Aspects of and Insights Into the Rigorous Validation, Verification, and Testing Processes for a Commercial Electromagnetic Field Solver Package

Ulrich Jakobus, *Fellow, IEEE*, Renier Gustav Marchand, and Daniël J. Ludick

***Abstract*—This paper focuses on rigorous validation, verification, and testing methodologies applied to a commercial electromagnetic software package to ensure that as accurate as possible results are given dependent on the accuracy of the solution method, for instance, whether a full-wave or approximate numerical method is used. In this paper, the general availability of reliable benchmark results such as analytical solutions, measurements, results from other codes and other numerical methods, and general benchmarking activities will be presented. The cross-validation aspects, once the benchmark results are available, will be discussed with respect to amongst other sequential runs compared with parallel multcore/cluster runs or, with and without, GPU acceleration. Internal consistency checks (which are a required but not necessary condition when assessing the accuracy) such as power budget, mesh size convergence, or boundary condition error estimates are also covered. Special emphasis is put on the validation of the actual computational model that is used as input to simulations. This is necessary, for example, because incomplete representation of real geometry might ignore small details that are needed for the specific quantity that is analyzed. Also, uncertainties with regards to material parameters or transition impedances could lead to discrepancies between the computed results and reality that are not to be attributed to the electromagnetic solution as such, but rather the model generation.**

***Index Terms*—Numerical methods, quality assurance (QA), testing, validation.**

I. Introduction

THE numerical modeling of real-life electromagnetic phenomena plays a significant part in the electromagnetic compatibility (EMC) community, amongst many others. Strenuous validation of the electromagnetic solution techniques that are used for the simulation is however required to place confidence in the results that are produced by both proprietary and also commercially available field solver packages.

Validation, verification, and testing will be investigated from the perspective of an actively developed commercial code where changes to existing algorithms are continuously made and new algorithms are regularly added. The terms validation and verification have different meanings to different people and it is therefore important to clarify the meanings as used in this paper. Validation of code will refer to the activity of comparing computed results to physical reality and trying to establish to what extent the computational method and implementation thereof compares with physical reality. This agrees with the definition of the American Institute of Aeronautics and Astronautics (AIAA) [1] as well as the concepts of "Individual software code implementation validation" and "Specific model validation" discussed in the IEEE 1597.1 standard [2]. Verification in this paper will refer to the consistency checks that an individual user should do when creating a computational model. These consistency checks include whether results are stable and convergent under mesh refinement and whether similar results are obtained using two or more different numerical methods. Verification may also refer to the internal checks done to ensure that the conceptual computational mathematics are correctly implemented (e.g., are the theoretical convergence rates achieved for a specific method?) [3]. This definition agrees with the definition for verification of the AIAA [1] and as discussed by Roache [4].

Testing in this paper will refer to the specific software quality assurance (QA) activities that ensure that the validated results are consistent between versions of the code across multiple platforms.

Standards such as the IEEE 1597 [2], [5], and the IEEE 1528 [6] are excellent sources to validate the correctness of computational codes. These standards and recommended practices include problems that might be challenging to solve and are representative of a certain class of problems that are solved by users of electromagnetic codes. The analytical problems defined in these standards can be used as test cases against which convergence studies can be done and hence supply extremely accurate verification examples. These verification examples are simple but not exhaustive and can be supplemented by artificial solutions generated by the method of manufactured solutions [7], [8]. Verification can also be done by looking at expected theoretical quantities such as boundary conditions between material interfaces or the power budget of the overall problem.

Testing/software QA is an active field in the computer science community. Various methods such as mutation testing [9] exist to investigate the code coverage and sufficiency of a test set.

Comparison between measured results and computed results is ultimately a subjective process that has to be done by an expert. In the past decade, the feature selective validation (FSV) has emerged as an attempt to introduce objectivity into this comparison process [10], [11]. FSV uses two components—the

Manuscript received May 8, 2013; revised August 27, 2013; accepted October 10, 2013. Date of publication January 24, 2014; date of current version July 29, 2014.

The authors are with the EM Software & Systems—S.A. (Pty) Ltd, Stellenbosch, 7600, South Africa (e-mail: jakobus@emss.co.za; rmarchand@emss.co.za; dludick@emss.co.za).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TEMC.2014.2299408

amplitude difference measure (ADM) and the feature difference measure (FDM)—to compare different results in a more objective way. A practical application of FSV will be shown for one of the examples considered later on.

The purpose of this paper is to present some aspects of the rigorous validation, verification, and testing processes that are used during the development of a commercial electromagnetic field solver package.

This paper is organized as follows: Section II discusses the use of reliable benchmark results obtained through analytical solutions, published data from other codes, published data from specific benchmarking activities, and measured results. Section III introduces the concept of cross-validations. It focuses on the use of different numerical techniques as well as different platforms/solution setups applied to the same problem set. Section IV deals with internal consistency checks as required (but not sufficient) conditions in verifying the numerical results for electromagnetic problems. These consistency checks include power budget checks, boundary condition checks, and numerical convergence checks. Automated regression testing is discussed in Section V as a means to assure quality of the numerical results obtained by techniques that are continually being extended in a changing commercial environment. In some cases, the results produced by solution techniques can be interpreted as inaccurate by incorrect modeling and/or inexperience, which is highlighted in Section VI. The paper is then concluded in Section VII.

II. Validation of Computational Results Using Known/Trusted Reference Data

The most natural way to confirm the accuracy of computational electromagnetic results is the comparison with established trusted results that can be obtained from various sources as outlined in the following subsections.

A. Validating Against Analytical Results for Selected Problems

Analytical results for certain problems are quite often regarded as the most trustworthy way of assessing the accuracy of the electromagnetic simulation results without being affected by measurement uncertainties and tolerances or the fact that the measurement equipment as such might distort the quantity to be measured (e.g., probe or cable). Unfortunately, such analytical results are only available for a very special class of problems, and one often uses certain approximations when evaluating such analytical results (e.g., numerical truncation of an infinite series of spherical Bessel functions for spherical problems). Nonetheless, these analytical solutions are an important factor in the overall validation process.

Probably, the most often used structures for the validation of 3-D field solvers using analytical results are spherical objects. These objects include perfectly conducting, dielectric, and different material nested structures where exact solutions in terms of the Mie series [12] exist for surface currents, near- and far-fields when excited with an incident plane wave.

Fig. 1 compares the numerical results of the method of moments (MoM) [13] solver with the analytical far-field radar cross section for a perfectly electric conducting (PEC) sphere given by

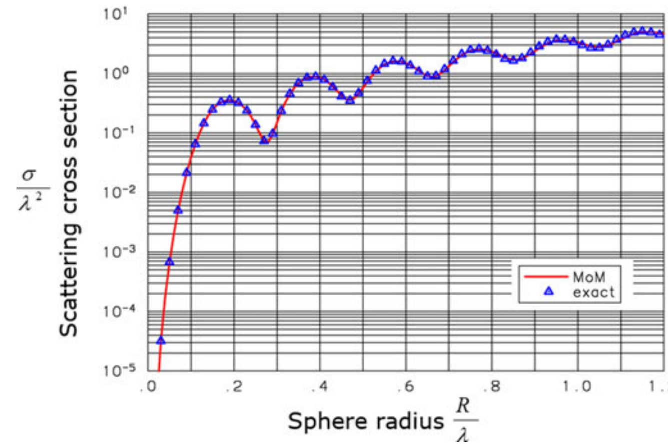

Fig. 1. Comparison of the MoM to the exact solution of the far-field scattering of a plane wave incident onto a PEC sphere with radius R normalized to the wavelength λ.

$$\sigma = \frac{\lambda^2}{4\pi} \cdot \left| \sum_{n=1}^{\infty} \frac{(-1)^n \, (2n+1)}{\hat{H}_n^{(2)}(\beta R) \; \hat{H}_n^{(2)'}(\beta R)} \right|^2 \tag{1}$$

where R is the radius of the sphere, $\beta = 2\pi/\lambda$ the wavenumber, λ the wavelength, and $\hat{H}_n^{(2)}$ is the spherical Hankel function of the second kind.

Analytical validation models can be extended to cover dielectric objects. Fig. 2 shows the electric near-field inside a homogeneous dielectric sphere with radius 90 mm. This is benchmark problem no. 1 of the working group for numerical methods of the German IEEE EMC chapter [14], [15]. For illustrative purposes, we used the popular Rao–Wilton–Glisson (RWG) [16] basis functions on flat triangular patches for the MoM solution as well as a scheme with higher order basis functions (HOBF) on curvilinear mesh elements [17], [18]. The RWG model uses flat triangular patches and consequently introduces geometrical modeling errors that cause the shape of the object to deviate from the exact sphere and which cause the boundaries of this meshed model to deviate slightly from $x = \pm 90$ mm. Results for the multiple multipole method (MMP) [19], [20] are also shown. Despite the surface boundary approximation effects for the MoM, the general error as compared to the analytical Mie series for the RWG result is below 1% and can be reduced even further by using a curvilinear mesh with HOBF. The MMP solution can be improved by using more terms for the multipole expansion.

