IEICE Communications Express, Vol.13, No.4, 138–141

ComEX

LETTER

An index-modulated FSCM systems with low complexity algorithm and enhanced data rate*

Li Li[1], **Zujun Liu**[1, a)], **and Yurong Guo**[1]

Abstract Frequency-Shift Chirp Modulation (FSCM) also known as LoRa modulation, enables long-distance transmission with low power consumption for Internet of Things (IOT), but has a limited transmission rate. Frequency-shift chirp spread spectrum system with index modulation (FSCSS_IM) introduces index modulation to improve the transmission rate of FSCM at the cost of high demodulation complexity. This letter propose an improved FSCSS_IM scheme to further improve the transmission rate of FSCM by adding indexes with equal values to the codebook. And then we design a low complexity demodulation algorithm which employs index matching to replace the index search matching algorithm in FSCSS_IM. Finally, we derive the approximate BER expression of the proposed scheme. The analysis and simulation results show that the transmission rate of the proposed scheme is up to 91.7% higher than that of the FSCM and 8.3% higher than that of the existing FSCSS_IM scheme, and the demodulation algorithm has the almost the same computational complexity as the FSCM and much lower than that of FSCSS_IM.

Keywords: IOT, chirp spread spectrum, noncoherent detection, index modulation

Classification: Fundamental theories for communications

1. Introduction

Low Power Wide Area Networks (LPWAN) allows ubiquitous connectivity between millions of sensors, allowing for the development of a large number of smart IoT applications, resulting in a smart lifestyle where everything is connected and controlled via the Internet [1]. As a derivative of chirp spread spectrum (CSS) technology, FSCM (Frequency-Shift Chirp Modulation) [2] that is the well-known LoRa modulation, has become one of the most popular LPWAN technologies due to its advantages of low power consumption and long-distance transmission. However, these advantages are gained at the cost of low transmission rates, which greatly limits the future IoT applications [3].

Towards this ends, many schemes has been proposed to improve the transmission rate of FSCM. E_LoRa [4] adds one additional bit to determine whether the FSCM signal is in phase or quadrature. Although it improves the transmission rate of FSCM system, the receivers need to accurately estimate the channel state information which greatly increases the demodulation complexity. SSK_LoRa [5] determines whether to transmit upchirp or downchirp by adding one additional bit information. However, it need to demodulate upchirp and downchirp at the same time, also resulting in a higher demodulation complexity.

Unlike most prior work, FSCSS_IM [6] maps the bit information to the index with unequal values, and then modulates each value of the index to the initial frequency offset of chirp. Each modulated FSCSS_IM symbol can transmit more $SF - 2$ bits than FSCM. However, FSCSS_IM needs to search and match index in the codebook during demodulation, which makes the demodulation complexity significantly high.

In this letter, we further explore the line of FSCSS_IM by adding the indexs with equal values into the codebook so as to further improve the transmission rate. Then we present a low complexity demodulation algorithm based on index matching by designing index ordering in codebook. In addition, we derive the approximate BER expression in AWGN channel and give the simulation results.

2. Basis of FSCM and FSCSS_IM

2.1 FSCM

FSCM is a chirp modulation via changing the initial frequency shift [2]. Assuming that the sampling rate is equal to the bandwidth, the discrete expression for the FSCM signal is:

$$x_m(k) = e^{j2\pi(\frac{k}{2N}-\frac{1}{2}+\frac{m}{N})k}, k = 0, 1, \ldots, N-1, \tag{1}$$

where $N = 2^{SF}$ and $SF \in \{7, 8, 9, 10, 11, 12\}$ is the spread factor, $m \in [0, N-1]$ is the decimal value of the SF bits information carried in a modulated symbol.

FSCM uses the non-coherent demodulation denoted as

$$x_{dem} = \arg\max(FFT(y_m(k) * conj(x_0(k)))). \tag{2}$$

The receiver multiplies the received signal $\boldsymbol{y}_m$ with the conjugation of the $\boldsymbol{x}_0$ (upchirp) where $x_0(k) = e^{j2\pi(\frac{k}{2N}-\frac{1}{2})k}$ is the basic FSCM signal, then performs the FFT operation, and finally selects the bin corresponding to the largest amplitude as the decimal value of the message (x_{dem}).

