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Iterative Vector-Based Localization in a Large Heterogeneous Sensor Network

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***Abstract*—This article proposes a novel iterative vector-based localization method in a large heterogeneous sensor network, where a subset of nodes possesses the capability to measure both distance and angle information, while the others are only limited to distance measurements. Unlike conventional vector-based positioning methods that assume all nodes can measure both distance and angle, our approach tackles a more realistic scenario where some nodes are limited to distance-only measurements. To address the challenges of the node localization in a heterogeneous sensor network, the proposed positioning method calculates vector information between the nodes that are not directly communicated and aligns it with a reference coordinate. In addition, the proposed method employs an iterative calculation, such as least-squares minimization, thereby achieving high positioning accuracy. Simulation results demonstrate that the proposed positioning method outperforms the conventional distance-based positioning method in environments with low angle measurement errors, exhibiting up to 44% higher positioning accuracy. Furthermore, the proposed positioning method shows 24% higher positioning accuracy compared with the conventional vector-based positioning method.**

***Index Terms*—Angle of arrival (AOA), communication range, distance, least-squares minimization (LSM), localization, positioning, vector.**

I. Introduction

KNOWING the positions of the nodes is important in many applications of wireless sensor network (WSN). This location information can be used to optimize communication resources (e.g., by assisting routing or reducing power consumption) and for surveillance purposes (e.g., monitoring the environment, conducting military surveillance, providing health care, tracking objects, and more) [1], [2], [3]. In such applications, the nodes are often distributed randomly, making it even more crucial to estimate their positions accurately. Global positioning system is a well-known and widely used positioning system, but it is not suitable for WSNs. Because it requires extra hardware, has high power consumption, and is unusable indoors. As a result, various alternative localization methods for node positioning have been studied in WSNs to achieve reliable positioning performance [2], [4].

In WSNs, positioning methods primarily rely on distance or connectivity information to estimate the positions of nodes. Depending on whether distance information is available or not, positioning methods are largely classified into two groups: range-based and range-free methods. Range-based methods utilize distance or angle information to calculate the locations of nodes, whereas range-free methods calculate the locations of nodes using only the connection information between nodes. Range-based methods are typically able to provide more accurate positioning compared with range-free methods.

The recent advancements in the multiple antenna technology have enabled the acquisition of dependable angle information between nodes, which can be employed for the purpose of computing node positions. Consequently, vector-based positioning methods that utilize both distance and angle information concurrently [5], [6], [7], [8] have been studied. Although these methods require additional resources, such as power consumption and hardware complexity, they enable more precise positioning.

Those conventional vector-based positioning methods assume that the angle of arrival (AOA) between every single pair of nodes in the network is known or measured. In the case of large-scale WSNs, it is challenging to obtain angle information between nodes that do not have direct communication links. To address this issue, a vector combination-based positioning method was proposed in [9], which is capable of operating in such environments. But, the process of simply combining vectors leads to the accumulation of vector errors that have an adverse impact on the positioning performance. Hence, in very large networks, there is a possibility that positioning methods based on vector combinations may exhibit inferior positioning performance.

In addition, conventional vector-based methods that utilize both distance and angle information typically assume that all nodes have the capability to measure both distance and angle information. However, this assumption may not always be practical, as certain nodes may be subject to constraints, such as limited space for multiple antennas or the use of cost-efficient hardware, rendering them incapable of measuring angle information.

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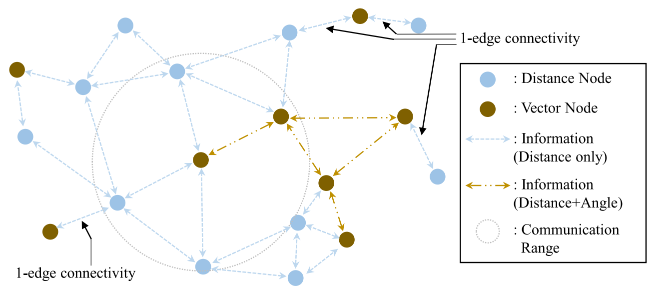

Fig. 1. Heterogeneous sensor network model in which vector nodes and distance nodes coexist simultaneously.

To overcome the limitations mentioned above in large WSNs, this article proposes a new node localization method in a large heterogeneous network environment where only some nodes can measure both distance and angle information. The proposed localization method is specifically designed for such network environments and is more effective than conventional vector-based positioning methods and conventional distance-based positioning methods.

The contributions of this article are summarized as follows.

1) This article proposes an iterative vector-based positioning method in a large network environment where some nodes are not directly communicated. Accurate positioning in a large network is possible through iterative calculation.
2) The proposed vector-based positioning method enables positioning of the nodes even in a heterogeneous network environment where only some nodes can measure both distance and angle information, which is more realistic and practical.
3) Extensive simulations are conducted in various environments. The performance of the proposed positioning method is evaluated in terms of distance error, AOA error, node density, network size, network topology, communication range, computational complexity, and the proportion of vector nodes.

II. System Model

The proposed positioning method employs both distance and angle information to calculate the positions of all nodes in a network. Assume that all nodes can measure distance information, but some of nodes can measure the angle of signals from their neighboring nodes. Vectors are generated using the measured distance and angle information, and the positions of nodes are estimated through iterative calculations based on least-squares minimization (LSM) using the generated vectors and measured distance information.

Fig. 1 illustrates the heterogeneous network model that consists of various nodes, where some nodes can measure the distance to neighboring nodes and other nodes can measure both distance and angle information with neighboring nodes. In this article, nodes that can measure both distance and angle from neighboring nodes are called vector nodes, whereas nodes that can only measure distance information from neighboring nodes are called distance nodes. Moreover, when the network is represented as a graph consisting of vertices and edges, it is called k-vertex-connected if it remains connected even after removing any $k-1$ vertices [10]. In addition, if there is no routing path to a particular neighboring node through other nodes, the connection with that neighboring node has the property of 1-edge connectivity, and if there are such connections between some nodes in the network, it is referred to as a network with 1-edge connectivity. The proposed positioning method is capable of operating in such environments.

It is assumed that the proposed positioning method operates in the following conditions.

1) The positions of nodes are computed in a 2-D space.
2) All nodes are connected to each other via a routing path.
3) All nodes are capable of measuring the distance to their neighboring nodes.
4) Only some nodes are capable of measuring their AOA from neighboring nodes.

III. Conventional Localization Methods

This section investigates the traditional range-based positioning methods that rely on the range information (i.e., distance and angle). These positioning methods are categorized into distance-based, angle-based, and vector-based methods. In addition, a detailed explanation of the iterative positioning method that utilizes LSM-based distance information is provided.

Prior to mentioning the traditional method, the measurement errors are briefly described. Distance and angle information can be obtained through direct communication between nodes, using methods, such as time of flight, received signal strength indicator, or beamforming. These measurements are indeed influenced by environmental factors (e.g., fading, multipath, and non-line-of-sight propagation). These factors can occur due to walls or objects present on the communication path, and in such an environment, the actual measured values of the signal differ from ideal measured values. As a result, this difference between the actual measured value and the ideal measured value results in errors in distance and angle information.