Another quantity of relevance for EMC problems is shielding. As an example, Fig. 3 shows the magnetic shielding factor at the center of a thin hollow sphere computed with the full-wave MoM and compared with the analytical formulas of Kaden [21]. The sphere is made of steel with a conductivity of $\sigma = 1.3 \times 10^6$ S/m, has a wall thickness of $d = 1$ mm and a radius of $r = 0.15$ m. Special MoM extensions are required to achieve such an excellent agreement between the MoM and the analytical Kaden formulas for such high shielding factors up to 260 dB

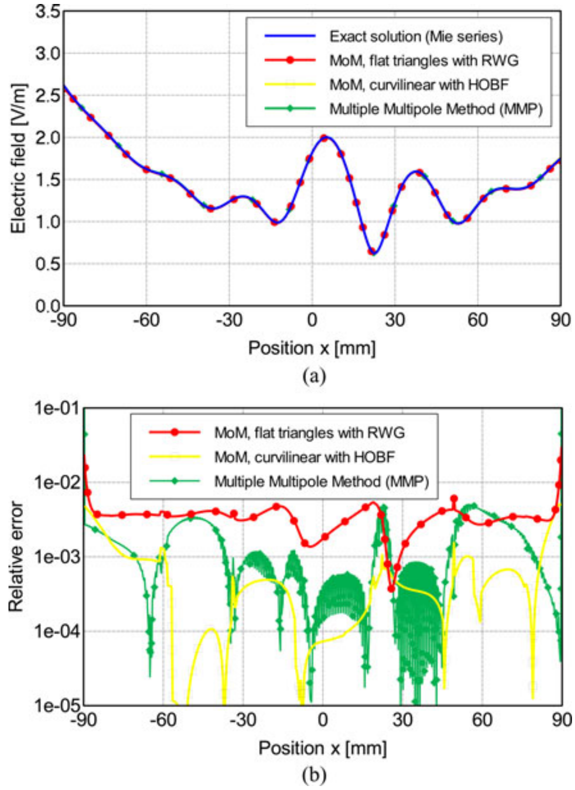

Fig. 2. Electric near-field inside a dielectric sphere with radius 90 mm and $\varepsilon_r = 44 - j19$ solved at 900 MHz. (a) Numerical MoM and MMP solutions compared to analytical Mie series, (b) relative error of numerical versus analytical results. (a) Electric near-field inside sphere. (b) Relative error.

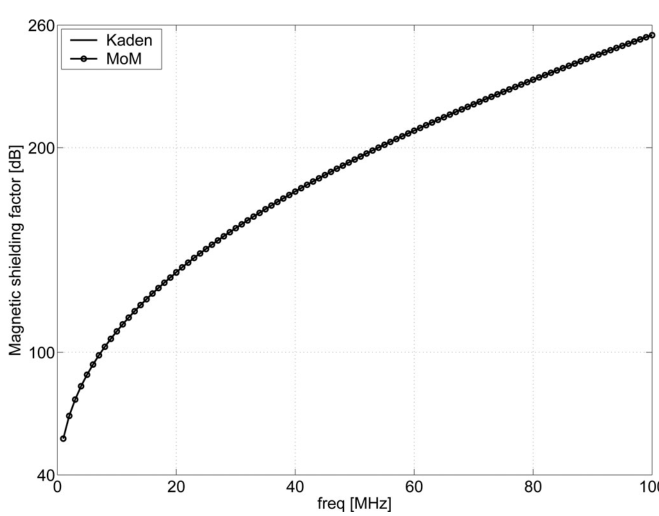

Fig. 3. Magnetic shielding factor in the center of a steel sphere of radius $r = 0.15$ m and wall thickness of $d = 1$ mm.

(decoupling of the internal and external regions and an analytical relationship describing penetration through the material, see [22] for some details).

One last example is considered for the validation of numerical results using analytical solutions. We look at validating the periodic boundary condition modeling for 3-D objects (used for instance when assessing shielding from body worn clothing by modeling a single elementary cell only and replicating this infinitely in two directions) by comparing the solutions to analytical results for 2-D infinite cylinders. The geometry is modeled with a 1-D periodic boundary condition, as illustrated in Fig. 4. The resultant scattering width (equivalent to the radar cross section RCS in 3-D) is compared in Fig. 5 to analytical results for an infinitely long cylinder (see e.g. [23]).

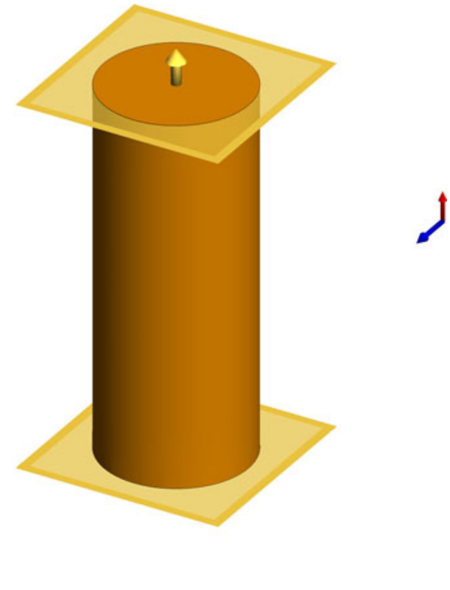

Fig. 4. Model of an infinitely long cylinder by using a finite cylindrical section and applying 1-D periodic boundary conditions at the top and bottom.

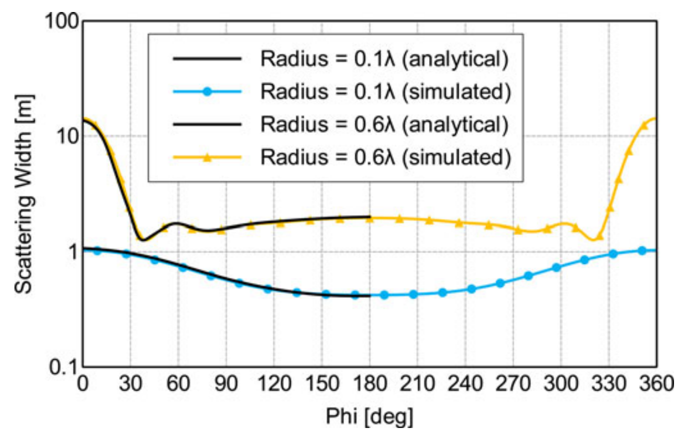

Fig. 5. Scattering width versus angle for infinitely long PEC cylinders of different radii. The solid lines on the left half represent the analytical solutions (see e.g. [23]) while the lines with markers represent numerical results computed from a 3-D MoM solution with periodic boundary conditions.

B. Validating Against Published Data in the Literature

Results generated by codes are often compared with results obtained and published in open literature, which can be theoretical, measured, or numerically simulated. Ideally, a combination of these should be used for validation. It is also preferable that simulated published results should be obtained using a different computational method than the method employed by the code that is being validated.

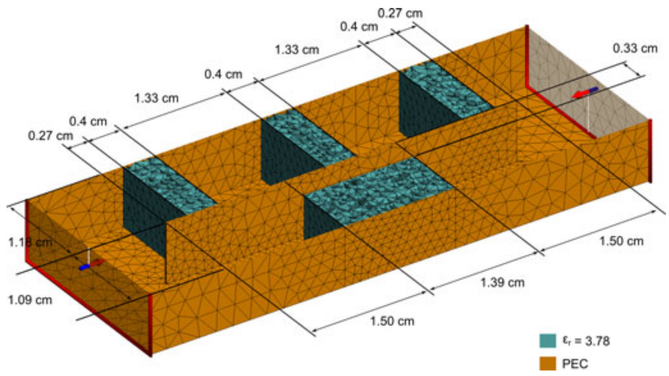

Fig. 6. Hybrid MoM/FEM model of a two-path cutoff filter as designed in [24].

