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A Method of Predicting Posture-related Pain Using Biomechanical Parameters for Patients with Lumbar Spinal Disc Herniation

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***Abstract*— Lumbar spinal disc herniation is a disease in which the protruding nucleus pulposus presses on the nerve due to actions that place loads on the disc, causing pain in the lower back and lower limbs. About 80% of treatments of disc herniation are conservative treatments, and although it is necessary to live with pain for a long time, there have been no studies that clearly define the relationship between pain and biomechanical parameters. In this study, we proposed a method of identifying biomechanical parameters that predict posture-related pain in patients with lumbar spinal disc herniation. The pain values were quantitatively evaluated by the Numerical Rating Scale (NRS) and the biomechanical parameters were analyzed by OpenSim. Lasso regression was performed to narrow down the biomechanical parameters that were related to pain and derive the mathematical model of the relationship. Therefore, many of the parameters of the obtained mathematical model were related to the lumbar spine and were consistent with areas that be related to lumbar spinal disc herniation.**

***Index Terms*- Lumbar spinal disc herniation, back pain, OpenSim, Lasso regression**

I. INTRODUCTION

In the Global Burden of Disease Study, low back pain ranks at the top of 289 diseases and injuries in the YLDs (Years Lived with Disability), which measures the number of years persons live in an unhealthy state [1]–[3]. More than 80% of the population will experience an episode of low back pain at some time during their lives [4]–[5]. One of the causes of low back pain is lumbar disc herniation. Statistics show that one-third of adults over the age of 20 have symptoms of herniated discs and 90% of herniation occurs in the lumbar and lumbosacral regions of the spine [6]. The intervertebral discs connect the spine and act as a shock absorber cushion. The thick outer part is called the annulus, and the inner gel-like part is called the nucleus. Lumbar spinal disc herniation is a disease that occurs when the annulus fibrosus cracks and the inner nucleus pulposus protrudes due to external pressure and is particularly likely to occur at the lumbar discs of L4-L5 and L5-S1 [6]–[7]. When the overload is applied to the intervertebral discs, the protruding nucleus pulposus presses on the nerve and causes pain. In addition to pain in the lower back, the pain may extend to the entire lower extremities, and muscle weakness, scoliosis, and nerve disorders occur. However, in about 80% of cases, the protruding nucleus pulposus is absorbed and lumbar spinal disc herniation heals naturally if conservative treatment like physical therapy is performed appropriately.

There have been many previous studies to detect lumbar disc herniation from magnetic resonance imaging (MRI) data [7]–[8]. In addition, some studies have focused on compressive forces, since the load on the intervertebral disc is directly related to symptoms. Studies on the relationship between disc compressive force and postures provide information on positions that patients should avoid [9]. In previous studies, disc compression forces have been estimated from various methods and sensors: back-mounted markers [10], accelerometers and bending sensors [5] [11], stress analysis using test body models [12], elastic models of the intervertebral discs [13], and modeling simulation software [14]–[15]. There is also research on the exoskeleton that reduces the disc compressive force on the intervertebral disc when people lift heavy things [16]–[18]. These studies could lead to the prevention of low back pain as well as to the recovery of lumbar spinal disc herniation. Although it is necessary to live with pain for a long time, no studies have clearly defined the relationship between pain and biomechanical parameters of patients with disc herniation. In addition, while pain is an important factor in the diagnosis and treatment of all diseases, it is still difficult to quantitatively evaluate pain because it is a subjective sensation. In this study, Lasso regression analysis was performed using quantitatively assessed pain values and biomechanical parameters. As a result, we were able to narrow down the biomechanical parameters related to pain and derive a mathematical model.

II. METHODS

Symptoms of lumbar spinal disc herniation vary greatly from person to person because various factors, such as the position of the herniated disc, muscle forces around the disc, and the patient's living environment, are related to pain. Therefore, we analyzed a subject. The subject has a herniated disc between L4-L5 on the left lumbar vertebra for 4 years. The sex, age, height and weight are 22 years old, 156.0 cm, and 46 kg, respectively.

