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LETTER

Analysis of outage probability for massive MIMO systems under multi-cell and imperfect CSI environments

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Abstract We analyze the outage probability for a massive multiple-input-multiple-output (MIMO) system employing maximum ratio transmission (MRT) under a multi-cell scenario. As cells become smaller, it will be even more necessary to analyze the outage probability considering interference from neighboring cells. Under a multi-cell environment, communication performance can be substantially compromised by channel estimation errors and inter-cell interference (ICI). We investigate the distribution of signal-to-interference and noise ratio (SINR) in a massive MIMO system and formulate a novel mathematical model for the outage probability. Simulation results show the validity of the derived mathematical model.
Keywords: MIMO, multicell, precoding, probability, imperfect CSI
Classification: Wireless communication technologies

1. Introduction

In recent years, the rapid proliferation of smartphones and the penetration of the internet of things (IoT) have led to an explosive increase in data traffic demand. In this context, massive MIMO has emerged as a pivotal technology for the next generation of wireless communication systems. Massive MIMO involves equipping the base station (BS) with a hundred or more large array antennas. By applying precoding techniques, the system can concurrently serve multiple users within the same time-frequency resource [1, 2]. While various precoding methods exist, MRT is a subject of extensive research due to its simplicity as a linear process capable of delivering high data rates and communication reliability [3, 4, 5].

SINR, a measure of transmission efficiency or rate, has been used in many existing studies to measure massive MIMO performance [6, 7, 8]. Outage probability, representing the probability of SINR falling below a predefined statistical threshold, is also a crucial metric for assessing system behavior and communication stability. In general, simulation approaches are practical in evaluating communication performance. However, the complexity of a system increases as the number of antennas increases, so massive MIMO requires an enormous amount of time for simulation. Given the anticipated increase in the number of antennas [9], it becomes increasingly important to analyze system behavior and establish a mathematical model.

Few existing studies have analyzed MRT precoding [10, 11]. In addition, these analyses assume a single-cell environment. However, as cell densification progresses further in the future, it will be necessary to perform analyses that take into account ICI, which has a significant impact on communication quality at cell boundaries. Furthermore, the precision of channel state information (CSI) estimation is crucial for precoding. The analysis in [10, 11] assumes that complete CSI is available. However, interference from other cell users in a multi-cell environment reduces the estimation accuracy. Therefore, it is essential to quantify how performance degrades in the presence of imperfect CSI [12, 13, 14]. This paper introduces a novel mathematical model for the outage probability of MRT precoding under a multi-cell environment, accounting for ICI and channel estimation errors. This analytical approach can simplify performance analysis and improve evaluation accuracy, providing valuable insights for developing next-generation wireless communication systems.

2. System model

Precoding is a preliminary signal processing on the base station side that enables simultaneous user multiplexing. Precoding can be divided into two types: linear precoding and nonlinear precoding. Linear precoding includes MRT, Zero-Forcing (ZF), and transmit Winner precoding. Among these, MRT is recognized for its ability to maximize the signal-to-noise ratio (SNR) through straightforward signal processing. In this paper, we investigate a massive MIMO system utilizing MRT. We consider downlink multi-user MIMO (MU-MIMO) under a multi-cell scenario, where each cell has a base station with $N_t (N_t \geq 100)$ antennas, N_u users, and N_r antennas per user. The channel vector $\boldsymbol{h}_{u,r} \in \mathbb{C}^{1\times N_t}$ represents the channel from the base station to the r-th antenna of the u-th user. Each entry in the channel vector follows an independent identical distribution (i.i.d) according to $\mathcal{CN}(0, 1)$, indicating a symmetric complex Gaussian distribution with zero mean and unit variance. The channel matrix $\boldsymbol{H}$ is defined as

$$\boldsymbol{H} = \left[\boldsymbol{h}_{1,1}^{\mathrm{T}}, \cdots, \boldsymbol{h}_{1,N_r}^{\mathrm{T}}, \boldsymbol{h}_{2,1}^{\mathrm{T}} \cdots, \boldsymbol{h}_{N_u N_r}^{\mathrm{T}}\right]^{\mathrm{T}}. \tag{1}$$

The precoding matrix is formed through the channel estimation matrix $\hat{\boldsymbol{H}} = [\hat{\boldsymbol{h}}_{1,1}^{\mathrm{T}}, \cdots, \hat{\boldsymbol{h}}_{N_u N_r}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^{N_u N_r \times N_t}$, which is obtained by the pilot signal. However, the pilot signal can be contaminated by both AWGN and ICI. Therefore, the channel estimation matrix $\hat{\boldsymbol{H}}$ is different from the actual