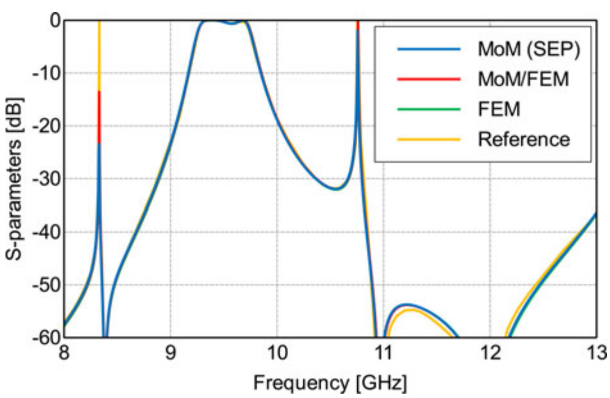

Fig. 7. Results for the insertion loss S_{21} of a two-path cutoff filter. Comparison between different numerical methods and reference results from [24].

A waveguide filter example with dielectric resonators from [24] shall be used as representative example. Theoretical and measured results are included in [24] which can be compared to other solution techniques. A hybrid MoM/FEM (Finite Element Method) model is shown in Fig. 6. The results for the insertion loss S_{21} for three methods hybrid MoM/FEM, standard MoM using the surface equivalence principle, and standard FEM are compared in Fig. 7 to the reference results from [24].

Unfortunately, in many cases, validations using published data fail due to an incomplete or inaccurate problem description (dimensions, material parameters etc.). If all data is available for reproduction, another uncertainty in published results is that typically only graphs are available, i.e., only a visual inspection is possible like done in Fig. 7 but no quantitative error analysis.

C. Validating Against Published Data From Specific Benchmarking Activities

There have been many attempts for general benchmarking of different codes, The definition of canonical problems where different numerical solutions from different groups using different solvers are available and in many cases also compared to measurements.

An example of a canonical problem is a monopole antenna on top of a table that couples into a coaxial cable underneath [25]. This example was first published by the Applied Computational Electromagnetics Society (ACES) as part of a special issue of its

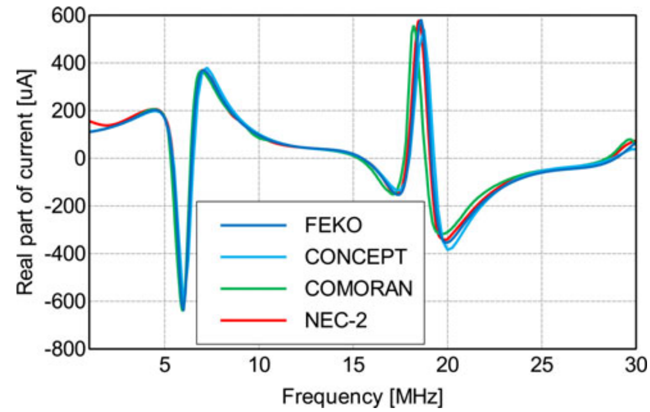

Fig. 8. Real part of the induced current due to a transmitting antenna coupling into a transmission line comparing different numerical codes (benchmark problem no. 3 from [14], [15], also in Section 5.3.1.2 of [5]).

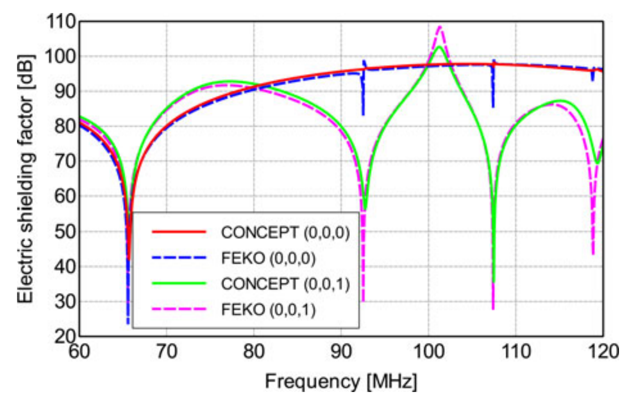

Fig. 9. Electric shielding factor for an enclosure with aperture at two different z positions comparing two MoM codes CONCEPT and FEKO (benchmark problem no. 5 from [14], [15]).

ACES Journal in 1990 that consisted of a collection of canonical problems. This problem subsequently became part (benchmark problem no. 4) of a set of EMC benchmark problems published by a working group on numerical methods of the German IEEE EMC Chapter [14], [15]. Later on, this problem was also included in Section V.3.2 of the IEEE Recommended Practice 1597.2 [5].

The plane wave incident upon a homogeneous dielectric sphere discussed above in Section II-A and Fig. 2 is benchmark problem no. 1 of the German IEEE EMC working group.

Benchmark problem no. 3 of the German IEEE EMC Chapter [14], [15] is also represented in Section V.3.1.2 of [5] and deals with the coupling of a transmitting antenna into an adjacent transmission line. Fig. 8 shows the real part of the induced current in the transmission line generated using four codes: Three independent MoM based codes namely NEC-2 [26], CONCEPT [27], and FEKO [28], and also one multiconductor transmission line (MTL) based code COMORAN [29].

Another interesting benchmark problem from an EMC perspective is shielding from an enclosure with a small aperture (benchmark problem no. 5 of the German IEEE EMC Chapter [14], [15]). The results for this problem are shown in Fig. 9 and compare the electric shielding factor at two locations inside the enclosure. The two MoM based solutions were computed using CONCEPT and FEKO.

Many other benchmarking activities for numerical codes exist, for instance for radar problems, an overview is given in [30] and the electromagnetic code consortium also has some benchmark models and results [31].

Numerical benchmarking and validation against measurements for different antenna types are conducted in the framework of the European Antenna Center of Excellence within SoftLAB (Software OnLine Antenna Benchmark), see [32]–[35] for details. Static and low-frequency modeling benchmark problems can specifically be found from the Compumag society available at [36].

D. Validating Against Measured Results

1) Validation of Computational Methods Using Measured Results: If analytical results, published benchmark data, or results from other trusted computational methods are not always available for specific problems, then validation using measured results is the only feasible option. Such measured results alone are not the ideal way to validate numerical methods due to a number of uncertainties (see discussions below), but are certainly a very valuable tool (reality is the ultimate benchmark). Validation against measured results of a specific new electromagnetic algorithm, application, or modeling process helps build confidence in a code base.

An EMC example of validation using measurements and the application of FSV shall be considered in the context of the interaction of electromagnetic fields with cables (radiation and/or irradiation). Full-wave models of such cables are often too large or complex in terms of run-time requirements and therefore MTL is often employed instead. The MTL is unfortunately limited to cables running close (within around $\lambda/10$) [37] of a conducting surface. To overcome this limitation, a combined MoM/MTL approach can be followed which models the current on the outer cable shield with MoM and only uses MTL for the inner cable interactions [38]. The models shown in Fig. 10 were used to investigate the effectiveness of the combined MoM/MTL method. The basic scenario considered is a RG58 coaxial cable running between two vertical metallic plates above a ground plane and interacting with an adjacent monopole antenna (either of the two can be excited, see Section IV-B for a discussion of cross-validation opportunities in terms of reciprocity). Two variations are considered, with the first being a single PEC ground plane (which the normal MTL can handle well) and the second being two separate PEC ground planes with a gap in between (where standard MTL fails and only the combined MoM/MTL can be applied).

Computed results are compared to measurements in Fig. 11 done in an open laboratory setup. This open laboratory setup causes ripple and as such is a good test for applying the FSV. The results agree well in general, as can be seen from visual inspection and is also confirmed through the FSV analysis in Fig. 12 (here exemplary done for the single ground plane case without gap).

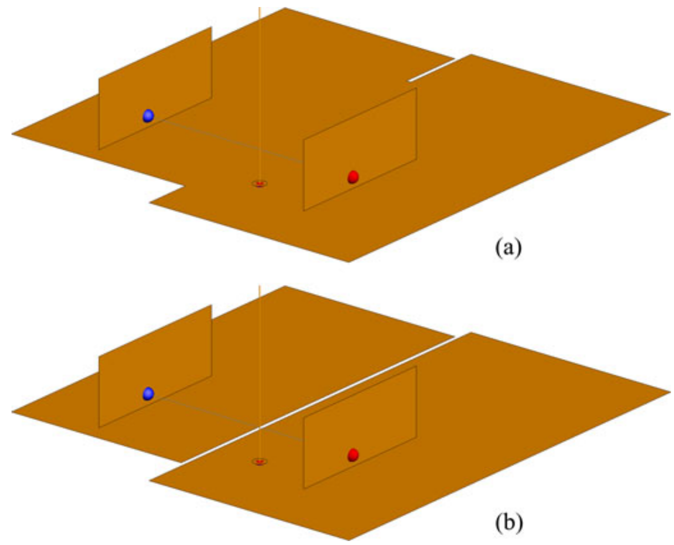

Fig. 10. Geometry setup of a shielded RG58 cable above different ground plane arrangements: (a) one single ground plane under the cable (solvable using MTL only) and (b) two separated ground planes with a slot in-between (not solvable using MTL only).