A. Pain Assessment

Pain assessment and calculating biomechanical parameters are performed separately because the degree of pain varies with physical condition and mood and is difficult to measure simultaneously with biomechanical parameters. Numerical Rating Scale (NRS) was used because it is simply a way of rating or quantifying pain, even on different days and in

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different moods. It is an evaluation method that expresses pain as a numerical scale from 0 to 10, where 0 is no pain and 10 is the maximum imaginable pain [19]. The pain was measured 12 times after a minute in each of the 10 postures (static standing, half sitting, crouching, sitting, tiptoeing, trunk flexion, trunk extension, standing on one leg, and bending knees). Since the degree of pain varies depending on the state of the body at the time, it is difficult to compare different postures using the average of the measured values. Therefore, the difference in pain values between each posture and the static standing posture was calculated based on the pain value in the static standing posture, and the average of the difference values for 12 times was calculated. Table I shows the results of pain values for the 10 postures.

TABLE I
RESULTS OF PAIN VALUES FOR THE 10 POSTURES

Posture	Pain value
Static standing	0.00
Half sitting	0.750
Bending knees	0.417
Standing on right leg	-0.167
Standing on left leg	1.75
Trunk extension	0.583
Sitting	1.42
Crouching	0.500
Tiptoeing	0.0833
Trunk flexion	2.75

B. Calculating Biomechanical Parameters by OpenSim

To calculate biomechanical parameters, we used the musculoskeletal model simulation software OpenSim. It is necessary to measure marker data and floor reaction force data. They were obtained using the motion capture system (from VICON in Oxford, UK) and force plates (BP400600 from Advanced Mechanical Technology, Inc. in Watertown, US). The simulation model was based on the full body model from Rajagopal et al. [20], and the markers were adjusted manually so that they were placed in the Plug-in-Gait model [21].The subject wore a suit with 39 markers and the marker set is based on the Plug-in-Gait model [21]. Scaling was performed in OpenSim to reproduce the subject's physique and to approximate the positions of the experimental markers in the static standing posture to the marker positions in the simulation model. First, the mass and length of the segments were adjusted. The subject's weight was used for mass. The distance of the markers in the simulation model was compared to the distance of the experimental markers in the static standing posture, and the ratio of two lengths was then used to create a model with the subject's specific segment lengths. Next, a weighting scheme was set to determine the degree to which each simulation model marker matched the experimental markers. Markers with small displacements, such as those on bones, were given higher weights, while markers on the thigh and lower leg were given lower weights. In Fig. 1, the measured postures are reproduced on OpenSim using data from the scaled model and the positions of the experimental markers. Inverse kinematics, inverse dynamics, static optimization, and joint reaction force calculations resulted in 39 joint angle, 39 joint torque, 97 muscle force, and 198 joint reaction force parameters.

C. Lasso Regression

Lasso regression analysis estimates the parameter values that minimize the sum of the mean squared error and weights to suppress overlearning [22]. Lasso regression analysis which allows limiting the number of explanatory variables was chosen because it is more practical to have fewer biomechanical parameters in physical therapy. The loss function J can be expressed in (1). The second term on the right side of (1) is called the L1 normalization term,

TABLE II
THE POSTURES OF TEST DATA AND THE RESULT OF MEAN SQUARED ERROR

i	Test data	Mean squared error L_i	
		Llinearly	Exponentially
1	Static standing	0.942	0.562
2	Half sitting	0.0168	0.0168
3	Bending knees	0.0560	0.0600
4	Standing on right leg	0.0308	0.0308
5	Standing on left leg	0.205	0.205
6	Trunk extension	0.129	0.129
7	Sitting	0.936	0.936
8	Crouching	0.427	0.305
9	Tiptoeing	0.885	0.676
10	Trunk flexion	6.51	6.31

which induces the weight coefficients to be 0 if the regression model includes many variables with low predictive abilities. Therefore, variables with high predictive abilities can be clearly selected from among many explanatory variables and the dimension can be compressed.

$$J = \sum_{i=1}^{n} \left(y^{(i)} - \hat{y}^{(i)} \right)^2 + \lambda \sum_{k=1}^{m} |w_k| \tag{1}$$

In (1), n is the number of data, m is the number of samples, y^i is the actual measured value of the i-th data, $\hat{y}^i$ is the predicted value of the i-th data, λ is the normalization parameter and w_k is the k-th weight coefficient.

