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Is Lyapunov exponent a reliable metric to detect dynamic stability in Parkinson's disease?

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***Abstract*— Idiopathic Parkinson's disease (PD) is the second most common neurodegenerative disorder worldwide. It affects the nervous system, causing motor and non-motor symptomatology. However, its clinical diagnosis remains dependent on the expertise of clinicians, as perceptual clinical scales are often used. Gait stability is one of the most relevant motor signs in PD. Nonetheless, it is usually not reported or quantified, possibly due to its unclear meaning and the high variability of metrics used in the literature. This work aims to identify a reliable and objective indicator that clinicians can use to assess patients in realistic contexts. We focused on the Largest Lyapunov Exponent (LLE), being the most common metric used in previous research works to quantify gait stability. The short and long-term LLEs were calculated in a group of 34 healthy and 42 participants diagnosed with PD. The long-term LLE extracted from the chest, right arm and right foot sensors showed statistical differences between subjects with PD and healthy control (HC) subjects, showing that the HC subjects are more stable than PD patients, whereas the short-term LLE showed the opposite results. Further investigation is required to clarify the reliability of this metric to detect and rate gait stability in people affected with PD.**

***Clinical Relevance*— This study is the first step towards the identification of an objective methodology to assess gait stability in clinical settings. Achieving this goal will contribute to improve the understanding and support the diagnosis of gait disorders that cause gait stability problems.**

I. Introduction

Idiopathic Parkinson's disease (PD) is a neurodegenerative disorder that affects the human nervous system. It is associated with neural loss in the substantia nigra of the brain, leading to the appearance of intracellular inclusions with aggregates of α-synuclein, known as Lewy bodies, and dopamine deficiency [1]. It is the second most common neurodegenerative disease worldwide, which affects about 10 million people, according to the World Health Organization (WHO).

The clinical symptoms of this disease are heterogenous, including motor and non-motor signs [1]–[3]. Typically, non-motor symptoms emerge ten or more years before the actual diagnosis of PD, which is usually based on motor symptoms in the so-called premotor or prodromal phase. By the time motor symptoms show up, 70% of dopaminergic neurons are to be dead [1]. As the disease advances, these symptoms increase in frequency and magnitude [4]. An early detection is crucial to delay the onset of symptoms and increase the patients' quality of life and survival. The only way to accurately diagnose PD is by looking for Lewy bodies in the brain by autopsy. Although the existence of several diagnostic criteria such as the UK Parkinson's Disease Society Brain Bank and the National Institute of Neurological Disorders and Stroke (NINDS), as well as follow-up scales like the Hoehn and Yahr Scale (H&Y) and the Movement Disorders Society-Unified Parkinson's Disease Rating Scale (MDS-UPDRS), no reliable objective method exists yet. The diagnosis still totally depends on the expertise of movement disorder specialists, who base their decisions on recognizing some combination of cardinal motor signs and responsiveness to medication [5], [6]. Even the most complex scale relies on the subjective criterion of a clinician, leading to the possibility of getting different scores when being evaluated by different specialists. In addition, there is also a part of the diagnosis process that depends on the patient, e.g., those questions regarding daily symptoms and response to treatment [7]. The lack of an explicit clinical diagnostic method has been shown to lead to about a 25% of diagnosis failure [5].

In conclusion, there is a clear need to improve PD clinical diagnosis and follow-up methods to make them more objective and accurate. The definition of metrics that objectively rate the performance of each task evaluated in the aforementioned scales may be a first step towards this goal. Although a broad spectrum of tasks needs to be covered, this research will focus on one important, yet particularly overlooked aspect: the dynamic stability of gait. More precisely, this work addresses the following scientific question: how is dynamic stability affected in PD?

II. Methods

Data from 34 healthy control (HC) subjects and 42 PD patients were collected. The experimental protocol was approved by the Hospital Universitario Gregorio Marañon's Review Board. People participating in this study met the following inclusion/exclusion criteria: PD subjects must be between 50 and 80 years old; have the ability to understand verbal instructions (score equal to or greater than 24 on the mini-mental state examination); have a H&Y score between I

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and III; preserve the ability to walk at least 10 meters without the use of technical aids; be in the off phase of the disease (4 hours after the last intake of medication); have been diagnosed of PD between 2 and 6 years ago; have stable pharmacological treatment without changes in the last 3 months. Subjects must not have been diagnosed with other types of Parkinsonism, have a history of neurological conditions other than PD (e.g., Stroke) and/or psychiatric or have undergone surgery for deep brain stimulation. HC subjects must have been age matched, not present any neurological or neurodegenerative disease, mayor health condition, movement disorder or injuries affecting movement and be able to walk without at least 10 meters without assistance.