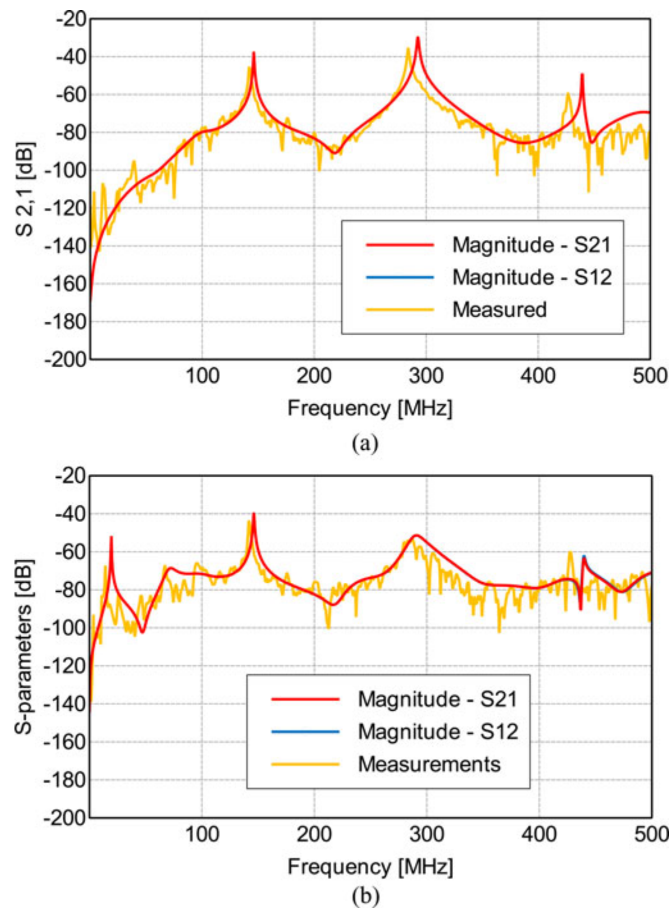

Fig. 11. Measured and computed results (both S_{21} and S_{12} with the MoM/MTL to check reciprocity) for the cable example of Fig. 10 with (a) the single PEC ground plane with no air gap and (b) the two PEC ground planes with an air gap separating them.

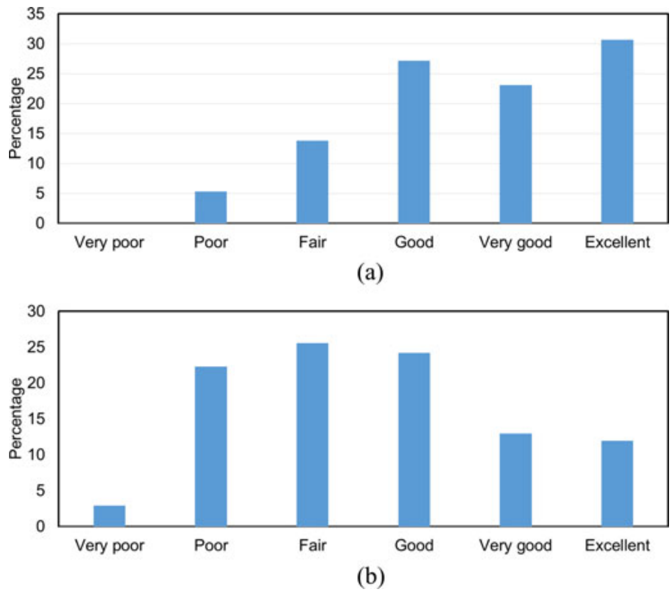

Fig. 12. FSV analysis for the measured versus computed results of Fig. 11 for the case of a ground plane with no gap: (a) the Amplitude Difference Measure (ADM = 0.259) and (b) the Finite Difference Measure (FDM = 0.551).

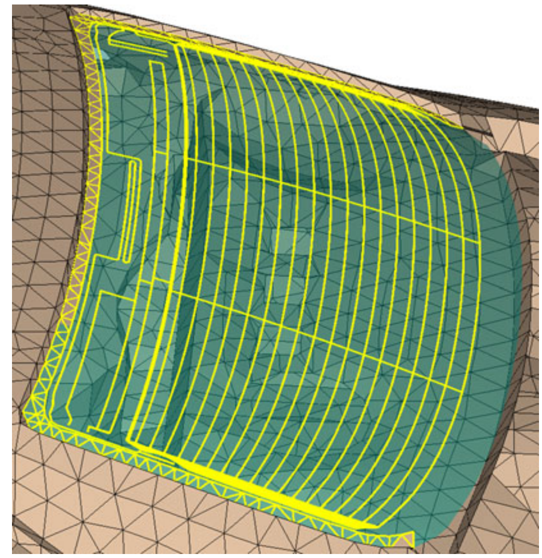

Fig. 13. Model of an integrated windscreen antenna (metallic elements embedded in dielectric glass layers).

2) Validation of the Computational Model From a Measurement Setup: Complex electromagnetic scenarios often have intricate features that cause considerable modeling uncertainty. In such a situation, measurements are often performed to verify the process of constructing a simulation model (level of details to be included etc.). As an example, a car with an integrated windscreen antenna is discussed. It is important to include a detailed model of the different windscreen layers for the purpose of windscreen antenna design but it is not important to include the car tires—these would of course be included for instance when modeling a tire pressure monitor system. It is very important to gain confidence in using the correct level of detail for a model. Over complication of a model leads to additional run-time that might be prohibitive for numerical optimizations or variational studies using different combinations of car models / windscreen types / antenna designs etc.

The example in Fig. 13 was generated and measured using the discussed measurement validation approach. Good agreement between measured and simulated magnitude and phase for the input reflection coefficient S_{11} is achieved in Fig. 14.

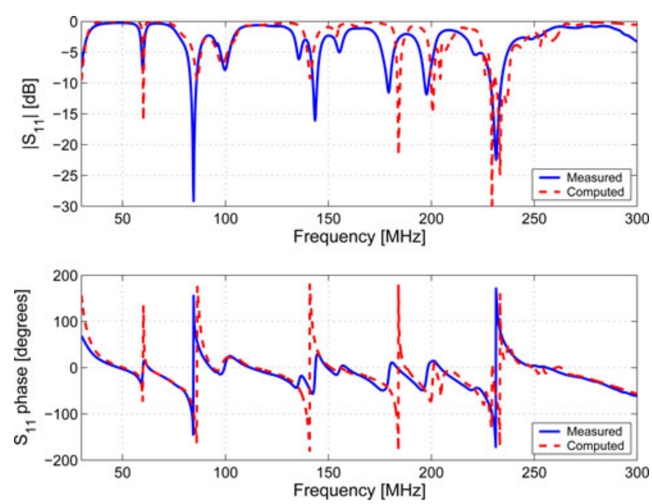

Fig. 14. Magnitude (top) and phase (bottom) of the input reflection coefficient S_{11} for the integrated windscreen antenna of Fig. 13.

Another example comparing measurements with numerical simulations to assess what must be included in the model and what can be ignored (e.g., linear motor to move the probe) is shown in Fig. 15. The electric near-field (basis for the specific absorption rate) is measured and computed inside a box filled with lossy dielectric tissue exposed to the near-field of a GSM base station antenna. The comparison of measurement versus simulations is shown in Fig. 16.

Sometimes, discrepancies occur when computational and measured results are compared which exceed "typical" measurement or modeling uncertainties (say in the order of a few percent). Such discrepancies are often the result of an over-simplified simulation environment or uncertainties with regards

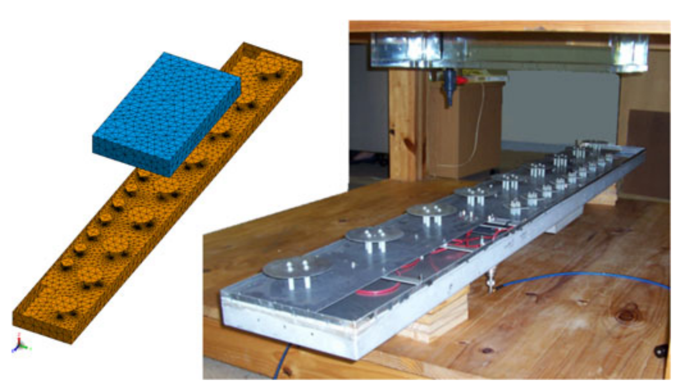

Fig. 15. GSM base station antenna close to a lossy dielectric box. Computational model (left) and measurement setup (right).