Table II shows the postures of test data, parameter i, and the result of Mean Squared Error (MSE). The 10 equations obtained by repeating the Lasso regression analysis 10 times with different test data are represented by (2) with $x_\circ$ as the variable and $a_{\circ,i}$ as the coefficient. When the posture of the test data is changed, the variables and the number of variables selected in each equation change. There are j variables $x_\circ$, where j is the number of all variables selected in the 10 equations. The coefficients of variables not used in each equation are set to 0.

$$y_i = a_{0,i}x_0 + a_{1,i}x_1 + a_{2,i}x_2 + \cdots + a_{j-1,i}x_{j-1} + a_{j,i} \tag{2}$$

III. Measurement of Biomechanical Parameters

Ten equations were combined into an equation. The following two methods were tried.

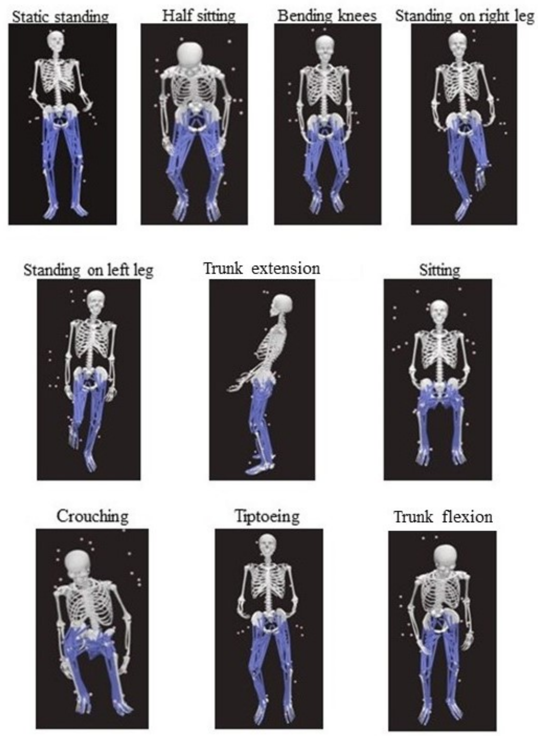

Fig. 1: Model reproduced in OpenSim

- Method 1: Taking the average of the coefficients
  Table Ⅱ shows that MSE is very large when the data of trunk flexion is used as the test data. Therefore, we averaged the 9 coefficients for each variable using the 9 equations other than the one using the data of trunk flexion as the test data. The resulting equation is shown in (3).

$$y = \frac{\sum_{i=1}^{9} a_{0,i}}{9} a_{0,i} x_0 + \frac{\sum_{i=1}^{9} a_{1,i}}{9} a_{1,i} x_1 + \cdots + \frac{\sum_{i=1}^{9} a_{j-1,i}}{9} a_{j-1,i} x_{j-1} + \frac{\sum_{i=1}^{9} a_{j,i}}{9} a_{j,i} \quad (3)$$

- Method 2: Using MSE L_i as weights
  The equation obtained so that the influence of the equations with the smaller MSE is stronger is shown in (4).

$$y = \frac{\frac{1}{{L_1}^2}}{\sum_{i=1}^{10} \frac{1}{{L_i}^2}} y_1 + \frac{\frac{1}{{L_2}^2}}{\sum_{i=1}^{10} \frac{1}{{L_i}^2}} y_2 + \cdots + \frac{\frac{1}{{L_i}^2}}{\sum_{i=1}^{10} \frac{1}{{L_i}^2}} y_{10} \quad (4)$$

Assuming that pain varies linearly or exponentially, the number of explanatory variables could be narrowed down to 17 and 24, respectively. The variables and their coefficients are shown in Table Ⅲ and Table Ⅳ. The three axes in OpenSim are shown in Fig. 2. The parameters with absolute coefficients larger than 0.01 are assumed to have strong effects as shown in gray. Assuming that pain varies linearly and exponentially, the parameters are as follows: lumbar rotation angle, left knee joint torque, right gluteus medius muscle force, right gluteus minimus muscle force, left long flexor digitorum longus muscle force, and left femur (hip side) joint reaction force. Assuming that pain varied linearly, left talus joint reaction force was also included. Fig. 3 shows scatter plots between measured and predicted pain values. The results for the tiptoeing posture were outliers, but the results for the other postures showed high predictive accuracy, regardless of how the pain varied and how the 10 equations were combined. In the present analysis, no significant differences were found between the two methods. More analysis data is needed to clarify the characteristics of each method.

