Corrections to "High-Throughput Random Access Using Successive Interference Cancellation in a Tree Algorithm"

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***Abstract*— In the above article, the authors propose d-ary SICTA and derive the expected conditional length of the collision resolution interval, optimal splitting probability and the maximum stable throughput (MST) for $d \geq 2$ under stationary ergodic packet arrivals. In this correction, we show that the premise of the analysis for $d > 2$ and consequentially the results presented for $d > 2$ do not hold.**

***Index Terms*— Collision resolution, maximum stable throughput, random access, and tree protocol.**

I. Correction to the Premise of the Analysis

The analysis presented in [1, Section IV-A] is performed for d-ary tree algorithms with successive interference cancellation (SIC), where $d \geq 2$. The premise of the analysis states that the length of the collision resolution interval (CRI) in slots, given that n users collided in the first slot, is (verbatim):

$$l_n = \begin{cases} 1, & \text{if } n = 0, 1 \\ \sum_{j=1}^{d} l_{I_j}, & \text{if } n \geq 2 \end{cases} \tag{1}$$

where I_j is the number of nodes that select the j-th group, $j \in \{1, 2, \ldots, d\}$. However, the premise of the analysis does not hold for $d > 2$. We illustrate the shortcomings of the analysis through an example with $n = 2$, depicted in Fig. 1. This example is similar to the example in [1, Fig. 3], only with $d = 3$ instead of 2. In Fig. 1a, node A selects the first group and node B selects the second group after a collision in slot 1. The receiver is able to decode the transmission occurring in slot 2 and after applying SIC, is also able to recover the remaining transmission in slot 1. As these two transmissions are the only ones in this tree, there is no need for further splitting, and the total duration of the collision resolution interval (CRI) is 2 slots. However, according to (1), the CRI length in this example should be

$$l_2 = l_1 + l_1 + l_0 = 3. \tag{2}$$

Note that the CRI would also have been 2 slots if node B had selected the third group. In fact, if $n = 2$ and the first group has a single node, like in the Fig. 1a, the length of the CRI will be 2 slots, irrespective of the value of the splitting factor d (given that $d \geq 2$). On the other hand, the example in Fig. 1b shows the case when the CRI length is indeed 3 slots and agrees with the formula (1). This happens only when no collided node selects the first group.

Thus, the conclusion drawn in [1] (verbatim)

$$(d-1)(L'_n - 1) = d(L_n - 1) \tag{3}$$

where L'_n and L_n are the expected conditional length of the CRI for the standard tree algorithm (STA) [2] and SICTA, respectively, does not hold for $d > 2$. This invalidates the subsequent analysis in [1] that exploited the results known for STA.

Generalizing the insights shown above, we write

$$l_n = \begin{cases} 1, & n = 0, 1 \\ \sum_{j=1}^{d_{\min}} l_{I_j}, & n \geq 2 \end{cases} \tag{4}$$

where $d_{\min}$ is the minimum value of $o \in \{1, \ldots, d\}$ for which the following holds

$$\sum_{j=1}^{o} I_j \geq n - 1. \tag{5}$$

The explanation of (4) is that the splitting process will stop as soon as there is a single user remaining from the original collision, no matter how many groups are left.

When $d > 2$, the expressions (4) can not be computed in the same manner as it can be done when $d = 2$. In particular, the summands in (4) are subject to the same recursion that holds for L_n, making the overall computation intractable.

II. Remarks on MST of SICTA With Gated Access

The above article [1] also provides results for MST of d-ary SICTA with gated access, claiming (i) that fair splitting is the optimal choice for any $d \geq 2$ and (ii) that binary fair splitting achieves the highest MST, see [1, Section IV-B]. However,