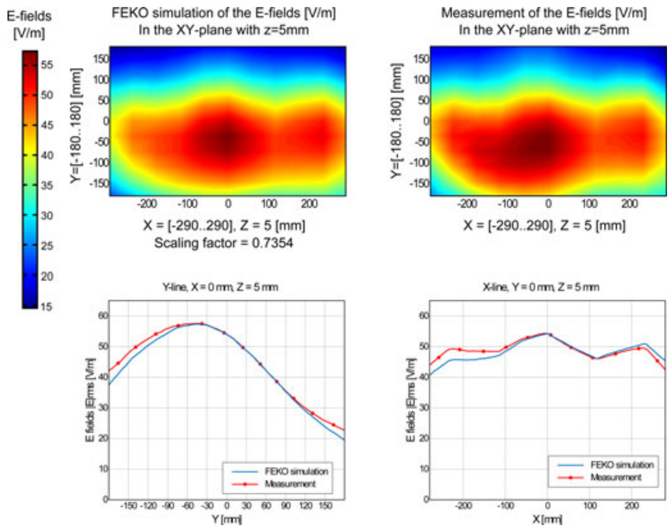

Fig. 16. Comparison of measurements to computational results for the electric near-field inside the lossy dielectric medium of Fig. 15.

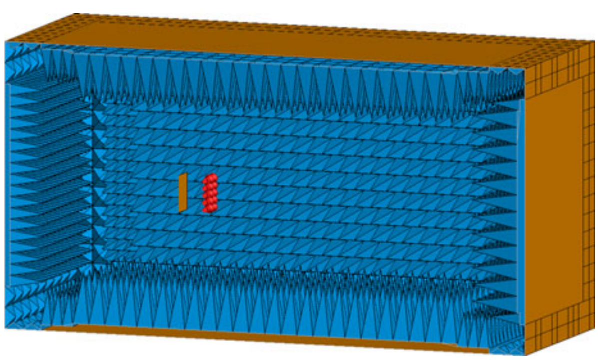

Fig. 17. Example of modeling a device under test inside of the measurement environment (anechoic chamber with pyramidal absorbers solved with FEM in [39]).

to dimensions/material parameters etc. It is therefore important to include environmental elements at least for selected simulations to assess their impact. This may even mean that the measurement setup must be included in the computational model. An example of the inclusion of the measurement environment is shown in Fig. 17 and discussed in [39]. The anechoic chamber was included in a specific electromagnetic modeling setup which also required a separate verification of the absorber model, as precise frequency dependent material parameters are often not readily available.

III. Cross-Validations Comparing Different Numerical Techniques and/or Different Solution Setups

Once the validity of some numerical algorithms to a certain class of problems has been established using the methods outlined previously, these results itself applied to similar problems can act as benchmark results to compare with. A special emphasis is here on the word similar: When, for instance, knowing that the results for a dielectric sphere in Fig. 2 are accurate enough within the desired accuracy levels, one might get similar confidence in the results when distorting the shape to an ellipsoid.

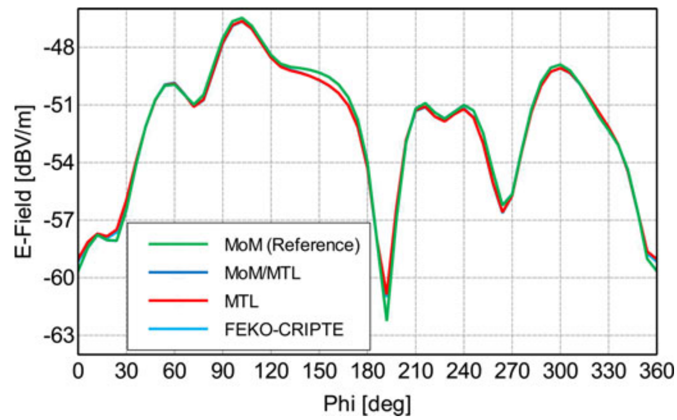

Fig. 18. Radiation from an L-shaped cable (see [38] for details). Comparison of different numerical methods involving full-wave MoM as reference and various transmission line modeling approaches.

TABLE I
Comparison of Memory and Run-Time for the Satellite Example of Fig. 19 (Intel Core2 6600 @ 2.4 GHz)

Method	Memory	Run-time
MLFMM	14.1 GByte	1842 sec
UTD	1.0 MByte	8.1 sec

When solving a dielectric cube instead, effects of sharp corners and edges might require a separate validation. Material parameters may change to extreme values of say $\mu_r = 30000$ for some ferrite materials or negative values for modeling meta-materials. In such cases, care must be taken for the generalization and validation should be done using known benchmarks/measurements.

A. *Cross-Validation Using Different Numerical Techniques*

Often when developing new computational methods, previous established ones can be used as benchmarks to assess their accuracy. These established methods can thus be regarded as a trusted reference for continuous verification and regression testing when assessing the accuracy of other methods. This was done for instance in [40], where a modal approach was "verified" against an established MoM code for shielding problems with small apertures.

The results from a similar cross-validation are shown in Fig. 18. The reference radiated near-field from an L-shaped cable above a ground plane has been computed using the MoM. This reference is compared to various transmission line modeling methods hybridized with the MoM.

In particular, when validating results based on high-frequency asymptotic methods, an approach can be used to solve selected structures on sufficiently large computers with full-wave methods and compare these with much faster and memory economical high-frequency solutions. For example, Fig. 19 shows a radiation pattern of an antenna mounted on a satellite (rectangular box with 1-m dimension) evaluated at 6 GHz where the multilevel fast multipole method (MLFMM) [41] is used as reference solution to validate the uniform theory of diffraction (UTD) [42]. The memory and run-time for both solutions are

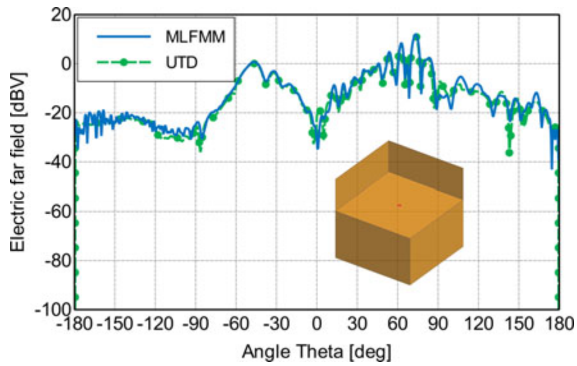

Fig. 19. Far-field radiation pattern of an antenna on a satellite comparing UTD (high-frequency asymptotic solver) with MLFMM (full-wave reference solution).

summarized in Table I showing the clear preference for the UTD in this case.

B. Cross-Validation Using Different Platforms and Solution Options

Numerical codes often have to be adapted to take advantage of different types of platforms such as parallel processing (OpenMP for shared memory environment and MPI for distributed clusters) and GPU computing [43] or even just using different types of CPUs with different mathematical libraries (e.g., Intel Math Kernel Library MKL versus AMD Core Math Library ACML).

Once a set of benchmark problems has been validated using other methods already described in this paper, these can be used as reference to validate other computational platforms, e.g., compare the results with GPU acceleration to those without, in-core versus out-of-core, sequential versus parallel, different CPU types etc.

IV. Internal Consistency Checks

While it is possible for certain numerical methods to obtain reliable *a priori* or *a posteriori* error estimates (see e.g., some discussions for FEM in [44]), it is not straightforward to develop such concepts for general numerical methods such as the UTD used in the example in Fig. 19.

A number of basic checks exist that can be applied generally to gain confidence in a solution (these are required but not sufficient conditions in establishing the accuracy of numerical results for electromagnetic problems).

A. Numerical Convergence Checks

Many computational electromagnetics methods are based on some or other discretization scheme. For instance, the MoM meshes wires into segments, surfaces into surface patches, and volumes into tetrahedral elements. For FDTD or FIT, a voxel type gridding is used, and FEM is also typically based on tetrahedral volume meshes.

Certain general rules for the size of these mesh elements can be derived as a function of the order of the underlying basis functions, the frequency, and material parameters. To gain a first level of confidence in these results, convergence should be observed when using finer mesh sizes, larger bounding boxes, smaller time steps, and/or increasing basis function orders (either locally or globally).

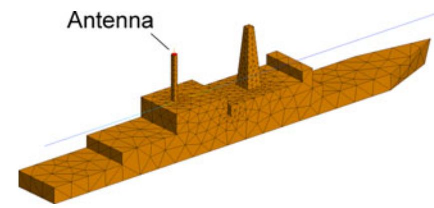

Fig. 20. Model of a transmitting antenna mounted on the mast of a ship radiating at 50 MHz with near-field calculation along the indicated line.