A. Distance-Based Methods

Traditional range-based localization methods in WSN typically rely on distance information to estimate the position of nodes using mathematical techniques (e.g., trilateration, LSM, etc.). For nodes that cannot communicate directly, the distances between them are calculated by adding the distances between intermediate nodes obtained through shortest path algorithms, such as Dijkstra or Floyd algorithms. However, these methods may result in inaccurate distance values, particularly in large or irregular networks, which negatively impact the position estimation performance. To address this issue, several studies have been conducted to improve the prediction of the nodes in environments where direct communication is not always possible. For precise localization purposes, more accurate calculations of the distance between nodes that are not directly connected [11], creating and combining local maps based on clusters [12], [13], [14], [15], [16], and iterative calculations with reliable distance values [17], [18], [19] are executed. Among these studies, LSM-based iterative calculation methods have been shown to provide reliable positioning performance.

The iterative calculation method, such as LSM, estimates the positions of nodes by minimizing the difference between the distances between the nodes in the current state and the actual distances between the nodes. In some cases, weights are assigned to the differences to indicate the reliability of the distance measurements. The weighted LSM formula for calculating the positions of all nodes [17] is expressed as

$$\min_{x_i, y_i} \sum_{i,j} w_{ij}(d_{ij} - \bar{d}_{ij})^2 \tag{1}$$

$$= \min_{x_i, y_i} \sum_{i,j} w_{ij} \left(\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} - \bar{d}_{ij} \right)^2$$

$$\text{for} \quad i = 1, 2, \ldots, N \tag{2}$$

where d_{ij} means the distance between nodes i and j at the positions of currently given nodes, and $\bar{d}_{ij}$ is the measured distance between node i and j.

The weight w_{ij} is assigned according to the reliability of the information and is expressed as

$$w_{ij} = \begin{cases} \frac{1}{h_{ij}^2} & \text{if } h_{ij} \leq h_{\text{ref}} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where h_{ij} is the number of hops between node i and j. h_{ref} denotes the refinement range. Note that h_{ij} is 1 if nodes i and j are directly connected and, h_{ij} is 2 if they are indirectly connected through intermediate nodes. As this calculation repeats until a certain limit is reached, the error of position estimation keeps decreasing.

Meanwhile, in the absence of good initial position information, iterative calculation methods may encounter issues, such as slow convergence and local minima. Using multidimensional scaling (MDS)-based localization methods, these issues can be resolved by providing good initial positions for nodes [17]. Classical MDS (CMDS) [20] estimates the positions of nodes using distance information. CMDS calculates the positions of all nodes at once using the property of the Euclidean inner product and eigenvalue decomposition. However, since each distance component affects the position of all nodes, there is a drawback that the positioning performance is significantly reduced in large or irregular networks. Despite this limitation, classical MDS (CMDS) can be a highly appealing technique for iterative LSM-based localization systems because it easily provides starting points for nodes. Based on the aforementioned advantages, the proposed localization technique in this article also utilizes CMDS.

These positioning methods rely solely on distance information, which poses a limitation in position estimation for a network with 1-edge connectivity. Inaccurately predicting the positions of such nodes can have a negative impact on the estimation of the positions of all nodes in the network. Thus, distance-based algorithms do not provide good positioning performance in a network with 1-edge connectivity.

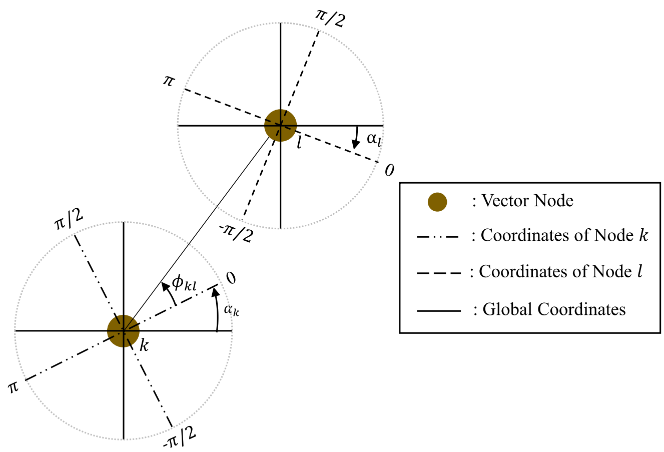

Fig. 2. Angles at node *k*: absolute angle, relative angle, and offset value.

B. Angle-Based Methods

To determine the angle from an adjacent node, beamforming or similar methods can be used. This technique requires extra hardware complexity and power consumption and extra spatial resource. Fig. 2 illustrates the angle of the signal from node l to node k. If node l is the transmitter and node k is the receiver, the absolute arrival angle of the signal at node k consists of a relative angle (i.e., AOA) ϕ_{kl} and an offset value (i.e., orientation) α_k. Since the AOA measurement is the relative angle in the local coordinate system of the receiving node, it is necessary to perform a coordinate transformation of all the local coordinate systems to express the measured AOAs at each node in a global coordinate system. In other words, the offset value α_k is computed as a result of coordinate transformation, then the absolute angle θ_{kl} can be calculated in the global coordinate system as

$$\theta_{kl} = \phi_{kl} + \alpha_k. \tag{4}$$

Although the offset value α can be directly measured using a sensor, such as a magnetic sensor, it may not always be possible to measure it for all nodes as some nodes may not be equipped with such a sensor.

Ash and Potter [21] suggested a robust angulation using subspace techniques (RAST) algorithm that utilizes mathematical techniques to estimate the orientations. The estimated orientations are based on the local coordinates of a specific node. Watanabe [22] presented a two-step optimization process based on LSM that utilizes AOA for 3-D localization.

Similar to distance-based methods, angle-based methods also face a limitation in accurately estimating the positions of nodes in a network with limited connectivity. In 1-edge connectivity networks, angle-based methods exhibit very poor positioning performance.

C. Vector-Based Methods

By utilizing vector information, which combines both distance and angle information, it is feasible to localize nodes with higher precision. Abreu and Destino [5] proposed a node localization method that estimates the positions of nodes by considering the distance information and the difference between the angles of the received signals from two nodes. Subsequently, similar studies have been conducted based on the above method, known as super MDS (SMDS) [6], [7], [8]. Compared with conventional distance-based algorithms, these methods exhibit improved node positioning performance. However, in large WSNs with some nodes that cannot communicate directly, SMDS-based positioning methods face practical challenges in estimating the position of nodes as those methods must be capable of calculating the difference in the angle of the received signal from two nodes.

When vectors consisting of distance and angle information between all node pairs in a network are projected onto a single coordinate system, the relative positions of nodes can be determined based on this vector information. However, since the AOA measured is the relative angle in the local coordinate system of the corresponding node, the resulting vector is also influenced by the local coordinate system at that node. To express the measured AOAs in a global coordinate system, coordinate transformation of all AOAs is required. If all nodes have a routing path to each other, the coordinate transformation of all AOAs can be conducted using mathematical techniques [21]. After the coordinate transformation of all AOAs, the vectors between all nodes can be recalculated in a single global coordinate system, thereby facilitating the estimation of the position of all nodes [9]. The method of recalculating vectors in this process is briefly introduced below.