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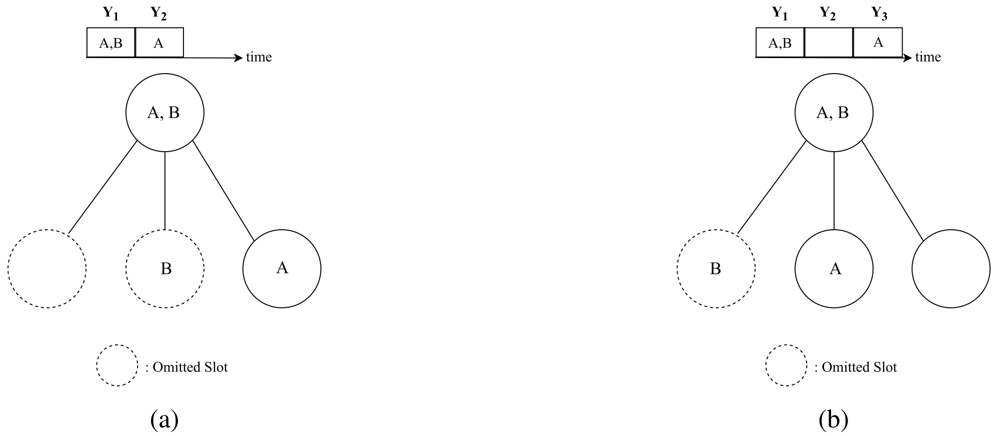

Fig. 1. Example of a collision resolution with $n = 2$ and $d = 3$: (a) The CRI is 2 slots when there is single node in the 1st sub-group. According to [1], this CRI should have been 3 slots. (b) The case when the CRI is indeed 3 slots.

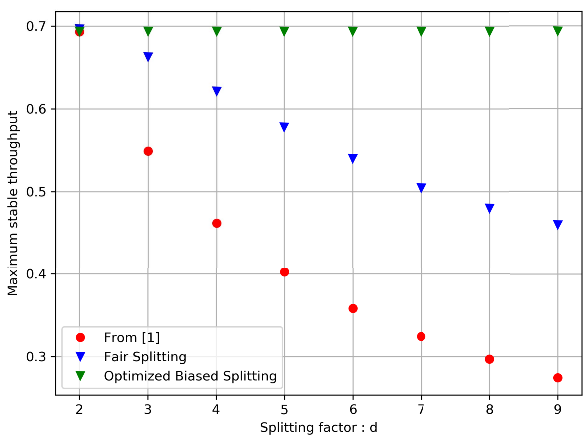

Fig. 2. Throughput performance of d-ary tree-algorithms with SIC as function of splitting factor d: red dots represent MST results from [1], blue and green triangles represent the mean throughput obtained via simulations for $n = 1000$ averaged over 10000 simulation runs with fair splitting and optimized biased-splitting, respectively.

these results do not hold for $d > 2$, a consequence of the fact that equation (3) does not hold for $d > 2$.

Fig. 2 compares the results for MST with the gated access presented in [1], shown in [1, Fig 6.], with the ones obtained through simulations by measuring the actual CRI length L_n. The number of nodes $n = 1000$ and the throughput for each run is calculated as n/L_n. The MST plotted in Fig. 2 is obtained by averaging the throughput of 10000 simulation runs. Such obtained throughput, for a high number of nodes n, tends to serve as a proxy for the MST for gated access,

$$T = \frac{1}{\lim_{n\to\infty} L_n/n} \tag{6}$$

see [1, Section IV-B], where T denotes the MST.

The mismatch between simulation results for fair-splitting and the analytical result in [1] given by $T \approx \ln(d)/d - 1$ for $d > 2$ can be verified in Fig. 2. The figure also shows that when the optimized biased-splitting is applied for $d > 2$, the MST has the same value[1] obtained for $d = 2$, which is $\ln 2$. For the optimized splitting probabilities we used the following values

$$p_j = \begin{cases} 0.5^j & j \in \{1, .., d-1\} \\ 0.5^{d-1} & j = d \end{cases} \tag{7}$$

where p_j is the probability that a node selects the j-th group, $j \in \{1, .., d\}$.

References

[1] Y. Yu and G. B. Giannakis, "High-throughput random access using successive interference cancellation in a tree algorithm," *IEEE Trans. Inf. Theory*, vol. 53, no. 12, pp. 4628–4639, Dec. 2007.

[2] J. Capetanakis, "Tree algorithms for packet broadcast channels," *IEEE Trans. Inf. Theory*, vol. IT-25, no. 5, pp. 505–515, Sep. 1979.

[1]Disregarding the precision loss due to the averaging over a finite number of realizations.