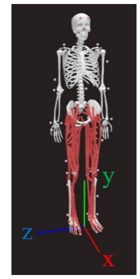

Fig. 2: Three axis on OpenSim

IV. Results and Discussion

Table Ⅲ and Table Ⅳ show that the variables narrowed down by Lasso regression analysis were more common around the lumbar spine, although some variables were obtained for areas distant from the lumbar spine, such as the ulna. In addition, among the biomechanical parameters shown in gray, the parameters of the lumbar rotation angle the and the gluteus medius and minimus muscle forces, which are involved in stabilizing the pelvis during standing and walking, are ideal parameters that be related to a lumbar spinal disc herniation. Furthermore, Fig. 3 shows a high correlation between the measured and predicted pain values. These findings indicate that a mathematical model including the lumbar spine angle and the gluteus medius and minimus muscle forces might predict posture-related pain.

This study had three limitations. First, the number of subjects was limited. By analyzing data from more subjects, different biomechanical parameters could be identified as posture-related features. Second, this study used only Lasso regression and did not compare the results with other regression models. Adapting multiple regression models to data from a larger number of subjects and measuring the validity of the features is needed in the future. Finally, this study did not measure pain during daily life. Posture-related pain could vary from day to day. Therefore, it is a future challenge to verify whether the identified parameters can also predict pain during daily life by utilizing IoT sensors.

TABLE III
COEFFICIENTS AND VARIABLES ASSUMING LINEAR VARIATION IN PAIN

Coefficients			Variables	
	Method 1	Method 2		
a_0	-7.10×10^{-4}	-1.47×10^{-6}	x_0	Angle of pelvis z-axis
a_1	0.00	1.08×10^{-6}	x_1	Angle of hip rotation
a_2	-1.53×10^{-1}	-1.84×10^{-1}	x_2	Angle of lumbar rotation
a_3	-3.41×10^{-1}	-3.55×10^{-1}	x_3	Left knee joint torque
a_4	2.78×10^{-2}	2.13×10^{-3}	x_4	Right gastrocnemius muscle force
a_5	1.59×10^{-2}	9.13×10^{-5}	x_5	Left gastrocnemius muscle force
a_6	-2.22×10^{-1}	-2.29×10^{-1}	x_6	Right gluteus medius muscle force
a_7	-9.93×10^{-2}	-1.15×10^{-1}	x_7	Right gluteus minimus muscle force
a_8	9.80×10^{-2}	1.37×10^{-1}	x_8	Left flexor digitorum longus muscle force
a_9	0.00	-6.95×10^{-7}	x_9	Left piriformis muscle force
a_{10}	1.69×10^{-3}	6.68×10^{-6}	x_{10}	Left soleus muscle force
a_{11}	0.00	1.55×10^{-7}	x_{11}	Right suture muscle force
a_{12}	4.69×10^{-2}	3.04×10^{-2}	x_{12}	Y-component of joint reaction force on the left femur (hip side)
a_{13}	0.00	2.03×10^{-7}	x_{13}	Y-component of joint reaction force on the left tibia (knee side)
a_{14}	-7.28×10^{-2}	-7.33×10^{-2}	x_{14}	Y-component of joint reaction force on the left talus
a_{15}	1.99×10^{-2}	1.54×10^{-4}	x_{15}	Y-component of joint reaction force on the right ulna
a_{16}	0.00	-6.99×10^{-7}	x_{16}	X-component of joint reaction force on the left ulna
a_{17}	8.22×10^{-1}	8.16×10^{-1}		