First, the fundamental relation matrix $\mathbf{B}$ is calculated as

$$\mathbf{B} = e^{i\mathbf{\Psi}} + 2\mathbf{I} \tag{5}$$

where $\mathbf{\Psi} = \mathbf{\Phi}^\top - \mathbf{\Phi} + \pi \mathbf{1}_N \mathbf{1}_N^\top$, $\mathbf{\Phi}$ is an N-by-N matrix representing the set of relative angles, $\mathbf{1}_N$ is a column vector with all its components equal to 1, and $\mathbf{I}$ is an identity matrix. In addition, the operation $e^{i\mathbf{\Psi}}$ signifies the elementwise exponentiation of the matrix $\mathbf{\Psi}$.

Subsequently, within matrix $\mathbf{B}$, the elements relating to pairs of nodes that are not directly communicated should be substituted with independently derived values. In the case that nodes k and l do not have direct communication, the element B_{kl} of the matrix $\mathbf{B}$ is computed as

$$B_{kl} = e^{i((\phi_{h(1)k} - \phi_{kh(1)} + \pi) + (\phi_{h(2)h(1)} - \phi_{h(1)h(2)} + \pi) + +\cdots+ (\phi_{lh(L)} - \phi_{h(L)l} + \pi))} \tag{6}$$

where L denotes the number of intermediate nodes between nodes k and l, while $h(1), \ldots, h(L)$ represent the indices of those intermediate nodes. These indices are ascertained through the utilization of the shortest path algorithm to determine the shortest path between nodes k and l.

After that, it is possible to estimate the absolute angle by means of the eigen-decomposition and the elementwise phase calculation technique as follows:

$$\mathbf{B} = \mathbf{Q_B} \Lambda \mathbf{Q_B}^\top \tag{7}$$

$$\hat{\boldsymbol{\alpha}} = \angle \boldsymbol{a}(\hat{\boldsymbol{\alpha}}) \tag{8}$$

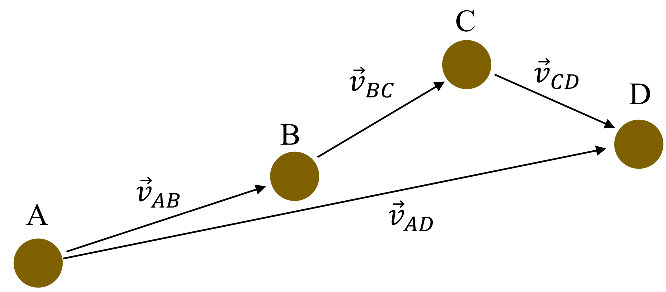

Fig. 3. Vector generation for 3-hop relation that is not directly communicated through vector combinations.

where $\mathbf{Q_B}$ refers to a matrix whose columns correspond to the eigenvectors of the matrix $\mathbf{B}$, and Λ denotes a diagonal matrix in which the corresponding eigenvalues are placed along the diagonal. $\angle \boldsymbol{a}(\hat{\boldsymbol{\alpha}})$ denotes the elementwise phase calculation of $\boldsymbol{a}(\hat{\boldsymbol{\alpha}})$, and $\boldsymbol{a}(\hat{\boldsymbol{\alpha}})$ refers to the eigenvector that corresponds to the largest eigenvalue of matrix $\mathbf{B}$. By replacing $\hat{\boldsymbol{\alpha}}$ in (4), it is possible to derive the estimated absolute arrival angles $\hat{\mathbf{\Theta}}$, expressed as a set in the global coordinate system ($\hat{\boldsymbol{\alpha}} = \{\alpha_k\}$, $\hat{\mathbf{\Theta}} = \{\hat{\theta}_{kl}\}$).

Fig. 3 depicts the method of computing the vector between nondirectly communicating nodes based on vector combination. The calculation for the vector from node A to node B is expressed as

$$\vec{v}_{AB} = d_{AB} \angle \hat{\theta}_{AB} \tag{9}$$

where the distance between nodes A and B is denoted by d_{AB}, and the absolute angle from node A to node B is presented by $\hat{\theta}_{AB}$. Since nodes A and D are not directly connected, the vector between nodes A and D can be computed through a vector combination as follows:

$$\vec{v}_{AD} = d_{AD} \angle \hat{\theta}_{AD} = \vec{v}_{AB} + \vec{v}_{BC} + \vec{v}_{CD}. \tag{10}$$

The path of the vector combination comprises intermediate nodes on the shortest path acquired by Dijkstra's algorithm. If the network consists of N nodes and all nodes are vector nodes, the vector combination results in $M = N(N-1)/2$ distinct vectors. Finally, the set of predicted vectors $\hat{\mathbf{V}}$ can be expressed as follows:

$$\hat{\mathbf{V}} = [\hat{\mathbf{v}}_1, \hat{\mathbf{v}}_2, \ldots, \hat{\mathbf{v}}_M]^\top \tag{11}$$

where the kth estimated vector is represented as $\hat{\mathbf{v}}_k = [d_k \cdot \cos\hat{\theta}_k, d_k \cdot \sin\hat{\theta}_k]^\top$. d_k and $\hat{\theta}_k$ denote the distance and the absolute angle of kth estimated vector, respectively. M denotes the overall count of node pairs.

IV. Proposed Localization Method

The previously discussed conventional vector-based localization method [9] lacks the ability to ensure precise positioning accuracy in large networks. This limitation arises from the direct conversion of vector information into location data, disregarding the reliability of the information, which consequently results in cumulative errors during the vector synthesis process. Moreover, a further constraint lies in the requirement that all nodes within the network must possess the capability to measure both distance and angle information with neighboring nodes.

To address these challenges, the proposed localization method uses a weighted and iterative approach in the positioning process. This enhancement aims to improve the localization performance specifically in large-scale networks. Also, the proposed localization method effectively facilitates the estimating of the positions in a heterogeneous network environment where only some of the nodes can measure both distance and angle information with neighboring nodes.

The proposed localization method determines the positions of nodes according to the following contents. The proposed localization method computes the vector information between each node by using the distance and angle information simultaneously. However, the vector values obtained in the previous process are reliant on the local coordinate system of certain nodes. If only some nodes in the network are vector nodes, one or more vector sets consisting of vector nodes communicating with each other will be formed. In such cases, the direct computations between vectors in different vector sets become intricate since each vector relies on the global coordinate system of the corresponding set of vectors to which it belongs.

In a network where only a subset of nodes is capable of measuring both distance and angle information, the process of recalculating all vectors based on a single global coordinate system is added. If every node in the network can measure both distance and angle information from neighboring nodes (i.e., vector nodes), the optimization function for proposed localization method is promptly employed to compute the positions of nodes that satisfy the integrated vector information. However, if some nodes in the network are vector nodes, while others can only measure distance information (i.e., distance nodes), aligning the vector information and the distance information is necessary. After this alignment process, the proposed optimization function is utilized to estimate the node positions by mixing the vector and distance information.