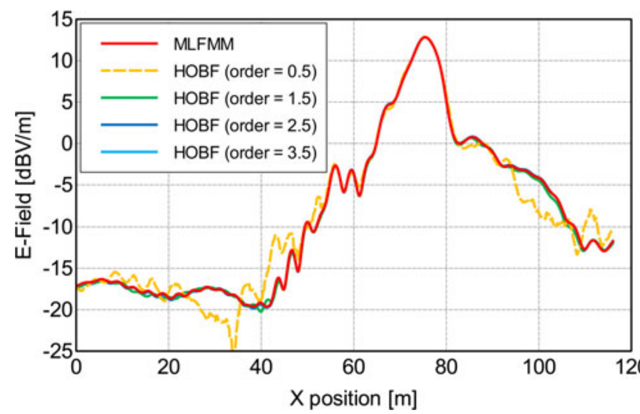

Fig. 21. Electric near-field along the length of the ship geometry in Fig. 20 calculated with MLFMM (reference solution) and MoM using HOBF of different orders.

An example is shown in Fig. 20 where we compute the near-field radiated by an antenna on a ship with MoM for various basis function orders using the same meshing. Near-field results for various basis function orders are compared in Fig. 21. Note that the mesh size is too coarse for the lowest order 0.5 and, under normal circumstances, our MoM solver will not allow this but abort with an error (here overridden).

For users of commercial software tools, the number of tunable parameters to assess the "numerical convergence" are typically limited to, for example, mesh sizes, order for basis functions ray launching angle for shooting and bouncing ray solvers, time step for time domain simulations etc. Developers of numerical electromagnetics codes have many more parameters at hand to vary during development, for instance, the number of terms to be included in series expansions, the termination criteria for iterative solvers or the number of quadrature points in Gaussian integration schemes.

For developers of computational electromagnetics codes, it is crucial to include internal checks to ensure that algorithms are not used beyond their range of validity. An example for this is shown in Fig. 22. There, the relative error is plotted as a function of the number of summation terms L for the addition theorem in the MLFMM using the exact Green's function as reference for the critical case of a highly lossy dielectric material. The standard MLFMM solution with one so-called "buffer

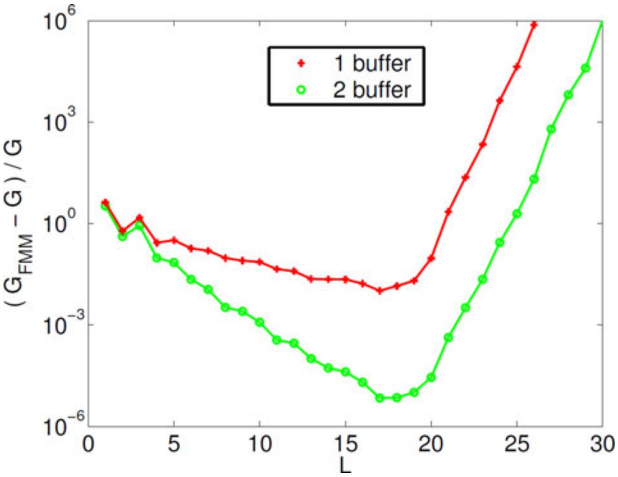

Fig. 22. Relative error in the MLFMM addition theorem for a lossy dielectric medium as a function of the number of summation terms L (see [45] for details).

box" (see [45] for details) cannot achieve good accuracy and by introducing two buffer boxes and selecting the number of terms L suitably, an accuracy close to 10^{-6} can be achieved.

A similar internal consistency check is the automatic detection of low-frequency breakdown for some high-frequency solvers.

B. Reciprocity Checks

Reciprocity is often a useful check to verify the plausibility of numerical results. If a system is passive and contains only reciprocal materials, the S-matrix must be symmetrical (i.e., $S_{21} = S_{12}$ etc.). An example of this was already given in Fig. 11 where a monopole antenna radiates and couples into a coaxial cable or the reciprocal case where the cable radiates and couples into the monopole antenna. The numerical treatment of these two scenarios within the MoM/MTL hybrid method used is rather different [38], [46], and results that closely match are required to ensure correctness but not a necessary proof thereof. For example, an error in the transfer impedance calculations would manifest itself symmetrically in both S_{21} and S_{12}).

C. Power Budget Checks

Due to the laws of energy conservation, the power that is fed into a system (e.g., by a voltage source) must be equal to the power loss in the system plus the power that is radiated into the far-field. These three quantities can be evaluated independently and can be used to establish an error measure:

1) Source power is dependent on the source type, for a voltage source, this is simply obtained from the feeding voltage and the current is obtained from the numerical solution at the source position.
2) Power loss is typically obtained through an integral over the lossy region, which could mean that a volume integral has to be solved (lossy dielectric bodies), a surface integral over the surface current density vector (skin effect loss for metallic surfaces with finite conductivity) or simply the loss power for lumped elements (resistors) is obtained from the current density.
3) The power radiated into the far-field can be obtained by integrating the normal component of the power density vector over a closed surface surrounding the object. Typically, this surface would be the far-field sphere (the numerical accuracy of such integrals must be established in itself, i.e., a sufficiently fine sampling density must be used or advanced interpolation techniques such as the usage of spherical mode representations can be considered).

For the ship configuration of Fig. 20, the power budget results are shown in Table II. It can be seen that the accuracy of power budget increases as basis function order is increased (as noted already, the mesh size is too coarse for basis functions of order 0.5, which is confirmed in the relatively large power budget error of 1.5 %.

TABLE II
COMPARISON OF THE POWER BUDGET CALCULATION FOR THE SHIP GEOMETRY OF FIG. 20 FOR THE DIFFERENT ORDERS OF BASIS FUNCTIONS OF FIG. 21

Method	Source power P_s	Far-field power P_{ff}	$\frac{\lvert P_s - P_{ff}\rvert}{P_s}$
MLFMM	2 W	2.00031 W	1.549×10^{-4}
MoM (order 0.5)	2 W	2.03002 W	1.479×10^{-2}
MoM (order 1.5)	2 W	1.99887 W	5.653×10^{-4}
MoM (order 2.5)	2 W	1.99953 W	2.350×10^{-4}

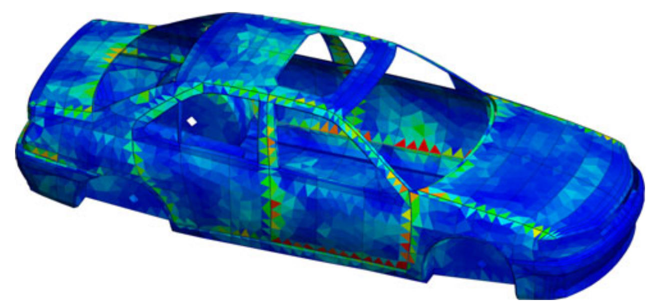

Fig. 23. Estimation of errors based on local charge variations for a plane wave incident on a car solved with MoM (can be used as basis for local mesh refinement strategies).

D. Boundary Condition Checks

The solution of electromagnetic problems must comply with certain boundary conditions. For instance, the tangential electric and magnetic field, $E_{\tan}$ and $H_{\tan}$, must be continuous across dielectric boundaries, or $E_{\tan}$ must be zero on PEC surfaces.

Some error estimates based on these boundary conditions are presented in [47] for the MoM, but the evaluation of these will increase run-time (N^2 scaling with N being the number of unknowns, thus quite significant). We have developed a faster approach based on the local variation of charges, which scales linear in N. It does not result in the "exact" boundary condition errors, but is still very useful to indicate regions of largest error in order to apply local mesh refinement strategies, see Fig. 23 for an example.

V. Automated Regression Testing

Once the accuracy of a numerical method has been established using any of the validation principles described in the previous sections or preferably a combination thereof, it is of crucial importance in the world of commercial software development to ensure that this is maintained in a changing environment.

Code is continually extended by adding new features and algorithms are made more efficient in terms of speed and memory requirements. Compilers and external math libraries are constantly evolving to be more efficient on specific CPU types. Upgrading of MPI libraries or DAPL drivers etc. used in parallel execution scenarios sometimes result in an unexpected behavior for specific cluster configurations.

All these updates and changes can introduce unexpected errors in general or only for specific hardware configurations / solver features.

It is therefore of vital importance to continuously (e.g., weekly automated test runs) ensure that previously validated results are still obtained in a large variety of environments and for many different configurations (as outlined in Section III-B). It must also be ensured that the set of validation examples is sufficiently large to cover all the different solver options (code coverage tools will assist in this process).