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channel matrix $\boldsymbol{H}$ and is expressed as follows [15]

$$\hat{\boldsymbol{H}} = \sqrt{1-\sigma_e^2}\boldsymbol{H} + \sigma_e \boldsymbol{E}, \tag{2}$$

where each entry of $\boldsymbol{E}$ is an independent identical distribution (i.i.d) according to $\mathcal{CN}(0,1)$ and represents an estimation error uncorrelated with $\boldsymbol{H}$. The correlation coefficient between the actual channel and the estimate, assumed to be uniform across all channels, is denoted by $\sqrt{1-\sigma_e^2}$. Therefore, a smaller $\sigma_e \in [0,1]$ indicates a higher correlation and better estimation accuracy. The definition of the normalized mean squared error (NMSE) for channel estimation is as follows:

$$NMSE = \frac{\mathbb{E}\left[|\boldsymbol{h}_{u,r} - \hat{\boldsymbol{h}}_{u,r}|^2\right]}{\mathbb{E}\left[|\boldsymbol{h}_{u,r}|^2\right]} = 2\left(1-\sqrt{1-\sigma_e^2}\right), \tag{3}$$

where $\mathbb{E}$ represents the expected value. In this model, where distinct data is transmitted for each user's antenna, the data symbol $\boldsymbol{X}$ is represented as:

$$\boldsymbol{X} = \left[x_{1,1},\cdots,x_{1,N_r},x_{2,1},\cdots,x_{N_u,N_r}\right]^{\mathrm{T}}. \tag{4}$$

The symbol power is normalized to $\mathbb{E}(|x_{u,r}|^2) = 1$. As MRT precoding involves multiplying the conjugate transpose of the channel estimation matrix $\hat{\boldsymbol{H}}$ by the data symbol $\boldsymbol{X}$, the resulting received signal matrix $\boldsymbol{Y}$ is given by:

$$\boldsymbol{Y} = \sqrt{\beta}\boldsymbol{H}\hat{\boldsymbol{H}}^{\mathrm{H}}\boldsymbol{X} + \sqrt{P\eta}\sum_{l=1}^{L}\boldsymbol{H}_l\boldsymbol{W}_l\boldsymbol{X}_l + \boldsymbol{N}, \tag{5}$$

where $(\cdot)^H$ denotes the matrix Hermitian transpose. The first term is the signal from within the cell. β is the coefficient for satisfying the transmit power constraint, and using the total transmit power P, it is expressed as $\beta = \frac{P}{N_t N_u N_r (1-\sigma_e^2)}$. The second term is ICI, where η is the inverse of the signal-to-interference ratio (SIR) [16] and $\boldsymbol{H}_l = [\boldsymbol{h}_{l(1,1)}^{\mathrm{T}},\cdots,\boldsymbol{h}_{l(N_u,N_r)}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^{N_u N_r \times N_t^{(l)}}$ is the inter-cell interference channel where each entry follows $\mathcal{CN}(0,1)$. Also, assuming that each cell has the same number of $N_u N_r$ transmit streams, $\boldsymbol{W}_l \in \mathbb{C}^{N_t^{(l)} \times NuNr}$ is the precoding matrix and $\boldsymbol{X}_l \in \mathbb{C}^{N_u N_r \times 1}$ is the inter-cell interference symbol. The third term, $\boldsymbol{N} = [n_{1,1},\cdots,n_{N_u,N_r}]^{\mathrm{T}} \in \mathbb{C}^{N_u N_r \times 1}$, represents additive white Gaussian noise (AWGN), where each entry follows $\mathcal{CN}(0,1)$. Consequently, the expression for the received signal at the r-th antenna of the u-th user is:

$$\begin{aligned} y_{u,r} &= \sqrt{\beta(1-\sigma_e^2)}\boldsymbol{h}_{u,r}\boldsymbol{h}_{u,r}^H x_{u,r} \\ &+ \sqrt{\beta(1-\sigma_e^2)}\sum_{\substack{m=1\\m\neq u}}^{N_u}\sum_{\substack{n=1\\n\neq r}}^{N_r}\boldsymbol{h}_{u,r}\boldsymbol{h}_{m,n}^H x_{m,n} \\ &+ \sqrt{\beta}\sigma_e\boldsymbol{h}_{u,r}\boldsymbol{E}^H\boldsymbol{X} + \sqrt{P\eta}\sum_{l=1}^{L}\boldsymbol{h}_{l(u,r)}\boldsymbol{W}_l\boldsymbol{X}_l + n_{u,r}, \end{aligned} \tag{6}$$

where the first term represents the desired signal, the second term represents the inter-user interference (IUI), the third term represents self-interference (SI), the fourth term represents ICI, and the fifth term represents noise.