2.2 FSCSS_IM

FSCSS_IM [6] improves the transmission rate of FSCM by introducing index modulation to modulate information on the initial frequencies of two superimposed upchirps. FSCSS_IM maps the information to the index $\{f_1, f_2\}$, where $f_1, f_2 \in [0, N-1]$ and $f_1 < f_2$. So there are $\boldsymbol{C}_N^2$ possible indexes. And the information an index can carry is $SF_{d1} =$

[1] School of Telecommunications Engineering, Xidian University, 2 South Taibai Road Xian, Shaanxi, CN 710071

* This work was supported in part by the National Natural Science Foundation of China under Grant 62171345.

a) liuzujun@mail.xidian.edu.cn (Corresponding author)

DOI: 10.23919/comex.2024XBL0011
Received January 17, 2024
Accepted January 29, 2024
Publicized February 16, 2024
Copyedited April 1, 2024

$\lfloor log_2 \boldsymbol{C}_N^2 \rfloor$, which is equal to $2 \cdot SF - 2$ when $SF \in [7, 12]$. The indexes of FSCSS_IM are sorted in the codebook as $\{0,1\}$,$\{0,2\}$,...,$\{0, N-1\}$,$\{1,2\}$,...,$\{N-2, N-1\}$.

Every SF_{d1} bits in FSCSS_IM is converted to a decimal value $f \in [0, N-1]$, and then the f-th index $\{f_1, f_2\}$ in the codebook is extracted. Each value in the index is modulated at the initial frequency of the upchirp signal, and the transmitted signal of FSCSS_IM is written by:

$$x_F(k) = \frac{E_s}{\sqrt{2}} \left[e^{j2\pi(\frac{k}{2N} - \frac{1}{2} + \frac{f_1}{N})k} + e^{j2\pi(\frac{k}{2N} - \frac{1}{2} + \frac{f_2}{N})k} \right]. \quad (3)$$

In the demodulation, the receiver performs the dechirp and FFT operations on the received signal and selects the two bins with the largest amplitude. After obtain the index by sorting bins, the receiver will search the codebook for a matching index and converts its position in the codebook to binary, and then the bit information is recovered.

3. Improved FSCSS with index modulation (ICSS_IM)

3.1 Modulation

The proposed ICSS_IM adds the indexes with equal values into the codebook, where the two values in the index is $f_1 \le f_2$, $f_1, f_2 \in [0, N-1]$. Thus, there are $\boldsymbol{C}_N^2 + N$ possible indexes such that each symbol can carry $SF_{d2} = \lfloor log_2(\boldsymbol{C}_N^2 + N) \rfloor$ bits information. And each symbol can carry $2 \cdot SF - 1$ bits which carries one more bit than FSCSS_IM.

The indexes of ICSS_IM are sorted in the codebook as $\{0,0\}$,$\{0,1\}$,$\{1,1\}$,...,$\{N-1, N-1\}$, which is shown in Fig. 1. The codebook contains only $2^{SF_{d2}}$ of $\boldsymbol{C}_N^2 + N$ possible indexes. Each SF_{d2} bits information converts to a decimal value f and then maps to the f-th index $\{f_1, f_2\}$ in the codebook. To ensure that the power of each transmitting symbol equal, the transmitting signal is given by:

$$x_I(k) = \begin{cases} \frac{E_s}{\sqrt{2}} \left(x_{f_1}(k) + x_{f_2}(k) \right), & f_1 < f_2 \\ E_s \times x_{f_1}(k), & f_1 = f_2 \end{cases} \quad (4)$$

where $x_{f_1}(k) = e^{j2\pi(\frac{k}{2N} - \frac{1}{2} + \frac{f_1}{N})k}$ and $x_{f_2}(k) = e^{j2\pi(\frac{k}{2N} - \frac{1}{2} + \frac{f_2}{N})k}$. We notice that in the case of $f_1 < f_2$, the transmitted signal is essentially identical to the transmitted signal of FSCSS_IM. Additionally, if $f_1 = f_2$, the transmitted signal is equivalent to FSCM.

3.2 Low complexity demodulation

The pseudo-code of the low complexity demodulation is described in Algorithm 1. The receiver first performs dechirp and FFT operations to obtain the frequency domain bins $\boldsymbol{f}_s^{'} = [0, 1, \ldots, N-1]$ and their corresponding amplitude

{0,0}				
{0,1}	{1,1}			
{0,2}	{1,2}	{2,2}		
...	...	...		
{0,N-2}	{1,N-2}	...	{N-2,N-2}	
{0,N-1}	{1,N-1}	...	{N-2,N-1}	{N-1,N-1}

Fig. 1 Order of indexs in the codebook.