The specifics of these proposed methods are presented and discussed below. Section IV-A outlines the approach for computing the positions of nodes in an environment where all nodes in the network are vector nodes. Section IV-B presents the method for determining the positions of nodes in heterogeneous networks where vector nodes and distance nodes coexist. Finally, Section IV-C proposes a node localization method with an adaptive weight determined based on the reliability of the measurement of the angle information.

A. Iterative Vector-Based Method for a Network With All Vector Nodes

The proposed method for node localization utilizes LSM computations to estimate the positions of nodes. Unlike conventional range-based methods that rely on distance values, this method employs vector values in the estimation process.

For instance, the conventional method relies on the distance information between nodes, as depicted in Fig. 4(a). Here, $\bar{d}_{x_{ij}}$ and $\bar{d}_{y_{ij}}$ represent the measured distance values on the x-axis and the y-axis between nodes i and j (referred to as an observed distance), respectively. Similarly, $d_{x_{ij}}$ and $d_{y_{ij}}$ represent the distance values on the x-axis and the y-axis between nodes i and j at the positions of the currently given nodes (referred to as a current distance), respectively. Also, $de_{x_{ij}}$ and $de_{y_{ij}}$ represent the differences between the current and the observed distance between nodes i and j on the x-axis and the y-axis, respectively. In the conventional approach, the positions of nodes are calculated in the direction of minimizing these differences.

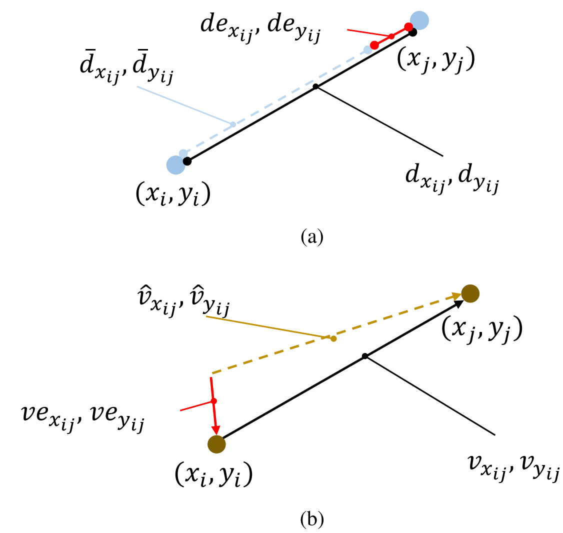

Fig. 4. Errors to be minimized in the conventional method and proposed method. (a) Errors to be minimized in distance-based method (conventional). (b) Errors to be minimized in vector-based method (proposed).

However, in the proposed positioning method, the positions of nodes are calculated by utilizing not only the distance information but also the vector information between nodes, as shown in Fig. 4(b). Here, $\hat{v}_{x_{ij}}$ and $\hat{v}_{y_{ij}}$ represent the values that project estimated vectors from node i to node j into the x-axis and y-axis, respectively, calculated by the measured distance and angle information (referred to as an observed vector). Similarly, $v_{x_{ij}}$ and $v_{y_{ij}}$ represent the values that project vectors from node i to node j into the x-axis and y-axis at the positions of the currently given nodes (referred to as a current vector), respectively. In addition, $ve_{x_{ij}}$ and $ve_{y_{ij}}$ represent the differences between the current and observed vector values between node i and j on the x-axis and the y-axis, respectively. In the proposed algorithm, the positions of the nodes are computed by minimizing these differences.

The LSM-based calculations for obtaining the positions of nodes using vector information are explained in detail below.

1) Optimization Function for a Network With All Vector Nodes: The proposed calculation method that employs vector information can effectively estimate the positions of the nodes. The proposed localization method for a network with all vector nodes requires that all nodes in the network can measure both distance and angle information with their neighboring nodes. If the number of nodes in the network is N, and all nodes are vector

Algorithm 1: Iterative Vector-Based Localization Method in a Network With all Vector Nodes.

Input:
$\boldsymbol{\Phi} = \{\boldsymbol{\Phi}_1, \boldsymbol{\Phi}_2, \ldots, \boldsymbol{\Phi}_N\}^\top, \boldsymbol{D} = \{\boldsymbol{D}_1, \boldsymbol{D}_2, \ldots, \boldsymbol{D}_N\}^\top$
Output: $\mathbf{X} = \{\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_N\}^\top$
($\boldsymbol{\Phi}_i$: column vector which means relative angles to node i
$\boldsymbol{D}_i$: column vector which means distances to node i)
1: Calculate a set of absolute angles $\hat{\boldsymbol{\Theta}} = \{\hat{\boldsymbol{\Theta}}_1, \hat{\boldsymbol{\Theta}}_2, \cdots, \hat{\boldsymbol{\Theta}}_N\}^\top$ by using (5)–(8)
2: Calculate a set of vectors $\hat{\mathbf{V}} = [\hat{\mathbf{v}}_1, \hat{\mathbf{v}}_2, \ldots, \hat{\mathbf{v}}_M]^\top$ by using (9)–(11)
3: Calculate initial values for positions of all nodes $\mathbf{X}$ by using CMDS
4: Update values for positions of all nodes $\mathbf{X}$ by using (12)
5: Repeat process 4 until the positions of all nodes converge

nodes, the positions of all nodes can be computed as follows:

$$\min_{x_i, y_i} \sum_{i,j} w_{ij} \sqrt{(v_{x_{ij}} - \hat{v}_{x_{ij}})^2 + (v_{y_{ij}} - \hat{v}_{y_{ij}})^2} \tag{12}$$

$$= \min_{x_i, y_i} \sum_{i,j} w_{ij} \sqrt{(x_i - x_j - \hat{v}_{x_{ij}})^2 + (y_i - y_j - \hat{v}_{y_{ij}})^2}$$

$$\text{for } i = 1, 2, \ldots, N \tag{13}$$

$$\text{where} \quad w_{ij} = \begin{cases} \frac{1}{h_{ij}^2} & \text{if } h_{ij} \leq h_{\text{ref}} \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

Repeating (12) for all nodes multiple times leads to a reduced difference between the estimated positions and the actual positions of the nodes. Before computing the above equation, techniques, such as CMDS, can be utilized to generate a good initial position for the nodes.

If h_{ref} is equal to 1, the position of a node is determined solely based on the information of its directly connected nodes. However, if h_{ref} is equal to 2, the information of the corresponding node and all the nodes within two hops are employed, and this increases the computational complexity.

The initial positions of the nodes were determined using CMDS in this article. The positions of the nodes were then computed by setting the refinement range to h_{ref} equal to 2 and iterating the optimization function 25 times for all nodes. The proposed localization method for a network with all vector nodes is expressed in Algorithm 1.