The data used in this study was collected during 2016-2020 at Hospital Universitario Gregorio Marañón, Madrid, Spain. The experimental protocol was the following. Prior to the experimentation, the participants signed a written informed consent, according to the declaration of Helsinki. Afterwards, the subject was instrumented with ten inertial measurement units (IMUs) (Tech-MCS, Technaid). The location of these sensors was the chest, the pelvis (lumbar region), the wrists, the thighs, the shanks, and the feet. The participants were asked to walk in a straight line for 15m at their self-selected velocity, turn 180° and return along the 15m path, three times.

To assess gait stability, the Largest Lyapunov Exponent (LLE) was calculated. LLE is a measure of the rate of separation of infinitesimally close trajectories of an attractor. An attractor is the reconstructed trajectory of a nonlinear dynamic system extracted from a time series (e.g., acceleration). LLE was calculated based on Rosenstein's method [8]. First, the attractor dynamics, that is, the evolution of the system's trajectory over time, was reconstructed from a time series, which, in this case, was the accelerometer signal of the IMUs (see Fig. 1). The reconstruction was carried out following Takens' delay-coordinate embedding technique: the delay-coordinate map from a d-dimensional smooth compact manifold to R^m, being the embedding dimension (m) > 2d, is a diffeomorphism on the manifold. This means that the reconstruction from the delayed time series has the same

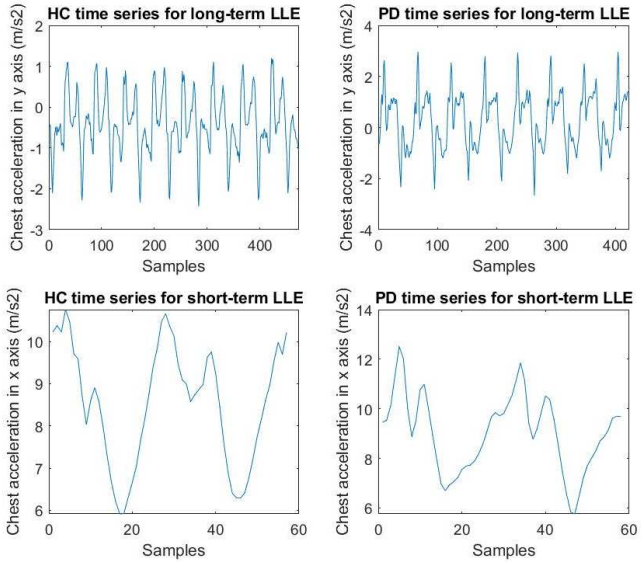

Figure 1. Time series used for long and short-term calculation for both cohorts.

behavior as the actual dynamical system [9]. This process is common to every nonlinear system analysis. Once the trajectory (X) was reconstructed, the LLE was calculated. For this matter, the nearest neighbor of each point on the trajectory was identified by looking for the point that minimizes the distance to the reference point X_i (1), where X_i is the phase space vector at discrete time i and $d_i(0)$ is the initial distance from the ith point to its nearest neighbor. ‖•‖ refers to the Euclidean norm.

$$d_i(0) = \|X_i - X_i\|, \quad (1)$$

Finally, the mean separation of the neighbors was calculated, and least squares used to fit a line to the data. The LLE represents the slope of this line.