$\boldsymbol{y}_s^{'} = [y_s^{'}(1), y_s^{'}(2), \ldots, y_s^{'}(N)]$, and then sorts $\boldsymbol{y}_s^{'}$ in a descending order as $\boldsymbol{y}_s$ and get their corresponding bins $\boldsymbol{f}_s$.

Before getting the initial index, we need to determine whether the two values in the initial index $\{f_1, f_2\}$ are equal or not. We notice that $y_s(2)$ is determined by only the power of the noise when $f_1 = f_2$ and the power of the signal and noise when $f_1 < f_2$. Therefore, we figure out the difference $D_1 = y_s(1) - y_s(2)$ and $D_2 = y_s(2) - y_s(3)$, if $D_1 > D_2 \cdot T$ (T is a threshold), the initial index is $\{f_s(1), f_s(1)\}$ with two equal values. Otherwise, the initial index is the sorted result of $f_s(1)$ and $f_s(2)$, i.e., $\{f_1, f_2\} = sort\{f_s(1), f_s(2)\}$. For each SF, the receiver can estimate the SNR through the preamble, and simply set a different threshold depending on whether the SNR is in the low or high range.

Depending on the order of the indexs in the codebook as shown in Fig. 1, we can derive the relationship between the index $\{f_1, f_2\}$ and the decimal value f of the bit information:

$$f = f_1 + \sum_{m=0}^{f_2} m. \quad (5)$$

Since the codebook contains only $2^{SF_{d2}}$ indexes, when the initial index is not in the codebook, i.e., $f > 2^{SF_{d2}}$, we extract the next bin in $\boldsymbol{f}_s$ and form a new index with $f_s(1)$, and we iteratively update the index until it is in the codebook. Finally the recovered bit information is obtained by converting the decimal f to $2 \cdot SF - 1$ binary bits.

Multipath channels will reduce the effective SNR of the system and lead to a significant BER performance loss. The matched filter described in [7] is employed to the algorithm 1 to mitigate the effects of multipath channel. Specifically, we keep the preamble DFT output $\boldsymbol{y}_p$, and then calculate the cyclic-correlation between $\boldsymbol{y}_p$ and the payload DFT output $\boldsymbol{y}_s^{'}$:

$$R(i) = \sum_{k=0}^{N-1} \left| y_p(k) \right| \left| y_s^{'} \left((k+i) \bmod 2^{SF} \right) \right|, \quad (6)$$

where $i = 0, 1, \cdots, N-1$ and $R(i)$ also be interpreted as the

Algorithm 1 Low complexity demodulation algorithm

Require: T, $x_0[k]$, $y[k]$ for $k = 0, 1, \ldots, N-1$
Ensure: y_b
Step1: Obtain the initial index.
$[\boldsymbol{y}_s^{'}, \boldsymbol{f}_s^{'}] \leftarrow |FFT(\boldsymbol{y} .* conj(\boldsymbol{x_0}))|$
$[\boldsymbol{y}_s, \boldsymbol{f}_s] \leftarrow sort(\boldsymbol{y}_s^{'})$
$D_1 = \boldsymbol{y}_s(1) - \boldsymbol{y}_s(2)$
$D_2 = \boldsymbol{y}_s(2) - \boldsymbol{y}_s(3)$
if $D_1 > D_2 * T$ **then**
  $[f_1, f_2] = [\boldsymbol{f}_s(1), \boldsymbol{f}_s(1)]$
else
  $[f_1, f_2] = sort[\boldsymbol{f}_s(1), \boldsymbol{f}_s(2)]$
end if
Step2: Obtain the final index.
$f = f_1 + \sum_{m=0}^{f_2} m$
while $f > 2^{SF_{d2}}$ **do**
  Update $[f_1, f_2]$
  $f = f_1 + \sum_{m=0}^{f_2} m$
end while
Step3: Recover the information.
$y_b = dec2bin(f, SF_{d2})$

Table I Complexity analysis of different scheme without index update

ICSS_IM	FSCSS_IM	FSCM
$O(2Nlog_2N + \frac{3}{2}N)$	$O(\frac{1}{8} \times N^2 + N\log_2 N + 3N)$	$O(Nlog_2N + 2N)$

k-th DFT outputs in the frequency domain. The direct DFT output $\boldsymbol{y}_s^{'}$ in the manuscript is replaced with $\boldsymbol{R}$.