For regression testing, it is often of more value to directly compare results with stored data from previous validation and verification runs (say using an older set of compilers) than using features like FSV. These comparisons rarely need expert judgment because exact (within reasonable numerical precision) agreement should be expected. Expert judgment is only needed when such comparisons fail within the set tolerances, which is then often the result of a coding error or compiler / third-party library bug, or in fact, a desired difference due to improving the numerical accuracy of some solvers (e.g., using more Gaussian quadrature points).

VI. Interpretation of the Validation Results in Case of Failure

No further action or investigations need to be done when applying the various concepts for establishing accuracy introduced in the previous sections of this paper and sufficient accurate results are obtained, for e.g., when comparing measurements with computational results that agree well. However, in many cases, for complex models, this agreement might not be obtained especially for inexperienced users of computational methods in electromagnetics. Often the cause for these discrepancies is not related to problems of the electromagnetic solver, but rather the computational model used as the input. This is mostly because the computational model does not represent the reality with sufficient detail. For instance, are all the subtle details of the geometry included which might have an impact on the electromagnetic results (e.g., slots at doors and connection resistances at the hinges for an automotive model) or are all the relevant material parameters known with sufficient accuracy (e.g., ε_r for tire rubber when modeling tire pressure monitoring systems embedded in tires).

In such cases, if the input data cannot be obtained with more reliability (e.g., measurement of material parameters), a sensitivity analysis might be a useful tool to establish a feeling of how certain results change versus certain assumptions made in setting up the computational model.

Another very important aspect when inaccurate results are obtained is the question whether sufficiently validated numerical methods against benchmarks are not stressed too much and used outside their range of applicability. An obvious example is high-frequency asymptotic methods, which work very efficiently at higher frequencies (compared to full-wave solutions, see e.g., the validation example in Fig. 19) and within the approximations underlying these formulations provide reasonably accurate results. However, when using such methods at frequencies which are too low, errors will just increase beyond acceptable levels. Thus, in the commercial software world particularly (where there are thousands of users using the codes for applications the developers might never have anticipated), it is of crucial importance to carefully assess the limits of each method with regards to the various input parameters (such as frequency) and, if required, establish safety limits and enforce them in the code (by giving the user errors and aborting or automatically switching to other algorithms etc.).

VII. Conclusion

The paper has introduced some of the concepts that we use in the validation, verification, and testing of a commercial field solver package. These activities reduce (and ideally eliminate) uncertainties and causes of error from the actual computation engine. However, the uncertainty whether a specific computational model represents reality (inclusion of all the necessary details, unknown material parameters, tolerances etc.) can only be assessed by comparisons or sensitivity type of analysis.

We believe that validation, verification, and automated testing plays a very critical role in today's fast changing environment (new computational algorithms and accelerations, but also advances in hardware such as multicore CPUs, GPU computing, and parallel processing using clusters). Without rigorous validation and testing strategies on an ongoing basis, the introduction of new platforms such as Intel Xeon Phi that require new compilers, new libraries, and new software programming paradigms, the extensions of computational algorithms, etc. could all lead to the unwanted introduction of errors.

It is impossible to test all the combinations of environments that thousands of users might use (e.g., all Infiniband driver versions with different Linux operating systems on clusters) or all types of problems customers might want to solve. For this reason, the inclusion of self-testing (internal consistency checks executed at run-time) is another concept that we have been following for many years to detect typical problems (floating point exceptions, out-of-bounds memory accesses but also electromagnetic features such as power budget, convergence checks, automatically changing preconditioner settings, etc.) in order to reduce (unfortunately not eliminate completely) uncertainties involved with the numerical modeling of electromagnetic problems.

REFERENCES

[1] *AIAA Guide for the Verification and Validation of Computational Fluid Dynamics Simulations*. Amer. Inst. Aeronaut. Astronaut., Reston, VA, USA, 1998.

[2] *IEEE standard for validation of computational electromagnetics computer modeling and simulations*, IEEE Std 1597.1-2008, May 2009.

[3] B. Archambeault and S. Connor, "Various approaches to validating full-wave simulations," in *Proc. IEEE Int. Symp. Electromagn. Compat.*, 2004, pp. 678–682.

[4] P. Roache, "Building PDE codes to be verifiable and validatable," *Comput. Sci. Eng.*, vol. 6, no. 5, pp. 30–38, Sep./Oct. 2004.

[5] *IEEE recommended practice for validation of computational electromagnetics computer modeling and simulations*, IEEE Std 1597.2-2010, Feb. 2011.

[6] *IEEE recommended practice for determining the peak spatial-average specific absorption rate (SAR) in the human head from wireless communications devices: Measurement techniques*, IEEE Std 1528-2003, 2003.

[7] R. G. Marchand and D. Davidson, "The method of manufactured solutions in 3D for the verification of computational electromagnetics," in *Proc. IEEE Antennas Propag. Soc. Int. Symp.*, 2012, pp. 1–2.

[8] R. G. Marchand, "The method of manufactured solutions for the verification of computational electromagnetic codes," Ph.D. dissertation, Dept. Elctr. Eng., Stellenbosch Univ., Stellenbosch, South Africa, Mar. 2013.

[9] Y. Jia and M. Harman, "An analysis and survey of the development of mutation testing," *IEEE Trans. Softw. Eng.*, vol. 37, no. 5, pp. 649–678, Sep./Oct. 2011.

[10] A. Duffy, A. Martin, A. Orlandi, G. Antonini, T. Benson, and M. Woolfson, "Feature selective validation (FSV) for validation of computational electromagnetics (CEM) part I: The FSV method," *IEEE Trans. Electromagn. Compat.*, vol. 48, no. 3, pp. 49–459, Aug. 2006.

[11] A. Duffy, A. Martin, A. Orlandi, G. Antonini, T. Benson, and M. Woolfson, "Feature selective validation (FSV) for validation of computational electromagnetics (CEM) part II: Assessment of FSV performances," *IEEE Trans. Electromagn. Compat.*, vol. 48, no. 3, pp. 460–467, Aug. 2006.

[12] G. Mie, "Beiträge zur Optik trüber Medien, speziell kolloidaler Metallösungen," *Annalen der Physik*, vol. 330, pp. 377–445, 1908.

[13] R. F. Harrington, *Field Computation by Moment Methods*. New York, NY, USA: Macmillan Company, 1968.

[14] V. Schulz and G. Mrozynski, " A benchmark catalog for numerical field calculations in the area of EMC," presented at the *20th Int. Zurich Symp. Electromagn. Compat.*, Zurich, Switzerland, Feb. 1999.

[15] G. Mrozynski, V. Schulz, and H. Garbe, "A benchmark catalog for numerical field calculations with respect to EMC problems," in *Proc. IEEE Int. Symp. Electromagn. Compat.*, Seattle, WA, USA, Aug. 1999, pp. 497–502.

[16] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propag.*, vol. 30, no. 3, pp. 409–418, May 1982.

[17] L. P. Zha, Y. Q. Hu, and T. Su, "Efficient surface integral equation using hierarchical vector bases for complex EM scattering problems," *IEEE Trans. Antennas Propag.*, vol. 60, no. 2, pp. 952–957, Feb. 2012.

[18] R. D. Graglia, A. F. Peterson, and F. P. Andriulli, "Curl-conforming hierarchical vector bases for triangles and tetrahedra," *IEEE Trans. Antennas Propag.*, vol. 59, no. 3, pp. 950–959, Mar. 2011.

[19] C. Hafner, *The Generalized Multipole Technique for Computational Electromagnetics*. Boston, MA, USA: Artech House, 1990.

[20] C. Hafner and L. Bomholt, "The 3 Electromagnetic Wave Simulator," in *3D MMP Software and User's Guide*. Chichester, U.K.: Wiley, 1993.

[21] H. Kaden, *Wirbelströme und Schirmung in der Nachrichtentechnik*. Berlin, Germany: Springer–Verlag, 1959.

[22] G. Bürger, H.-D. Brüns, and H. Singer, "Simulation of thin layers in the method of moments," in *Proc. 11th Int. Zurich Symp. Electromagn. Compat.*, Mar. 1995, pp. 339–344.

[23] C. A. Balanis, *Advanced Engineering Electromagnetics*. New York, NY, USA: Wiley, 1989.

[24] H. Shigesawa, M. Tsuji, T. Nakao, and K. Takiyama, "Two-path cutoff waveguide dielectric resonator filters," *IEEE Trans. Microw. Theory Tech.*, vol. 37, no. 7, pp. 1105–1112, Jul. 1989.

[25] T. H. Hubing and J. F. Kauffman, "Modeling electrically small, thin surfaces with wire grids," *Proc. Appl. Comput. Electromagn. Soc. J.*, vol. 5, no. 1, pp. 19–24, Mar. 1990.