B. Iterative Vector-Based Method for a Heterogeneous Network

In practice, it is challenging to ensure that all nodes in a network can measure both distance and relative angle information with neighboring nodes, which is required for the proposed localization method for a network with all vector nodes. When a network contains both vector nodes and distance nodes, one or more vector sets, which refer to sets of vector nodes with routing paths only among vector nodes, are formed. The calculation to find the offset value α_k of each node must be carried out

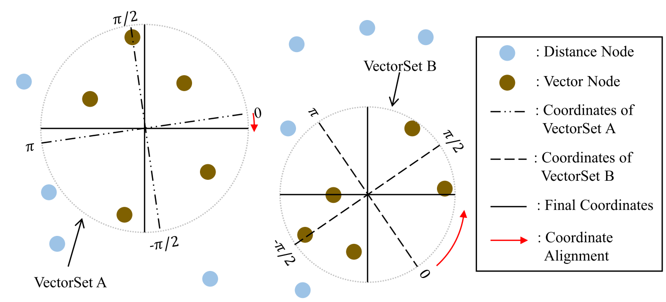

Fig. 5. Vector rotation through coordinate alignment in a network where vector nodes and distance nodes are distributed together.

separately for each set of vectors, as the vectors calculated belong to the coordinate system of the vector set to which the corresponding nodes belong. The vectors obtained through the estimation of offset value α_k represent values on the global coordinate system of each vector set. Therefore, it is necessary to align them with a reference coordinate system to facilitate the integration of vector and distance information. This alignment process involves rotating the vectors, as shown in Fig. 5. This process is crucial, even if there is only one vector set, to ensure the proper use of vector and distance information together.

In this study, the aligning of each vector was carried out by comparing the observed vectors and the current vectors, which are the vectors at the positions of currently given nodes. The positions of nodes obtained through the optimization function for the proposed localization method for a heterogeneous network, which combines the vector and distance information, are continuously updated. This means that the process of aligning the corresponding vectors must be repeatedly performed whenever the optimization function is performed. Even if all nodes can measure both distance and angle, the positions of nodes can be calculated using the proposed localization method for a heterogeneous network. In this case, the proposed localization method for a network with all vector nodes, and a heterogeneous network exhibit equivalent positioning performance. When vector nodes and distance nodes are distributed together, the processes of rotating all vector information into one reference coordinate system and calculating the positions of nodes are described below.

1) Aligning Observed Vectors: To align the set of observed vectors $\hat{\mathbf{V}}$, a rotation matrix $\mathbf{A}_V$ is required at every iteration. The rotation matrix $\mathbf{A}_V$ is needed to convert a vector in the corresponding vector set into a vector of one reference coordinate system, which can be obtained through Procrustes analysis [23] excluding scaling. For faster convergence, the initial position of the node can be calculated using techniques, such as CMDS.

The combined value of the set of current vectors $\mathbf{V}$ and the set of observed vectors $\hat{\mathbf{V}}$ undergoes singular vector decomposition, which is carried out as

$$\left(\mathbf{J} \cdot \mathbf{V}\right)^\top \cdot \hat{\mathbf{V}} = \mathbf{U}_V \cdot \mathbf{S}_V \cdot \mathbf{V}_V \tag{15}$$

where $\mathbf{J} = \mathbf{I} - \frac{1}{N}(\mathbf{1} \cdot \mathbf{1}^\top)$ represents a centering matrix, $\mathbf{I}$ is the identity matrix, $\mathbf{1}$ is a vector of ones, $\mathbf{U}_V$ is an orthogonal 2×2

matrix, $\mathbf{S}_V$ is a nonnegative 2×2 diagonal matrix, and $\mathbf{V}_V$ is an orthogonal 2×2 matrix.

The rotation matrix $\mathbf{A}_V$ can be derived by

$$\mathbf{A}_V = \mathbf{V}_V \cdot \mathbf{U}_V^\top. \tag{16}$$

The set of aligned-observed vectors $\tilde{\mathbf{V}}$ can be computed as

$$\tilde{\mathbf{V}} = \mathbf{V} \cdot \mathbf{A}_V. \tag{17}$$

The process of aligning vectors is carried out on each vector set independently. Upon completion of the alignment process, all aligned vectors are then depicted in a single reference coordinate system.

2) Optimization Function for a Heterogeneous Network: An optimization function for a heterogeneous network is proposed to estimate the positions of nodes by modifying the optimization function for a network with all vector nodes. Given a network containing N nodes, the positions of all nodes can be determined as follows:

$$\min_{x_i, y_i} \left(\sum_{\{i,j\} \subset \mathbb{V}} w_{ij} \sqrt{(v_{x_{ij}} - \tilde{v}_{x_{ij}})^2 + (v_{y_{ij}} - \tilde{v}_{y_{ij}})^2} + \sum_{\{i,k\} \not\subset \mathbb{V}} w_{ik} \sqrt{(d_{x_{ik}} - \bar{d}_{x_{ik}})^2 + (d_{y_{ik}} - \bar{d}_{y_{ik}})^2} \right) \quad \text{for} \quad i = 1, 2, \ldots, N \tag{18}$$

where $\mathbb{V}$ represents the sets of vector nodes. Similar to the proposed localization method for a network with all vector nodes, the positions of nodes are updated using the optimization function mentioned above for all nodes. As the positions of nodes are updated, the current vectors obtained at the positions of given nodes change, and the process of aligning observed vectors must be repeated. The position refinement process (i.e., optimization function for the proposed localization method) is repeated until the appropriate positions of nodes are calculated. The process of the proposed localization method for a heterogeneous network can be described using Algorithm 2.

C. Iterative Vector-Based Method With an Adaptive Weight for a Heterogeneous Network

To consider more practical scenarios and evaluate the performance of the proposed method, the proposed method employs an adaptive weight that represents the reliability of the measurement of angle information. In environments with high angle errors, the measured vector value may deviate considerably from the actual value. In such environments, a higher weight should be given to distance information than vector information. The appropriate weight can be determined in advance through simulations in the desired environment (e.g., modifying the number of nodes in the network or changing the size of the network while also varying the distance and angle error). This weight is calculated based on when it yields good positioning accuracy on average during multiple simulations in the corresponding environment. To implement this approach, (18) is modified by adding an adaptive weight K as follows:

Algorithm 2: Iterative Vector-Based Localization Method With an Adaptive Weight in a Heterogeneous Network.

Input:
$\mathbf{\Phi} = \{\mathbf{\Phi}_1, \mathbf{\Phi}_2, \ldots, \mathbf{\Phi}_N\}^\top, \boldsymbol{D} = \{\boldsymbol{D}_1, \boldsymbol{D}_2, \ldots, \boldsymbol{D}_N\}^\top$
Output: $\mathbf{X} = \{\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_N\}^\top$
($\mathbf{\Phi}_i$: column vector which means relative angles to node i
$\boldsymbol{D}_i$: column vector which means distances to node i)
1: Calculate all sets of absolute angles $\hat{\mathbf{\Theta}} = \{\hat{\mathbf{\Theta}}_1, \hat{\mathbf{\Theta}}_2, \cdots, \hat{\mathbf{\Theta}}_N\}^\top$ by using (5)–(8)
2: Calculate all sets of vectors $\hat{\mathbf{V}} = [\hat{\mathbf{v}}_1, \hat{\mathbf{v}}_2, \ldots, \hat{\mathbf{v}}_M]^\top$ by using (9)–(11)
3: Calculate initial values for positions of all nodes $\mathbf{X}$ by using CMDS
4: Align the all sets of observed vectors $\hat{\mathbf{V}}$ by using (15)–(17) on each vector set
5: Update values for positions of all nodes $\mathbf{X}$ by using (18) (or (19): if an adaptive weight can be used)
6: Repeat processes 4-5 until the positions of all nodes converge