The data processing protocol was the following. First, each trial was segmented to separate the two straight line walks from the turn. Then, gait was segmented in steps using the knee flexion/extension angles, considering that heel strike occurs at the time of maximum extension of the knee. A threshold was defined following the strategy proposed in [10] and used to detect the peaks of the signal. Each minimum after a maximum corresponds to heel strike. After segmenting the data, the first and the last steps of each recording were removed, corresponding to gait initiation/finalization or turn preparation/recovery. Finally, the LLE was calculated. There are two types of LLEs, depending on the length of the input signal: the short-term and the long-term LLE. In the case of gait, the short-term LLE refers to the LLE calculated over 1-2 strides and the long-term LLE to the one calculated over 4 or more strides [11]. In this case, both were calculated. Three different observations were considered, corresponding to the accelerometer signal in the X, Y and Z directions from the IMU data recorded from each of the ten sensors placed on the subject's body. Statistical analyses were performed to evaluate whether significant difference exists in the LLE of PD patients when compared to HC subjects. In general, nonparametric tests were applied since the data did not meet the assumptions of normality or equality of variances. Shapiro-wilk and Levene's tests, respectively, were performed to assess these two aspects. If both assumptions were met, an independent samples t-test was performed. If the normality assumption was met, but not the equal variance assumption, the Welch's test was performed. When neither of the two premises was met, the Kruskal-Wallis test was performed.

III. Results

The statistical analysis showed that there is a significant increase in the long-term LLE of PD subjects for the chest acceleration signal in the Y axis (Fisher's, $p < 0.013$), right arm acceleration signal in the Z axis (Fisher's, $p = 0.072$) and right foot acceleration signal in the Z axis (Kruskal-Wallis, $p = 0.001$). No significant differences were found in the long-term LLE for any other body part or axis. In the case of the short-term LLE, a significant decrease was found in the PD group for the chest acceleration signal in the X axis (Kruskal-Wallis, $p < 0.015$), the lumbar acceleration signal in the Y axis (Kruskal-Wallis, $p < 0.039$), the right arm acceleration signal in the X axis (Kruskal-Wallis, $p = 0.005$), the right shank acceleration signal in the X (Fisher's, $p = 0.024$) and the Y axis (Kruskal-Wallis, $p = 0.29$), the left arm acceleration signal in the X (Kruskal-Wallis, $p = 0.034$) and Y axis (Kruskal-Wallis,

p = 0.013), the left shank acceleration signal in the X axis (Kruskal-Wallis, p = 0.001) and the left thigh acceleration signal in the X axis (Kruskal-Wallis, p = 0.02), as well as a significant increase in the short-term LLE of PD subjects for the left foot acceleration signal in the Y axis (Kruskal-Wallis, p = 0.002) and the left thigh acceleration signal in the Y axis (Kruskal-Wallis, p = 0.015). These results, expressed in terms of mean and standard deviation, are summarized in Table 1, where significant values are marked with an asterisk and highlighted in blue and yellow in the case of a significant increase of the LLE of PD with respect to HC and a significant decrease of the LLE of PD compared to HC, respectively.

IV. Discussion

In this study, we evaluated the effect of PD on gait stability, looking for biomarkers capable of objectively assess it clinically.

In the case of the long-term LLE, significant statistical difference was only found in three signals: the chest acceleration signal in the Y axis, the right arm acceleration signal in the Z axis and the right foot acceleration signal in the Z axis. As for the short-term LLE, significant differences were found in 13 acceleration signals: the right arm in the X axis, the right shank in the X and Y axis, the left arm in the X and Y axis, the left shank in the X axis, the left thigh in the X axis, the left foot in the Y axis and the left thigh in the Y axis. As it can be seen, the relevant body parts and directions are different for the long and short-term LLE. For the long-term LLE, the chest and right arm and foot were the most meaningful locations of the IMUs while for the short-term LLE all the body parts in both sides of the body were able to classify PD patients and HC. Regarding the axis directions, the Z axis was the most relevant for the long-term LLE whereas in the case of the short-term LLE they were the X and Y axes. These results suggest that long and short-term LLE provide different information. This hypothesis is in line with authors in [11], where they state that the short and long-term LLEs should not be treated equally. In this study, the authors suggest that long-term LLE is a measure of gait complexity while short-term LLE is an index of stability. However, in the literature some studies use long-term LLE as a measure of gait stability [12]–[20]. Another difference found between the results of the long and short-term LLE is their behavior. We observed an increase in the long-term LLE in PD patients with respect to the HC, for all the signals that presented significant differences between groups. However, for the short-term LLE, contrary to our expectations, the observations were mostly the opposite. Short-term LLE of HC was significantly higher than that of PD subjects' short-term LLE for 10 out of the 13 signals that presented significant differences between groups. These latter results suggest that the gait of PD patients is more stable than that of HC since a positive LLE indicates that the system is chaotic [8], that is, the bigger the value of the LLE, the more unstable the gait is. These results contradict those obtained for the long-term LLE, which show that PD subjects have more unstable gait than HC subjects. This contradiction may be due to the fact that the signal used to calculate the short-term LLE comes from only one stride, which may be too small to get accurate LLE estimations. This leads to another inconsistency: it has been acknowledged that issues exist when calculating the LLE of small signals, while, at the same time, the short-term LLE is still the most employed metric in literature. This problem has also been identified in a previous study [21] in which the authors estimated the minimum number of steps needed to obtain a faithful LLE while accurately distinguishing between two populations. They observed that at least 50 steps are needed to obtain a faithful LLE, and 70 to distinguishing between populations. This research highlights that calculating the short-term LLE is unreliable, at least in the case of gait signals. In our current study, even the signals used to calculate the long-term LLE did not reach this reliability limit, being 15 the mean number of steps per capture. The high standard deviation values obtained (see Table 1), especially in