3. Outage probability analysis

3.1 IUI and ICI distribution

The sum of IUI power and ICI power is given by:

$$I = \frac{P}{N_t N_u N_r}\sum_{\substack{m=1\\m\neq u}}^{N_u}\sum_{\substack{n=1\\n\neq r}}^{N_r}|\boldsymbol{h}_{u,r}\boldsymbol{h}_{m,n}^H|^2 + P\eta\sum_{l=1}^{L}|\boldsymbol{h}_{l(u,r)}\boldsymbol{W}_l|^2, \tag{7}$$

where $\boldsymbol{h}_{l(u,r)}$ and $\boldsymbol{W}_l$ are uncorrelated and $\boldsymbol{W}_l$ is normalized to $1/N_u N_r$ for the magnitude of each vector. In that case, equation (7) is a sum of positive random variables. Therefore, by applying the central limit theorem, the distribution of equation (7) can be approximated by a gamma distribution. The probability density function (PDF) is then expressed as:

$$f_I(y) = \phi(k,\theta) = \frac{y^{k-1}\exp(-\frac{y}{\theta})}{\Gamma(k)\theta^k}, \tag{8}$$

where Γ is the gamma function, k is the shape parameter, and θ is the scale parameter. To derive the parameters k and θ, we obtain the expectation and variance of I. Here, we normalize IUI as follows.

$$U = \frac{1}{N_t}\sum_{\substack{m=1\\m\neq u}}^{N_u}\sum_{\substack{n=1\\n\neq r}}^{N_r}|\boldsymbol{h}_{u,r}\boldsymbol{h}_{m,n}^H|^2. \tag{9}$$

Then we see that U is the sum of $N_u N_r - 1$ of each stream whose distribution is $\phi(1,1)$ and whose stream correlation is $1/Nt$. Therefore, the PDF of U is

$$f_U(y) = (1-\zeta)\sum_{i=0}^{\infty}\zeta^i\phi(y;N_u N_r + i - 1, \nu), \tag{10}$$

where

$$\zeta = \frac{N_u N_r - 1}{\sqrt{N_t} + N_u N_r - 2}, \quad \nu = \frac{\sqrt{N_t}-1}{\sqrt{N_t}}.$$

Considering the same for ICI, the expected value and variance of equation (8) can be obtained as follows

$$\mathbb{E}[I] = \frac{P(N_u N_r - 1)}{N_u N_r} + P\eta L, \tag{11}$$

$$\begin{aligned} \mathrm{Var}[I] &= \frac{P^2(N_u N_r - 1)}{N_u^2 N_r^2} + \frac{P^2(N_u N_r - 1)(N_u N_r - 2)}{N_t N_u^2 N_r^2} \\ &+ \frac{P^2\eta^2 L}{N_u N_r} + \frac{P^2\eta^2 L(N_u N_r - 1)}{N_t N_u N_r}. \end{aligned} \tag{12}$$

Then k and θ can be derived as follows.

$$k = \frac{N_t\chi^2}{N_t\xi + (N_u N_r - 1)(\xi - 1)}, \tag{13}$$

$$\theta = \frac{P(N_t\xi + (N_u N_r - 1)(\chi - 1))}{N_t N_u N_r \chi}, \tag{14}$$

where

$$\chi = N_u N_r + \eta L N_u N_r - 1, \quad \xi = N_u N_r + \eta^2 L N_u N_r - 1. \tag{15}$$

3.2 Desired signal distribution

Normalize the amplitude D of the desired signal as follows

$$\sqrt{D} = \frac{1}{N_t}|\boldsymbol{h}_{u,r}\boldsymbol{h}_{u,r}^{\mathrm{H}}|. \tag{16}$$

Then Eq. (16) follows a gamma distribution $\phi(N_t, 1/N_t)$. Therefore, the expected value and variance of the desired signal power D is as follows

$$\mathbb{E}[D] = 1 + \frac{1}{N_t}, \tag{17}$$

$$\mathrm{Var}[D] = \frac{4}{N_t} + \frac{10}{N_t^2} + \frac{6}{N_t^3}. \tag{18}$$