$$\min_{x_i, y_i} \left(\sum_{\{i,j\} \subset \mathbb{V}} \left\{ K w_{ij} \sqrt{(v_{x_{ij}} - \tilde{v}_{x_{ij}})^2 + (v_{y_{ij}} - \tilde{v}_{y_{ij}})^2} + (1-K) w_{ij} \sqrt{(d_{x_{ij}} - \bar{d}_{x_{ij}})^2 + (d_{y_{ij}} - \bar{d}_{y_{ij}})^2} \right\} + \sum_{\{i,k\} \not\subset \mathbb{V}} w_{ik} \sqrt{(d_{x_{ik}} - \bar{d}_{x_{ik}})^2 + (d_{y_{ik}} - \bar{d}_{y_{ik}})^2} \right) \quad \text{for} \quad i = 1, 2, \ldots, N \tag{19}$$

where the adaptive weight K ranges from 0 to 1, where a K closer to 1 indicates a high degree of accuracy in the measured AOA, while a K closer to 0 indicates low accuracy in the measured AOA, which makes it unreliable. Performing node localization using the proposed localization method with an adaptive weight follows a process similar to that of Algorithm 2, with the only difference being the use of (19) in the fifth step of the algorithm instead of (18).

V. Simulation Results

To evaluate the performance of the proposed localization method, we conducted extensive simulations in MATLAB and compared it with conventional positioning methods.

A. Simulation Environments

To evaluate the positioning performance, we compared the proposed positioning method with existing positioning methods, such as CMDS, MDS-MAP(C,R), and enhanced hybrid localization system (EHLS) [9], while varying various environmental factors (e.g., distance error, angle error, the proportion of vector nodes, size of the network, and the number of nodes and communication range). In addition, the positioning accuracy of the proposed localization method was analyzed by changing

TABLE I
DEFAULT VALUES OF ENVIRONMENTAL FACTORS FOR SIMULATIONS

Environmental factors	**Values**
Number of nodes	100
node distribution area (m^2)	20×20
communication range (m)	5
distance noise factor nf	0.1
s.t.d of angle error σ_ϕ (°)	5
Number of iterations	25
refinement range h_{ref}	2
proportion of vector nodes (%)	100

the computational complexity (e.g., the number of iterations and refinement range), with the aim of evaluating performance based on computational complexity. In order to compare the proposed positioning method and the conventional vector-based method EHLS in a large network, we simulated several scenarios in an environment where all nodes are vector nodes. Also, a heterogeneous scenario was selected to show that the proposed method works well even in such heterogeneous environments. Meanwhile, the proposed method is not compared with the conventional angle-based method. The angle-based method (e.g., RAST) has a very high positioning error due to the node with the 1-edge connectivity [9]. The angle-based method needs each node to connect with at least two neighboring nodes to calculate the position. Since, for a large network, there are likely to be 1-edge connectivities in some parts of the network, huge positioning errors occur in large networks. In the simulations, the distance values between nodes are assumed to be measured in a multiplicative noise environment where the distance measurement error increases with actual distance as

$$\bar{d}_{ij} = \acute{d}_{ij} + \acute{d}_{ij} \times N(0, \mathrm{nf}^2) \tag{20}$$

where $\bar{d}_{ij}$ and $\acute{d}_{ij}$ represent the measured distance and actual distance between nodes i and node j, respectively. The distance noise factor nf is a scalar component corresponding to the distance error.

In addition, a Gaussian noise environment is assumed for measuring the angle between nodes as

$$\bar{\phi}_{ij} = \acute{\phi}_{ij} + N(0, \sigma_\phi^2) \tag{21}$$

where $\bar{\phi}_{ij}$ and $\acute{\phi}_{ij}$ represent the measured angle and actual angle (i.e., measured AOA and actual AOA), respectively. σ_ϕ is the standard deviation of angle error. Although environmental factors may vary during simulations, the values of these factors used in the simulations are given in Table I. It is assumed that nodes are randomly distributed within a predefined area. As for the positioning accuracy (positioning error $e_{\mathbf{P}}$), the difference between the actual position of each node and the position estimated by the proposed localization method is calculated as

$$e_{\mathbf{P}} = \frac{\sum_{i=1}^{N} \sqrt{(\breve{x}_i - \acute{x}_i)^2 + (\breve{y}_i - \acute{y}_i)^2}}{N} \tag{22}$$

where $(\breve{x}_i, \breve{y}_i)$ and $(\acute{x}_i, \acute{y}_i)$ represent the final estimated and actual positions of node i, respectively. N refers to the total number of nodes within the simulation environment.

To ensure the accuracy of the results, the simulations were repeated 1000 times for each simulation environment, and the positioning error values were evaluated by taking the average of the results of these simulations. Also, in order to obtain the best results, the positioning performance of the proposed positioning method was calculated using the method with adaptive weight in all simulations.

B. Cramer–Rao Lower Bound (CRLB) for Iterative Vector-Based Localization

To determine the theoretical performance limits of the algorithm's positioning accuracy, the CRLB was computed. In all simulations, the Cramer-Rao Lower Bound (CRLB) of the positioning error was calculated using the numerical integration method in MATLAB. The joint probability density function of the distance and angle information [24] is redefined according to the multiplicative noise environment used in this article.

The joint probability density function of the distance and angle information between nodes i and j, denoted as $\mathbf{g}_{ij} = [\bar{d}_{ij}, \bar{\phi}_{ij}]$, is expressed as

$$f(\mathbf{g}_{ij}; \boldsymbol{\zeta}_i, \boldsymbol{\zeta}_j) = \frac{1}{\sqrt{2\pi d_{ij}^2 \mathrm{nf}^2}} \cdot \mathbf{exp}\left[-\frac{1}{2d_{ij}^2 \mathrm{nf}^2}(\bar{d}_{ij} - d_{ij})^2\right]$$
$$\cdot \frac{1}{2\pi\sigma_\phi^2} \cdot \mathbf{exp}\left[-\frac{1}{2\sigma_\phi^2}(\bar{\phi} - \mathbf{arctan}(\tfrac{y_i - y_j}{x_i - x_j}))^2\right] \tag{23}$$

where $\boldsymbol{\zeta}_i$ and $\boldsymbol{\zeta}_j$ are the 2-D coordinates of node i and j, respectively, which can be represented as (x_i, y_i) and (x_j, y_j).

The Fisher information matrix (FIM) $\mathbf{F}$ yields

$$\mathbf{F} = E\left[\left(\frac{\partial \log f(\mathbf{g}_{ij};\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_j)}{\partial \mathbf{P}}\right)\left(\frac{\partial \log f(\mathbf{g}_{ij};\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_j)}{\partial \mathbf{P}}\right)^{\top}\right] \tag{24}$$

where $\mathbf{P}$ is the 2-D coordinate for all nodes in the network.

