\| = \delta_{4r}(\mathcal{A}).
\end{aligned}$$

Applying (1.2), (1.3) to (1.1), we obtain

$$\begin{aligned}
\|\mathcal{P}_{\Psi'}(X_0 - \widehat{X})\|_F \ge{}& (1 - 2\delta_{4r}(\mathcal{A}))\|X_0 - \widehat{X}\|_F \\
&- 2\sqrt{1+\delta_{2r}(\mathcal{A})}\|\nu\|_{\ell_2^p}
\end{aligned}$$

which implies

$$\begin{aligned}
\frac{\|\mathcal{P}_{\Psi'}(X_0 - \widehat{X})\|_F}{\|X_0 - \widehat{X}\|_F} \ge{}& \underbrace{1 - 2\delta_{4r}(\mathcal{A})}_{=\alpha} \\
&- \underbrace{2\sqrt{1+\delta_{2r}(\mathcal{A})}}_{=\beta} \cdot \underbrace{\frac{\|\nu\|_{\ell_2^p}}{\|X_0 - \widehat{X}\|_F}}_{=\omega}.
\end{aligned}$$

If $\alpha - \beta\omega < 0$, then

$$\|\mathcal{P}_{\Psi'}^{\perp}(X_0 - \widehat{X})\|_F \le \|X_0 - \widehat{X}\|_F < \frac{2\sqrt{1+\delta_{2r}(\mathcal{A})}}{1 - 2\delta_{4r}(\mathcal{A})}\|\nu\|_{\ell_2^p}. \tag{1.4}$$

Since $\delta_{4r}(\mathcal{A}) \le 0.065$, (1.4) is rewritten as

$$\|\mathcal{P}_{\Psi'}^{\perp}(X_0 - \widehat{X})\|_F < 2.373\|\nu\|_{\ell_2^p}$$

which implies the desired inequality.

It remains to consider the other case, $\alpha - \beta\omega \ge 0$. By the Pythagorean theorem, we have

$$\begin{aligned}
\frac{\|\mathcal{P}_{\Psi'}^{\perp}(X_0 - \widehat{X})\|_F}{\|X_0 - \widehat{X}\|_F} &= \sqrt{1 - \left(\frac{\|\mathcal{P}_{\Psi'}(X_0 - \widehat{X})\|_F}{\|X_0 - \widehat{X}\|_F}\right)^2} \\
&\le \underbrace{\sqrt{1 - (\alpha - \beta\omega)^2}}_{(*)}.
\end{aligned}$$

Since $g(t) = \sqrt{1 - t^2}$ is a convex function in t, a tangent line at $t = t_0$ for $t_0 \in [0, 1)$ provides an upper bound on $g(t)$, i.e.,

$$h(t) \triangleq \frac{-t_0}{\sqrt{1 - t_0^2}} \cdot t + \frac{1}{\sqrt{1 - t_0^2}} \ge g(t), \quad \forall t \in [0, 1].$$

Therefore, $(*)$ is further bounded from above by

$$(*) \le \frac{-t_0\alpha + 1}{\sqrt{1 - t_0^2}} + \frac{t_0\beta\omega}{\sqrt{1 - t_0^2}} \tag{1.5}$$

for any $t_0 \in [0, 1)$. For example, for given α, we can choose t_0 as the minimizer $t_0 = \alpha$ of the first summand in (1.5). Then, substituting $\delta_{2r}(\mathcal{A}) \le \delta_{4r}(\mathcal{A}) \le 0.065$ into α and β yields the desired inequality.

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REFERENCES

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