TABLE IV
COEFFICIENTS AND VARIABLES ASSUMING EXPONENTIAL PAIN

Coefficients			Variables	
	Method 1	Method 2		
a_0	-2.33×10^{-3}	-3.68×10^{-6}	x_0	Angle of pelvis z-axis
a_1	4.11×10^{-3}	2.52×10^{-3}	x_1	Angle of pelvis rotation
a_2	0.00	6.50×10^{-7}	x_2	Angle of hip rotation
a_3	-3.26×10^{-2}	-3.38×10^{-2}	x_3	Angle of lumbar rotation
a_4	-8.75×10^{-2}	-8.12×10^{-2}	x_4	Left knee joint torque
a_5	0.00	2.51×10^{-7}	x_5	Left elbow joint torque
a_6	6.19×10^{-3}	3.18×10^{-3}	x_6	Left ankle joint torque
a_7	1.69×10^{-3}	2.66×10^{-6}	x_7	Right gastrocnemius muscle force
a_8	-2.53×10^{-2}	-2.58×10^{-2}	x_8	Right gluteus medius muscle force
a_9	-7.57×10^{-4}	-2.55×10^{-4}	x_9	Left gluteus medius muscle force
a_{10}	-3.16×10^{-2}	-4.47×10^{-2}	x_{10}	Right gluteus minimus muscle force
a_{11}	3.32×10^{-2}	4.27×10^{-2}	x_{11}	Left flexor digitorum longus muscle force
a_{12}	0.00	-5.37×10^{-7}	x_{12}	Left piriformis muscle force
a_{13}	-2.32×10^{-3}	-6.80×10^{-3}	x_{13}	Right greater adductor muscle force
a_{14}	-3.45×10^{-3}	-6.46×10^{-5}	x_{14}	Right iliopsoas muscle force
a_{15}	7.34×10^{-4}	1.94×10^{-6}	x_{15}	Right suture muscle force
a_{16}	2.27×10^{-2}	3.39×10^{-2}	x_{16}	Y-component of joint reaction force on the left femur (hip side)
a_{17}	0.00	-1.02×10^{-8}	x_{17}	Z-component of joint reaction force on the right tibia (knee side)
a_{18}	0.00	1.24×10^{-7}	x_{18}	Z-component of joint reaction force on the left tibia (knee side)
a_{19}	3.96×10^{-4}	1.05×10^{-6}	x_{19}	Z-component of joint reaction force on the right talus
a_{20}	2.47×10^{-3}	-1.54×10^{-3}	x_{20}	Y-component of joint reaction force on the left talus
a_{21}	1.14×10^{-3}	2.41×10^{-5}	x_{21}	Z-component of joint reaction force on the left talus
a_{22}	5.58×10^{-3}	8.77×10^{-6}	x_{22}	Y-component of joint reaction force on the right ulna
a_{23}	0.00	1.73×10^{-7}	x_{23}	Z-component of joint reaction force on the left ulna
a_{24}	2.14×10^{-1}	2.15×10^{-1}		

V. CONCLUSIONS

In this study, we proposed a method of Lasso regression analysis using quantitatively evaluated pain values and biomechanical parameters obtained by OpenSim to identify the relationship between pain caused by postures and biomechanical parameters. As a result, we were able to find biomechanical parameters that were related to pain. This research finding can contribute to the treatment of patients with lumbar spinal disc herniation to avoid posture-related pain.

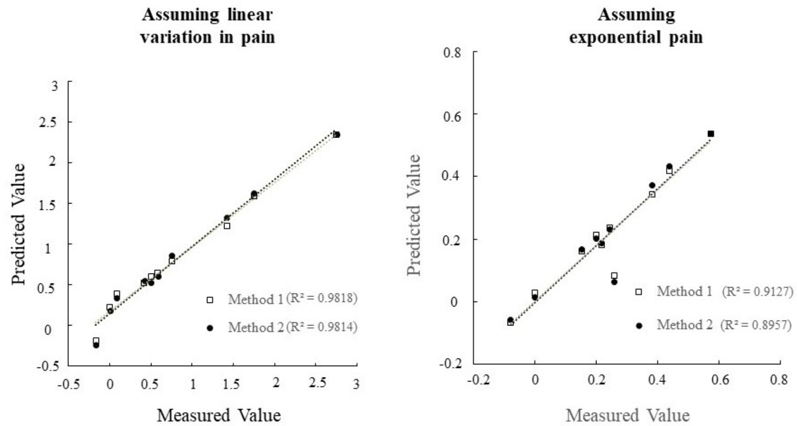

Fig. 3: Scatter plots depicting the relationship between the measured and predicted pain values

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