3.3 Complexity comparison and analysis

Table I summaries the computational complexity of FSCM, FSCSS_IM and the proposed ICSS_IM. FSCM multiply the received signal with the conjugation of $\boldsymbol{x}_0$, which has a computational complexity of $O(N)$; and then perform an FFT operation with a computational complexity of $O(N\log_2 N)$; the bin with the largest amplitude is extracted finally by the operation with $O(N)$ computational complexity.

FSCSS_IM also performs the dechirp and FFT operation, and then it extracts initial index and removes them from the frequency domain result, which results in a computational complexity of $O(2N)$. FSCSS_IM searches the codebook for matching initial index, and therein the computational complexity is $O(\frac{1}{2} \times 2^{2SF-2}) = O(\frac{1}{8} \times N^2)$.

ICSS_IM uses an index matching formula with the computational complexity of $O(\frac{N}{2} + 1)$ to map the index to the decimal value, replacing the existing search matching operation in FSCSS_IM. ICSS_IM sorts the magnitude of the FFT result before obtaining the index, which yields with a computational complexity of $O(N\log_2 N)$.

The demodulation computational complexity of the proposed ICSS_IM and FSCM is much lower than that of the FSCSS_IM. In addition, although the computational complexity of ICSS_IM is slightly higher than that of FSCM, it has great advantage in terms of transmission rate.

3.4 BER expression in AWGN channels

The symbolic error rate (SER) of ICSS_IM is equivalent to the probability of improperly demodulating the index, i.e.,

$$P_s = P_{e|f}P_{nov} + P_{e|\hat{f}}P_{ov}, \tag{6}$$

where $P_{nov} = \frac{C_N^2}{C_N^2+N} = \frac{N-1}{N+1}$ represents the probability that two values in the index are unequal, and $P_{e|f}$ is the probability of index detection error under this condition. $P_{ov} = \frac{N}{C_N^2+N} = \frac{2}{N+1}$ represents the probability of the equal values and $P_{e|\hat{f}}$ is the probability of index detection error.

Given an index $\{f_1, f_2\}$ with $f_1 < f_2$, the average symbol error probability can be expressed as the average probability of recognizing noise as a signal, i.e.,

$$P_{e|f} = \Pr\left[\max_{i,i\neq f_1,f_2}(\rho_i) > \min_{f,f\in\{f_1,f_2\}}(\beta_f)\right], \tag{7}$$

where $\rho_i = |\phi_i|$, ϕ_i is a complex Gaussian noise, then ρ_i depicts a Rayleigh distributed random variable with the cumulative function as:

$$F_{\rho_i}(\rho) = 1 - \exp\left[-\frac{\rho^2}{2\sigma^2}\right], \tag{8}$$

where $\sigma^2 = N_0/2$ is the variance and N_0 is the single-sided noise power spectral density. $\hat{\rho} = \max_{i,i\neq f_1,f_2}(\rho_i)$ is the maximum of the $2^{SF} - 2$ independent and equally distributed Rayleigh random variables which follows a Gaussian distribution.

The power of each chirp signal is $E_s/2$ for unequal index values. So $\beta_f = \left|\sqrt{E_s/2} + \phi_f\right|$ accordingly follows a Rician distribution. The equivalent SNR of chirp spread systems is high and the Rician distribution for high SNR can be approximated with a Gaussian distribution [8]. Therefore, the probability density function for β_f can be approximated as:

$$f_{\beta_f}(\beta) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{\left(\beta - \sqrt{E_s/2}\right)^2}{2\sigma^2}\right], \tag{9}$$

where $\sqrt{E_s/2}$ is the mean for β_f.

The joint probability of two Gaussian distributions can be equivalent to Q function [9]. From (7), (8) and (9), the symbolic error rate $P_{e|f}$ can be expressed as:

$$\begin{aligned} P_{e|f} &= 1 - \int_0^{\infty}\left[\left(\int_0^{\beta} f_{\rho_i}(\rho)\,d(\rho)\right)^{2^{SF}-2} \cdot f_{\beta_f}(\beta)\right]d(\beta) \\ &\approx Q\left(\sqrt{E_s/N_0} - 1.28\cdot\sqrt{SF} + 0.4\right) \end{aligned} \tag{10}$$