3.3 SI distribution

Define S as follows

$$S = \frac{\sigma_e^2}{N_t N_u N_r (1-\sigma_e^2)} |\boldsymbol{h}_{u,r}\boldsymbol{E}^{\mathrm{H}}|^2. \quad (19)$$

The expected value and variance of S can then be derived as follows.

$$\mathbb{E}[S] = \frac{\sigma_e^2}{1-\sigma_e^2}, \quad (20)$$

$$\mathrm{Var}[S] \fallingdotseq \frac{1}{N_u N_r}\left(\frac{\sigma_e^2}{1-\sigma_e^2}\right)^2. \quad (21)$$

In massive MIMO system, assume $\sigma_e << 1$. Then, from Eq. (21), we see that SI takes an almost constant value.

3.4 SINR distribution and outage probability

Based on the above, SINR is expressed using Eq. (6).

$$\mathrm{SINR} = \frac{PN_t}{N_u N_r} \cdot \frac{D}{1+I+PS}. \quad (22)$$

Equation (18) implies that as N_t approaches infinity, the variance of D becomes zero, indicating that the desired signal power in massive MIMO becomes deterministic with a large number of transmit antennas. Furthermore, the denominator predominantly influences the variance by I. Consequently, we attribute expected values to D and S, treating I as a random variable following the distribution outlined in equation (8). The outage probability pertains to the likelihood of the SINR dropping below a specified threshold. Let γ_{th} denote this threshold; hence, the outage probability P_{out} is

$$\begin{aligned} P_{out} &= \mathbb{P}\left[\frac{PN_t}{N_u N_r} \cdot \frac{1+\frac{1}{N_t}}{1+I+\frac{P\sigma_e^2}{1-\sigma_e^2}} < \gamma_{th}\right], \\ &= \mathbb{P}\left[I > \frac{P(N_t+1)}{\gamma_{th} N_u N_r} - \frac{P\sigma_e^2}{1-\sigma_e^2} - 1\right], \end{aligned} \quad (23)$$

where $\mathbb{P}$ means the probability. Using equation (8), P_{out} can be derived as follows

$$\begin{aligned} P_{out} &= \int_\rho^\infty \frac{y^{k-1}\exp(-\frac{y}{\theta})}{\Gamma(k)\theta^k} dy \\ &= \frac{\Gamma(k, \frac{\rho}{\theta})}{\Gamma(k)}, \end{aligned} \quad (24)$$

where $\rho = \frac{P(N_t+1)}{\gamma_{th} N_u N_r} - \frac{P\sigma_e^2}{1-\sigma_e^2} - 1$ and $\Gamma(a,b) = \int_b^\infty t^{a-1}e^{-t}dt$ is the upper incomplete gamma distribution.

4. Performance comparison

Within this section, we conduct a comparison between the analytical model and simulation results. Unless explicitly stated otherwise, our assumptions include a base station with 256 antennas, 8 users, each equipped with 2 antennas, an average transmit power (SNR) of 10 dB, a threshold of 10 dB in each cell, and a total of 4 cells.

Figure 1 compares simulation and theoretical results for the PDF sum of IUI and ICI power when $\eta = 0, 0.1, 0.2$. Figure 1 shows that the simulation and theoretical results are tightly matched. We can also see that the distribution of the interfering power is large. This suggests the necessity for an analysis that not only considers deterministic values but also incorporates the distribution aspect. Figures 2 and 3 compare analytical and simulation results for outage probability when $\eta = 0, 0.1, 0.2$ and NMSE = 0%, 5%, 10% (σ_e = 0%, 22.22%, 31.22%). Figure 2 illustrates the relationship between the outage probability and the number of transmit antennas. The graph indicates a notable improvement in the outage probability with the increasing number of transmit antennas. Furthermore, it highlights the significant variability in the outage probability, influenced by both the SIR and channel estimation error. At (η, NMSE) = (0, 0%), (0, 5%), (0, 10%), (0.1, 0%), (0.1, 5%), (0.1, 10%), (0.2, 0%), (0.2, 5%), (0.2, 10%), the outage probability falls below 0.5 for the number of transmit antennas 160, 170, 180, 210, 220, 230, 260, 265, 275, respectively. This em-