TABLE 1. Mean and standard deviation (SD) of the LLE of all the signals analyzed. Statistically significant values are marked with and asterisk and highlighted in blue and yellow in the case of a significant increase of the LLE of PD with respect to HC and a significant decrease of the LLE of PD compared to HC, respectively.

Axis	Long-term LLE		Short-term LLE		Body segment		Long-term LLE		Short-term LLE	
	Healthy	Parkinson	Healthy	Parkinson			Healthy	Parkinson	Healthy	Parkinson
X	-0.47 (±1.28)	-0.74(±1.58)	-2.91 (±3.16)*	-3.63 (±3.42)*	Chest	Lumbar	-0.11 (±1.34)	-0.58 (±2.03)	-2.99 (±3.96)	-3.85 (±4.11)
Y	-0.57 (±1.88)*	0.49 (±2.32)*	0.87 (±3.51)	-8.16 (±3.65)			1.61 (±2.19)	0.89 (±2.55)	3.15 (±3.95)*	1.35 (±4.18)*
Z	0.03 (±2.05)	-0.02 (±2.31)	-3.81 (±3.15)	-3.22 (±4.05)			-0.63 (±2.)69	-0.49 (±2.66)	0.03 (±3.21)	-0.55 (±3.79)
	Right						Left			
X	-0.84 (±1.78)	-0.83 (±2.04)	-2.77 (±3.02)*	-3.22 (±3.06)*	Arm		-0.63 (±1.81)	-0.65 (±1.85)	-3.03 (±3.13)*	-3.44 (±3.30)*
Y	0.55 (±2.03)	0.87 (±2.41)	-3.58 (±2.91)	-3.23 (±3.26)			1.11 (±2.54)	0.56 (±2.96)	-2.69 (±3.30)*	-2.85 (±3.48)*
Z	1.50 (±2.23)*	1.95 (±2.35)*	2.10 (±3.04)	2.08 -(±3.10)			2.00 (±2.17)	2.44 (±2.42)	-2.03 (±3.11)	-1.30 (±3.38)
X	-0.48 (±1.75)	-0.63 (±1.89)	-3.13 (±4.07)*	-3.28 (±4.10)*	Thigh		-0.67 (±1.77)	-0.67 (±2.06)	-3.83 (±4.48)*	-3.96 (±4.23)*
Y	-0.28 (±1.75)	0.006 (±2.13)	1.53 (±3.29)	1.31 (±3.76)			-0.74 (±2.08)	-0.40 (±2.36)	2.13 (±3.34)*	2.59 (±3.73)*
Z	-0.46 (±2.46)	-0.49 (±2.34)	1.86 (±4.02)	1.63 (±3.92)			-0.36 (±2.52)	0.07 (±2.37)	1.51 (±4.08)	0.89 (±4.21)
X	-0.90 (±1.92)	-0.56 (±1.93)	-3.98 (±4.64)*	-4.24 (±4.71)*	Shank		-0.87 (±1.83)	-0.85 (±2.05)	-3.82 (±4.28)*	-4.23(±4.51)*
Y	0.50 (±2.71)	0.30 (±2.44)	2.72 (±3.95)*	-2.65 (±3.66)*			1.22 (±2.62)	0.51 (±2.07)	2.64 (±3.98)	3.04 (±3.79)
Z	0.18(±1.85)	0.22 (±1.94)	0.30 (±3.32)	-0.50 (±3.70)			0.41 (±2.07)	-0.15 (±2.39)	1.04 (±3.82)	0.19 (±3.71)
X	0.23 (±1.84)	0.60 (±1.80)	1.23 (±6.37)	1.18 (±5.70)	Foot		0.09 (±2.55)	0.31 (±2.38)	1.77 (±5.67)	1.86 (±1.93)
Y	0.65 (±2.14)	0.60 (±2.39)	2.38 (±5.00)*	3.06 (±4.78)*			0.37 (±2.26)	0.50 (±1.96)	3.26 (±4.57)*	3.72 (±4.40)*
Z	0.28 (±2.39)*	1.07 (±1.68)*	1.58 (±6.27)	0.78 (±7.24)			0.54 (±1.90)	0.91 (±2.23)	3.42 (±5.83)	2.29 (±6.62)