The FIM $\mathbf{F}$ can be expressed in the form of submatrices [25] as

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{\mathbf{x},\mathbf{x}} & \mathbf{F}_{x,y} \\ \mathbf{F}_{x,y} & \mathbf{F}_{y,y} \end{bmatrix} \tag{25}$$

where (i, l)th elements of the submatrices in $\mathbf{F}$ are given as

$$\mathbf{F}_{x,x_{il}} = \begin{cases} \sum -E\left(\frac{\partial^2 \log f(\mathbf{g}_{ij};\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_j)}{\partial x_i^2}\right) & i = l \\ -E\left(\frac{\partial^2 \log f(\mathbf{g}_{ij};\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_j)}{\partial x_i^2}\right) & i \neq l \end{cases} \tag{26}$$

$$\mathbf{F}_{x,y_{il}} = \begin{cases} \sum -E\left(\frac{\partial^2 \log f(\mathbf{g}_{ij};\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_j)}{\partial x_i y_i}\right) & i = l \\ -E\left(\frac{\partial^2 \log f(\mathbf{g}_{ij};\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_j)}{\partial x_i y_i}\right) & i \neq l \end{cases} \tag{27}$$

$$\mathbf{F}_{y,y_{il}} = \begin{cases} \sum -E\left(\frac{\partial^2 \log f(\mathbf{g}_{ij};\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_j)}{\partial y_i^2}\right) & i = l \\ -E\left(\frac{\partial^2 \log f(\mathbf{g}_{ij};\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_j)}{\partial y_i^2}\right) & i \neq l. \end{cases} \tag{28}$$

According to the CRLB theorem, the minimum value of the squared positioning error for node i is determined as

$$E((\breve{x}_i - \acute{x}_i)^2 + (\breve{y}_i - \acute{y}_i)^2) \geq (\mathbf{F}_{x,x_{ii}})^{-1} + (\mathbf{F}_{y,y_{ii}})^{-1} \tag{29}$$

where $(\mathbf{F}_{x,x_{ii}})^{-1}$ and $(\mathbf{F}_{y,y_{ii}})^{-1}$ are the diagonal terms of the inverse of the submatrices $\mathbf{F}_{x,x}$ and $\mathbf{F}_{y,y}$, respectively.

By Cauchy–Schwarz inequality, (29) is derived to

$$(\mathbf{F}_{x,x_{ii}})^{-1} \geq \frac{1}{\mathbf{F}_{x,x_{ii}}} \tag{30}$$

$$(\mathbf{F}_{y,y_{ii}})^{-1} \geq \frac{1}{\mathbf{F}_{y,y_{ii}}}. \tag{31}$$

Therefore, the CRLB of the positioning error of node i can be approximated as

$$e_{\mathbf{P}i,\text{CRLB}} = \sqrt{\frac{1}{\mathbf{F}_{x,x_{ii}}} + \frac{1}{\mathbf{F}_{y,y_{ii}}}} \tag{32}$$

and, given distance and angle information, the CRLB of the positioning error of all nodes $e_{\mathbf{P}_{\text{CRLB}}}$ is as follows:

$$e_{\mathbf{P}_{\text{CRLB}}} = \frac{\sum_{i=1}^{N} e_{\mathbf{P}i}}{N}. \tag{33}$$

Meanwhile, the probability density function of the local probability information between node i and j is given by

$$f(\bar{d}_{ij}; \boldsymbol{\zeta}_i, \boldsymbol{\zeta}_j) = \frac{1}{\sqrt{2\pi d_{ij}^2 \text{nf}^2}} \cdot \mathbf{exp}\left[-\frac{1}{2d_{ij}^2 \text{nf}^2}(\bar{d}_{ij} - d_{ij})^2\right]. \tag{34}$$

If vector and distance nodes are distributed together, (26) and (28) are replaced with the following expressions:

$$\mathbf{F}_{x,x_{il}} = \begin{cases} \sum_{j\in\mathbb{V}} -E\left(\frac{\partial^2 \log f(\mathbf{g}_{ij};\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_j)}{\partial x_i^2}\right) & \\ +\sum_{j\in\mathbb{D}} -E\left(\frac{\partial^2 \log f(\rho_{ij};\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_j)}{\partial x_i^2}\right) & i = l \\ -E\left(\frac{\partial^2 \log f(\mathbf{g}_{ij};\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_j)}{\partial x_i^2}\right) & i \neq l \text{ and } j \in \mathbb{V} \\ -E\left(\frac{\partial^2 \log f(\rho_{ij};\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_j)}{\partial x_i^2}\right) & i \neq l \text{ and } j \in \mathbb{D} \end{cases} \tag{35}$$

$$\mathbf{F}_{y,y_{il}} = \begin{cases} \sum_{j\in\mathbb{V}} -E\left(\frac{\partial^2 \log f(\mathbf{g}_{ij};\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_j)}{\partial y_i^2}\right) & \\ +\sum_{j\in\mathbb{D}} -E\left(\frac{\partial^2 \log f(\rho_{ij};\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_j)}{\partial y_i^2}\right) & i = l \\ -E\left(\frac{\partial^2 \log f(\mathbf{g}_{ij};\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_j)}{\partial y_i^2}\right) & i \neq l \text{ and } j \in \mathbb{V} \\ -E\left(\frac{\partial^2 \log f(\rho_{ij};\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_j)}{\partial y_i^2}\right) & i \neq l \text{ and } j \in \mathbb{D} \end{cases} \tag{36}$$

where $\mathbb{V}$ and $\mathbb{D}$ denote the set of vector nodes and distance nodes, respectively.

By using the FIM derived from the combination of vector and distance information, we can determine the CRLB of the positioning error, even when vector and distance nodes are distributed together in an environment.

C. Performance Analysis

1) Distance Error: Extensive simulations were conducted in order to evaluate the positioning performance of the proposed positioning method, with varying distance measurement errors. Fig. 6 presents the positioning error of existing methods (CMDS, MDS-MAP(C,R), EHLS) and the proposed method for distance noise factors (nf) ranging from 0.005 (average error for distance at 5 m is 0.2 m) to 0.2 (average error for distance at 5 m is 0.8 m) in environments with low angle errors ($\sigma_\phi = 5°$: average error for angle is 4°). Results indicate that the proposed method achieves a lower positioning error than existing positioning methods in all nf values. The proposed method demonstrates a 15% improvement compared with the conventional distance-based iterative positioning method MDS-MAP(C,R) in environments with lowangle errors and a 22% improvement compared with the conventional vector-based positioning method EHLS. Notably, the proposed method demonstrates a 44% improvement in performance compared with the conventional positioning method MDS-MAP(C,R) in environments with high distance errors (nf = 0.2).