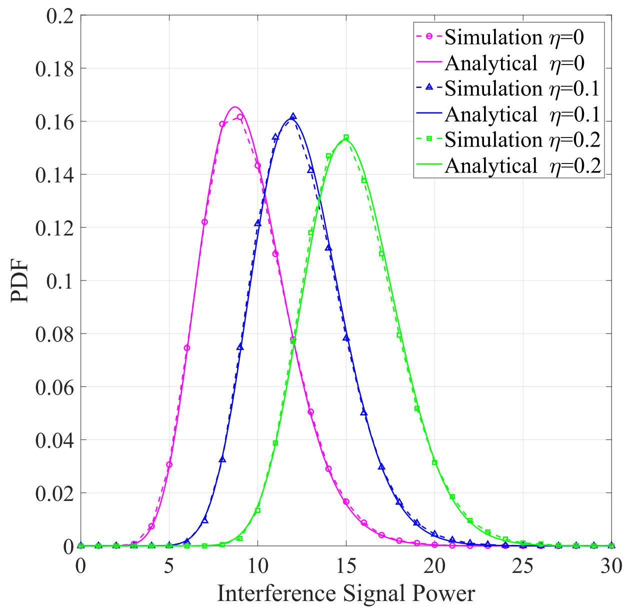

Fig. 1 Interference power distribution

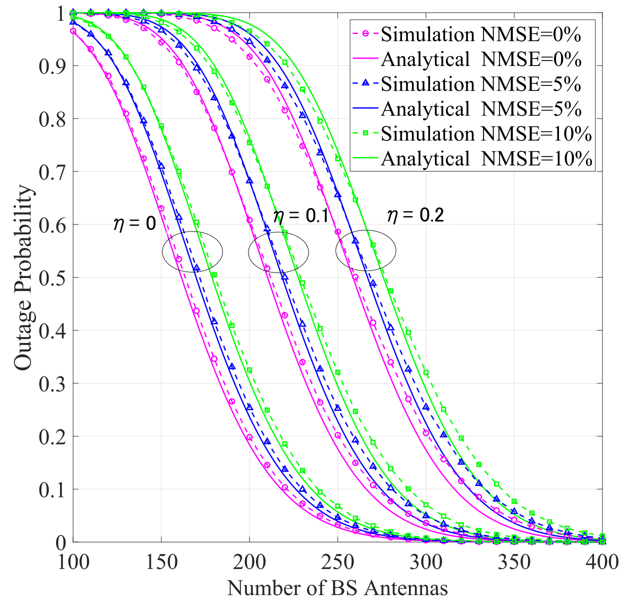

Fig. 2 Outage probability per number of BS antennas

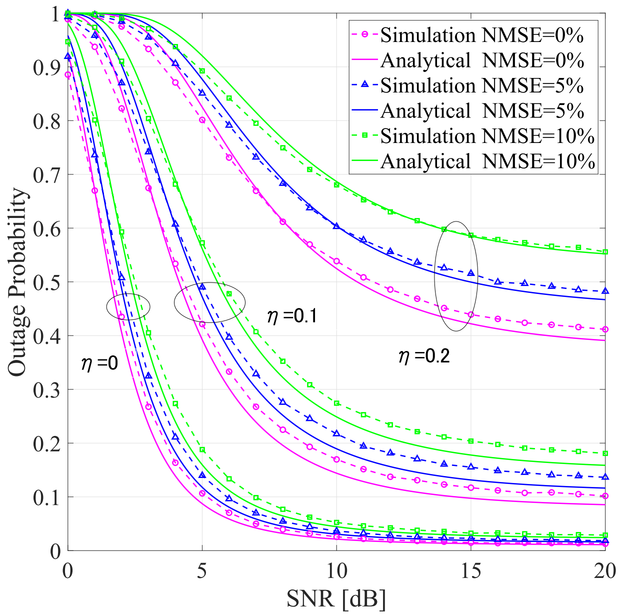

Fig. 3 Outage probability per SNR

phasizes the requirement for an analysis that considers ICI and channel estimation errors. In Fig. 3, the relationship between outage probability and SNR is depicted. Notably, the graph reveals an enhancement in outage probability with rising SNR. However, unlike the scenario with the number of transmit antennas, the outage probability does not asymptotically approach zero with increasing SNR. This divergence is attributed to the concurrent increase in IUI with escalating SNR. Consequently, augmenting the number of transmit antennas is the most effective means to ameliorate the outage probability.

5. Conclusion

This paper analyzed the SINR distribution in a downlink multi-user massive MIMO system employing MRT precoding in a multi-cell environment. We proposed a mathematical model for outage probability, and our derived model demonstrated high consistency with simulation results. The investigation also underscored the necessity for a mathematical model incorporating ICI and channel estimation error considerations. This model holds potential utility in determining crucial parameters such as the number of transmit antennas and transmit power during the prediction of communication environments and the system's design.

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