Given an index with $f_1 = f_2$, the average symbol error probability is equivalent to FSCM which is given in [8], i.e, $P_{e|\hat{f}} = Q\left(\sqrt{2E_s/N_0} - 1.28\cdot\sqrt{SF} + 0.4\right)$, and the symbol error rate of ICSS_IM can be expressed as:

$$\begin{aligned} P_s &= \frac{N-1}{N+1} \times Q\left(\sqrt{E_s/N_0} - 1.28\cdot\sqrt{SF} + 0.4\right) \\ &+ \frac{2}{N+1} \times Q\left(\sqrt{2E_s/N_0} - 1.28\cdot\sqrt{SF} + 0.4\right), \end{aligned} \tag{11}$$

The relationship between BER and SER for chirp spread systems is given in [10], so the BER of the ICSS_IM is approximated as:

$$P_e = \frac{2^{SF_{d2}-1}P_s}{2^{SF_{d2}} - 1} \approx 0.5 \times P_s. \tag{12}$$

4. Numerical results

In this section, we provide the simulation results of the BER performance and throughput for FSCM, FSCSS_IM and the proposed ICSS_IM. We set $B = 125$ KHz, the payload is 20 bytes, the SNR range is $[-30, -2]$ dB. And then the simulation results are averaged over 3000 runs.

Figure 2 provides the simulation results to compare the BER performance of the proposed ICSS_IM and FSCSS_IM. And the approximated BER derived in (14) is also given in Fig. 2. The results of the theoretical derivation are very close to the simulation results, which validates the theoretical analysis. The BER performance of FSCSS_IM is slightly better than that of ICSS_IM, because ICSS_IM has a certain error probability when judging whether the index

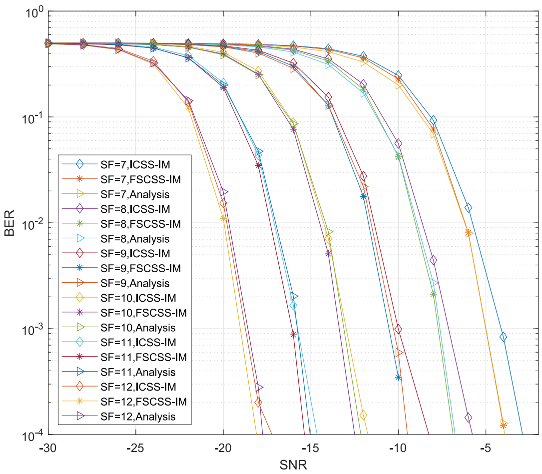

Fig. 2 The BER performance comparison for FSCSS_IM and ICSS_IM over AWGN channel.

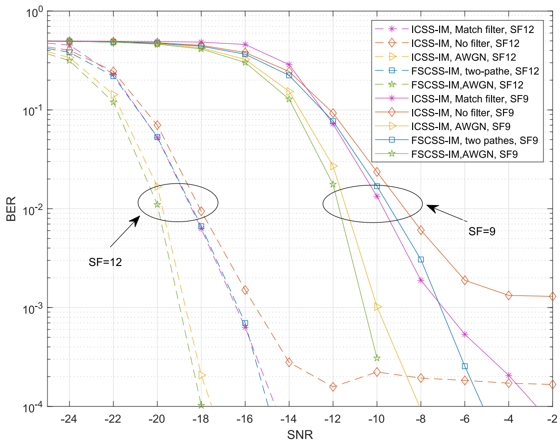

Fig. 3 The BER performance comparison for FSCSS_IM and ICSS_IM over 2-path channel.

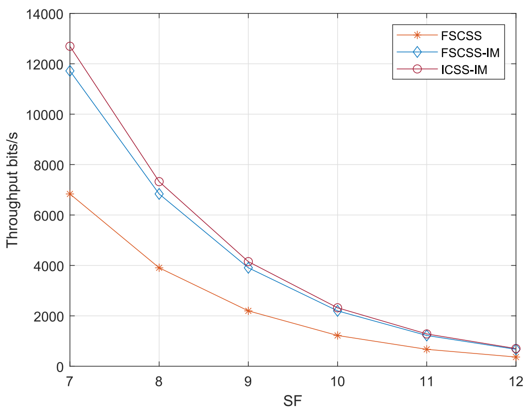

Fig. 4 Comparison of different schemes for throughput performance, when SF=7, SNR=-2dB.

values are equal due to the influence of noise.