the case of the short-term LLE, confirm the unreliability of this metric, particularly when the signal is short. Further studies ensuring that enough steps are recorded should be carried out, thus guaranteeing reliable LLE values. Also, other measures of stability should be used or proposed, allowing the analysis of gait signals without the constraints of LLE.

V. Conclusion

This study evaluated gait stability of PD patients and HC subjects, using the long and short-term LLE calculated on IMU data from straight walking trials, performed by 34 HC subjects and 42 PD patients. Acceleration signals in three axis directions from 10 different body parts were recorded, summing 30 signals analyzed per participant.

We found that both the long and short-term LLE are capable to distinguish between PD patients and HC. In most cases, the long-term LLE was higher for the first group than for the latter, indicating that gait is more unstable in PD patients than in HC. This means that, as expected, PD affects gait stability. However, contradictory results were obtained with the short-term LLE. Furthermore, it was also observed that the LLE is a very sensible metric that can be affected by several factors, such as noise and signal length.

The location of the sensor chosen to calculate the LLE does not seem to be trivial and the best location for the purpose of assessing gait stability should be furtherly studied. Our hypothesis is that, for gait stability evaluation, the best location is the trunk, either the chest or the lumbar region. The trunk is where the center of mass is usually found and the most stable segment during gait. Also, the most relevant axis direction should be investigated. Correlation analysis between axes and locations can help to approach both issues.

The reliability of the LLE to assess gait stability should also be further studied. Meanwhile, other metrics can be explored to support the outcomes extracted with the LLE. These may include other nonlinear dynamics features that complement the information provided by LLE. In this study, only LLE was considered to evaluate gait stability due to its extensive usage in the literature, but there are other common metrics like the margin of stability, the center of mass displacement, etc. This future validation study will include external perturbation to induce unstable gait in HC subjects and compare it to their natural gait. Future efforts should also aim to find a minimal sensor setup to facilitate its use in the clinical setting.

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References

[1] W. Poewe *et al.*, "Parkinson disease," *Nat Rev Dis Primers*, vol. 3, pp. 1–21, Mar. 2017, doi: 10.1038/nrdp.2017.13.

[2] L. V. Kalia and A. E. Lang, "Parkinson's disease," *The Lancet*, vol. 386, no. 9996. Lancet Publishing Group, pp. 896–912, Aug. 29, 2015. doi: 10.1016/S0140-6736(14)61393-3.

[3] S. Sveinbjornsdottir, "The clinical symptoms of Parkinson's disease," *Journal of Neurochemistry*. Blackwell Publishing Ltd, pp. 318–324, Oct. 01, 2016. doi: 10.1111/jnc.13691.

[4] S. Fahn, "Description of Parkinson's Disease as a Clinical Syndrome," *Ann N Y Acad Sci*, vol. 991, no. 1, pp. 1–14, 2003.

[5] Eduardo Tolosa, Gregor Wenning, and Werner Poewe, "The diagnosis of Parkinson's disease," *Lancet Neurol*, vol. 5, no. 1, pp. 75–86, 2006.

[6] J. Jankovic, "Parkinson's disease: Clinical features and diagnosis," *Journal of Neurology, Neurosurgery and Psychiatry*, vol. 79, no. 4. BMJ Publishing Group, pp. 368–376, 2008. doi: 10.1136/jnnp.2007.131045.