[26] G. J. Burke and A. J. Poggio, Numerical Electromagnetics Code (NEC)—Method of Moments. Lawrence Livermore National Laboratory, Livermore, CA, USA, Rep. UCID 18834, 1981.

[27] H. Singer and H. Brüns, *CONCEPT-II*, (2004). [Online]. Available: http://www.tet.tu-harburg.de/concept

[28] (2012, Aug.). *FEKO Suite 6.2—Field Computations Involving Bodies of Arbitrary Shape*, EM Software & Systems—S.A. (Pty) Ltd, Stellenbosch, South Africa, [Online]. Available: http://www.feko.info

[29] P. Kralicek and F. Sabath, "Considering EMI based on multiconductor transmission line," in *Proc. COST 243 Workshop*, Paderborn, Germany, Apr. 1997, pp. 9–12.

[30] A. C. Woo, H. T. G. Wang, and M. J. Schuh, "Benchmark radar targets for the validation of computational electromagnetics programs," *IEEE Antennas Propag. Mag.*, vol. 35, no. 1, pp. 84–89, Feb. 1993.

[31] A. Greenwood, " Electromagnetic code consortium benchmarks," Air Force Research Laboratory, Kirtland Air Force Base, Albuquerque, NM, USA, Tech. Rep. AFRL-DE-TR-2001-1086, Dec. 2001.

[32] G. Vandenbosch, R. Gillard, and M. Sabbadini, "The antenna software initiative (ASI): ACE results and EuRAAP continuation," *IEEE Antennas Propag. Mag.*, vol. 51, no. 3, pp. 85–92, Jun. 2009.

[33] A. Vasylchenko, Y. Schols, W. D. Raedt, and G. Vandenbosch, "Quality assessment of computational techniques and software tools for planar-antenna analysis," *IEEE Antennas Propag. Mag.*, vol. 51, no. 1, pp. 23–38, Feb. 2009.

[34] R. Gillard and G. Vandenbosch, " Benchmarking of optimally used commercial software tools for challenging antenna topologies part 1: The SoftLAB benchmark environment," presented at the 7th Eur. Conf. Antennas Propag., Gothenburg, Sweden, Apr. 2013.

[35] G. A. E. Vandenbosch, R. Gillard, F. Demuynck, A. Edquist, T. Wittig, M. Kiyani, J. Huysamen, U. Jakobus, W. Simon, A. Wien, B. Kolundzija, and M. Pavlovic, " Benchmarking of optimally used commercial software tools for challenging antenna topologies part 2: The 2012–2013 run," presented at the *7th Eur. Conf. Antennas Propag.*, Gothenburg, Sweden, Apr. 2013.

[36] International Compumag society—testing electromagnetic analysis methods (T.E.A.M.)," (2013). [Online]. Available: http://www.compumag.org/jsite/team.html

[37] G. Andrieu, L. Kone, F. Bocquet, and B. P. J.-P. Demoulin, "Multiconductor reduction technique for modeling common-mode currents on cable bundles at high frequency for automotive applications," *IEEE Trans. Electromagn. Compat.*, vol. 50, pp. 175–184, Feb. 2008.

[38] M. Schoeman and U. Jakobus, " Numerical solution of complex EMC problems involving cables with combined field/transmission line approach," *Int. J. Electromagn. Compat.*, pp. 78–88, Apr. 2011.

[39] D. Campbell, G. Gampala, C. J. Reddy, M. Winebrand, and J. Aubin, "Modeling and analysis of anechoic chamber using CEM tools," in *Proc. 34th Annu. Symp. Antenna Meas. Techn. Assoc.*, Bellevue, WA, USA, Oct. 2012, pp. 157–162.

[40] V. Rajamani, C. F. Bunting, M. D. Deshpande, and Z. A. Khan, "Validation of modal/MoM in shielding effectiveness studies of rectangular enclosures with apertures," *IEEE Trans. Electromagn. Compat.*, vol. 48, no. 2, pp. 348–353, May 2006.

[41] W. C. Chew, J.-M. Jin, E. Michielssen, and J. Song, Eds., *Fast and Efficient Algorithms in Computational Electromagnetics*. Boston, MA, USA: Artech House, 2001.

[42] R. G. Kouyoumjian and P. H. Pathak, "A uniform geometrical theory of diffraction for an edge in a perfectly conducting surface," *Proc. IEEE*, vol. 62, no. 11, pp. 1448–1461, Nov. 1974.

[43] E. Lezar and U. Jakobus, " GPU accelerated electromagnetic simulation with FEKO," presented at the Int. SuperComput. Conf., Hamburg, Germany, Jun. 2012.

[44] J. R. Stewart and T. J. R. Hughes, "A tutorial in elementary finite element error analysis: A systematic presentation of *a priori* and *a posteriori* error estimates," *Comput. Methods Appl. Mech. Eng.*, vol. 158, pp. 1–22, 1998.

[45] J. J. van Tonder and U. Jakobus, " Fast multipole solution of metallic and dielectric scattering problems in FEKO," in *Proc. 21st Annu. Review Prog. Appl. Comput. Electromagn.*, Apr. 2005, pp. 511–514.

[46] C. Nucci and F. Rachidi, "On the contribution of the electromagnetic field components in field-to-transmission line interaction," *IEEE Trans. Electromagn. Compat.*, vol. 37, no. 11, pp. 505–508, Nov. 1995.

[47] F. G. Bogdanov and R. G. Jobava, "Examination of boundary conditions performance for estimating accuracy of MoM solutions on square plate benchmark geometry using triangle doublet basis functions," *Microw. Opt. Technol. Lett.*, vol. 39, no. 3, pp. 193–196, Nov. 2003.

Ulrich Jakobus (F'13) received the Dipl.-Ing. and Dr.-Ing. degrees in electrical engineering from the University of Stuttgart, Stuttgart, Germany, in 1991 and 1994, respectively. He received the Habilitation degree from the same university in 1999.

From 1991 to 2000, he was employed at the Institut für Hochfrequenztechnik, University of Stuttgart, Germany, where he became a Privatdozent for Radio Frequency Technology. There, his main areas of research included numerical techniques in electromagnetics, antennas, electromagnetic compatibility, and bioelectromagnetics. Of particular interest were hybrid methods combining the method of moments with high-frequency asymptotic techniques and this research laid the foundations for the commercial electromagnetics code FEKO. Since 2000, he has been the Director of EM Software and Systems-S.A. (Pty) Ltd. in Stellenbosch, South Africa, and as a FEKO Product Manager focusing on the development of FEKO. He also founded EM Software and Systems GmbH in Böblingen, Germany, in 2002 and has since been a Director of this entity. He is a Member of the URSI commission B, the German VDE/ITG, and a Member of the Applied Computational Electromagnetics Society, where he was also elected Fellow in 2013 and currently serves on the Board of Directors.

Dr. Jakobus has published extensively on computational electromagnetics and related topics, and received several awards for his research, including the ACES 1996 Outstanding Paper Award and the Heinz–Maier Leibnitz prize 1998 by the Deutsche Forschungsgemeinschaft.

Renier Gustav Marchand received the B.Sc. degree in computer science and physics, the B.Eng. degree in electronics (*cum laude*), the M.Sc. degree (*cum laude*), andthe Ph.D. degree from the University of Stellenbosch, Stellenbosch, South Africa, in 2002, 2004, 2007, and 2013, respectively.

During his M.Sc. research, he investigated the application of the FETI method in the total/scattered-field decomposition and the application of the FETI to the time-domain. During his Ph.D., his primary focus was on the verification of computational code and more specifically the application of the MMS. From 2007 to 2009, he worked as a Researcher at the Faculty of Process Engineering, University of Stellenbosch, South Africa, where he investigated EM aspects of mineral processing of mining ores. He is currently part of the Quality Assurance team for the computational electromagnetic software package, FEKO.

Daniël J. Ludick was born in the Free State, South Africa, on July 30, 1985. He received the B.Eng. degree in electrical and electronic engineering with computer science (*cum laude*) and the M.Sc.Eng. degree in electronic engineering (*cum laude*) from the University of Stellenbosch, Stellenbosch, South Africa, in 2007 and 2009, respectively. He is currently working toward the Ph.D. degree in electronic engineering at the University of Stellenbosch.

His Master's thesis was focussed on the efficient analysis of focal plane arrays for the Square Kilometre Array radio telescope using electromagnetic simulation techniques. He is currently part of the development team for the computational electromagnetic software package, FEKO. His main research interests include computational electromagnetic simulation, domain decomposition techniques, and antenna design.