Figure 3 presents the BER performance comparisons for the FSCSS_IM and ICSS_IM systems with $SF = 9$ and $SF = 12$ over the two-path channel with impulse response $h(k) = \sqrt{0.8}\delta(k)+\sqrt{0.2}\delta(k-1)$ and the AWGN channel. The results indicate that all systems over the two-path channel is inferior to that in the AWGN channel. After employing the matched filter, the BER performance of the proposed system improved significantly and is close to that of the FSCSS_IM system.

Figure 4 shows the throughput of FSCM, FSCSS_IM and ICSS_IM at SNR=-2dB. The throughput of all schemes decreases as SF increases, because when SF increases by 1, the symbol length doubles but the information carried increases by only 1bit. Compared with FSCM, the transmission rate of ICSS_IM can be increased by 85.7% at $SF = 7$ and 91.7% at $SF = 12$, respectively. And ICSS_IM increased the transfer rate of FSCSS_IM by 8.3% at $SF = 7$ and 4.5% at $SF = 12$ respectively. Obviously, ICSS_IM is superior to FSCM and FSCSS_IM in terms of transmission rate.

5. Conclusions

Inspired by FSCSS_IM, we proposed an improved scheme called ICSS_IM of which transmission rate is further increased by adding indexes with equal values into the codebook. According to the order of indexes in codebook, the index matching is utilized to replace the existing search algorithm, which greatly reduced the demodulation complexity. Furthermore, we derived the approximate BER expression of the proposed ICSS_IM in AWGN channels and validated it by simulations. The analysis and the simulation results demonstrate that the proposed scheme can achieve a higher transmission rate than FSCM and FSCSS_IM, but with the almost same computational complexity as FSCM.

References

[1] X, Qi, Z. Wen, K. Yu, K. Murata, K. Shibata, and T. Sato, "Content-oriented disaster network utilizing named node routing and field experiment evaluation," *IEICE Trans. Inf. & Syst.*, vol. E102-D, no. 5, pp. 988–997, 2019. DOI: 10.1587/transinf.2018NTP0009

[2] L. Vangelista, "Frequency shift chirp modulation: The LoRa modulation," *IEEE Signal Process. Lett.*, vol. 24, no. 12, pp. 1818–1821, Dec. 2017. DOI: 10.1109/lsp.2017.2762960

[3] A. Lavric and V. Popa, "Internet of Things and LoRa^{TM} low-power wide-area networks: A survey," 2017 International Symposium on Signals, Circuits and Systems. (ISSCS), pp. 1–5, July 2017. DOI: 10.1109/isscs.2017.8034915

[4] T. Elshabrawy, P. Edward, S. Elzeiny, and M. Ashour, "On the different mathematical realizations for the digital synthesis of LoRa-based modulation," European Wireless 2019 25th European Wireless Conference, pp. 1–6, May 2019. DOI: 10.1109/wimob.2019.8923211

[5] M. Hanif and H.H. Nguyen, "Slope-shift keying LoRa-based modulation," *IEEE Internet Things J.*, vol. 8, no. 1, pp. 211–221, Jan. 2021. DOI: 10.1109/jiot.2020.3004318

[6] M. Hanif and H.H. Nguyen, "Frequency-shift chirp spread spectrum communications with index modulation," *IEEE Internet Things J.*, vol. 8, no. 24, pp. 17611–17621, Dec. 2021. DOI: 10.1109/jiot.2021.3081703

[7] Y. Guo and Z. Liu, "Time-delay-estimation-liked detection algorithm for LoRa signals over multipath channels," *IEEE Wireless Commun. Lett.*, vol. 9, no. 7, pp. 1093–1096, July 2020. DOI: 10.1109/lwc.2020.2981597

[8] T. Elshabrawy and J. Robert, "Closed-form approximation of LoRa modulation BER performance," *IEEE Commun. Lett.*, vol. 22, no. 9, pp. 1778–1781, Sept. 2018. DOI: 10.1109/lcomm.2018.2849718

[9] R.D. Nowak, "Wavelet-based Rician noise removal for magnetic resonance imaging," *IEEE Trans. Image Process.*, vol. 8, no. 10, pp. 1408–1419, Oct. 1999. DOI: 10.1109/83.791966

[10] M.K. Simon and M.-S. Alouini, *Digital Communication Over Fading Channels*, 2nd ed., Wiley, Hoboken, NJ, USA, 2005. DOI: 10.1002/0471715220