[7] E. Rovini, C. Maremmani, and F. Cavallo, "How wearable sensors can support parkinson's disease diagnosis and treatment: A systematic review," *Frontiers in Neuroscience*, vol. 11, no. OCT. Frontiers Media S.A., Oct. 06, 2017. doi: 10.3389/fnins.2017.00555.

[8] M. T. Rosenstein', J. J. Collins, C. J. de Luca, and P. E. Rapp, "A practical method for calculating largest Lyapunov exponents from small data sets," *Physica D*, vol. 65, no. 1–2, pp. 117–134, 1993.

[9] Z. Liu, "Chaotic time series analysis," *Math Probl Eng*, vol. 2010, 2010, doi: 10.1155/2010/720190.

[10] Jiang Shuo, Wang Xingchen, Kyrarini Maria, and Gräser Axel, "A Robust Algorithm for Gait Cycle Segmentation," *European Association for Signal Processing. European Association for Signal Processing*, pp. 31–35, 2017.

[11] P. Terrier and R. Fabienne, "Maximum Lyapunov exponent revisited: Long-term attractor divergence of gait dynamics is highly sensitive to the noise structure of stride intervals," *Gait Posture*, vol. 66, pp. 236–241, 2018.

[12] H. G. Kang and J. B. Dingwell, "Effects of walking speed, strength and range of motion on gait stability in healthy older adults," *J Biomech*, vol. 41, no. 14, pp. 2899–2905, Oct. 2008, doi: 10.1016/j.jbiomech.2008.08.002.

[13] S. Tajali, M. Mehravar, H. Negahban, J. H. van Dieën, M. J. Shaterzadeh-Yazdi, and R. Mofateh, "Impaired local dynamic stability during treadmill walking predicts future falls in patients with multiple sclerosis: A prospective cohort study," *Clinical Biomechanics*, vol. 67, pp. 197–201, Jul. 2019, doi: 10.1016/j.clinbiomech.2019.05.013.

[14] F. Reynard and P. Terrier, "Local dynamic stability of treadmill walking: Intrasession and week-to-week repeatability," *J Biomech*, vol. 47, no. 1, pp. 74–80, Jan. 2014, doi: 10.1016/j.jbiomech.2013.10.011.

[15] P. M. McAndrew, J. M. Wilken, and J. B. Dingwell, "Dynamic stability of human walking in visually and mechanically destabilizing environments," *J Biomech*, vol. 44, no. 4, pp. 644–649, Feb. 2011, doi: 10.1016/j.jbiomech.2010.11.007.

[16] P. Terrier and O. Dériaz, "Non-linear dynamics of human locomotion: Effects of rhythmic auditory cueing on local dynamic stability," *Front Physiol*, vol. 4 SEP, 2013, doi: 10.3389/fphys.2013.00230.

[17] E. H. Sinitksi, K. Terry, J. M. Wilken, and J. B. Dingwell, "Effects of perturbation magnitude on dynamic stability when walking in destabilizing environments," *J Biomech*, vol. 45, no. 12, pp. 2084–2091, Aug. 2012, doi: 10.1016/j.jbiomech.2012.05.039.

[18] E. A. F. Ihlen, T. Goihl, P. B. Wik, O. Sletvold, J. Helbostad, and B. Vereijken, "Phase-dependent changes in local dynamic stability of human gait," *J Biomech*, vol. 45, no. 13, pp. 2208–2214, Aug. 2012, doi: 10.1016/j.jbiomech.2012.06.022.

[19] M. Piórek, H. Josiński, A. Michalczuk, A. Świtoński, and A. Szczęsna, "Quaternions and joint angles in an analysis of local stability of gait for different variants of walking speed and treadmill slope," *Inf Sci (N Y)*, vol. 384, pp. 263–280, Apr. 2017, doi: 10.1016/j.ins.2016.08.069.

[20] L. Bizovska, Z. Svoboda, M. Janura, M. C. Bisi, and N. Vuillerme, "Local dynamic stability during gait for predicting falls in elderly people: A one-year prospective study," *PLoS One*, vol. 13, no. 5, May 2018, doi: 10.1371/journal.pone.0197091.

[21] V. Smith Hussain, C. W. Frames, and T. E. Lockhart, "Length of Time-Series Gait Data on Lyapunov Exponent for Fall Risk Detection," *Int J Progn Health Manag*, vol. 12, no. 4, Aug. 2021, doi: 10.36001/ijphm.2021.v12i4.2917.