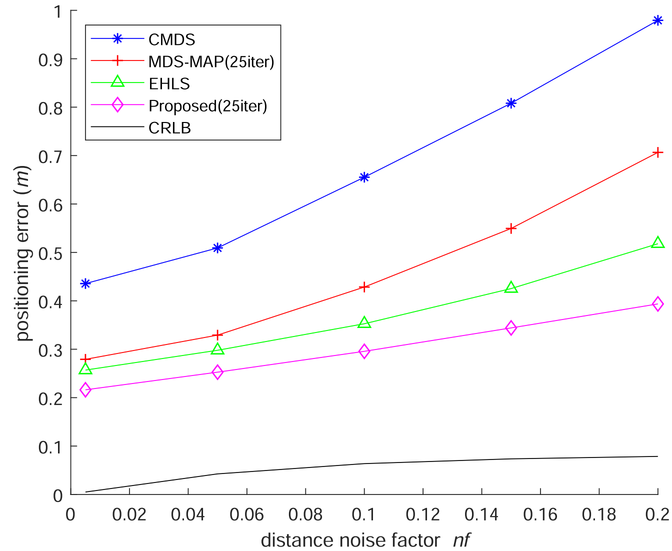

Fig. 6. Effect of distance measurement error in an environment where each node can measure distance and angle information from one's neighboring nodes ($N = 100$, $\sigma_\phi = 5°$).

2) Angle Error: Additional simulations are performed to investigate the impact of angle measurement error on the performance of the proposed method. Fig. 7 displays the positioning error of the proposed method and existing methods, while varying the standard deviation of angle error σ_ϕ from 2° (average error for angle is 1.6°) to 13° (average error for angle is 10.4°) in nf = 0.1 (average error for distance at 5 m is 0.4 m) environment. The proposed method outperforms the existing methods for all values of the standard deviation of angle error in the presented distance noise factor (nf = 0.1) environment. Specifically, the proposed method offers 52% higher positioning accuracy than the conventional iterative range-based positioning method MDS-MAP(C,R) in environments with sufficiently low angle measurement errors ($\sigma_\phi = 2°$).

3) Node Density: It is worth evaluating the impact of the number of nodes or node density on the positioning performance of the proposed method. Varying the number of nodes is considered within an area of 20×20 m^2. Fig. 8 illustrates the positioning error of the existing methods and the proposed positioning method while varying the number of nodes N from

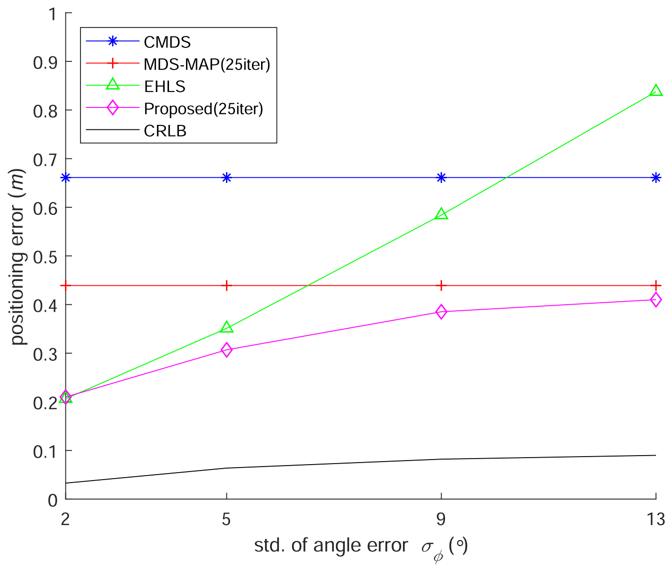

Fig. 7. Effect of angle measurement error in an environment where each node can measure distance and angle information from one's neighboring nodes ($N = 100$, nf $= 0.1$).

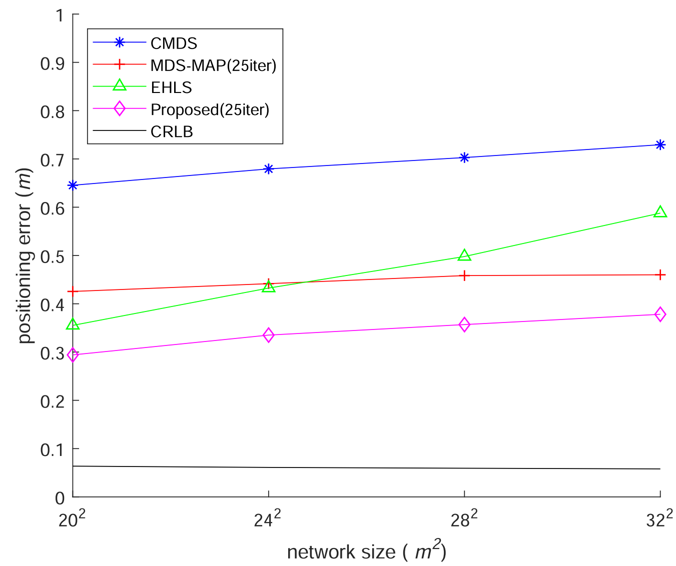

Fig. 9. Effect of network size in an environment where each node can measure distance and angle information from one's neighboring nodes (nf $= 0.1$, $\sigma_\phi = 5°$).

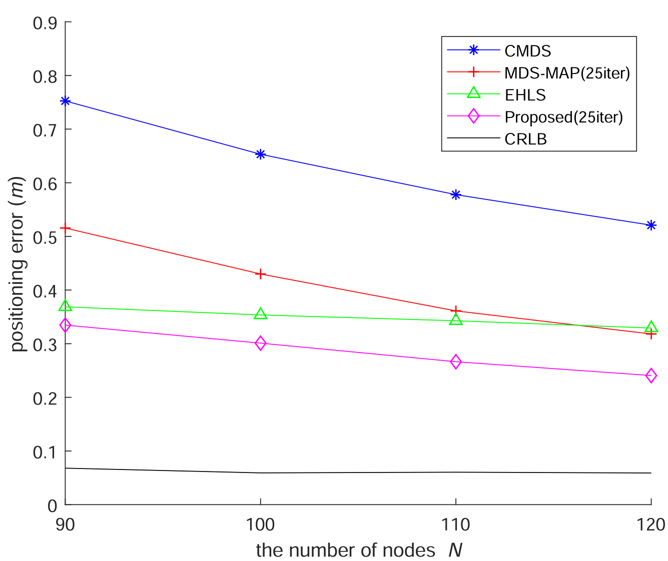

Fig. 8. Effect of node density in an environment where each node can measure distance and angle information from one's neighboring nodes (nf $= 0.1$, $\sigma_\phi = 5°$).

90 to 120 in environments with nf $= 0.1$ and $\sigma_\phi = 5°$. It is confirmed that as the density of nodes increases, all compared positioning methods exhibit a reduction in positioning error. This is due to the increase in the number of directly connected neighboring nodes per node, leading to an increase in information for node positioning. Compared with existing distance-based methods, the proposed method exhibits superior performance in predicting the positions of nodes with fewer neighboring nodes. Specifically, setting the number of nodes to 64 in the simulation can lead to a scenario where some nodes in the network have only one neighboring node (i.e., a network with 1-edge connectivity). Even in such a situation, the proposed positioning method exhibits a 35% enhancement in positioning performance compared with the conventional positioning method MDS-MAP(C,R). In addition, the proposed method demonstrates a positioning performance improvement of over 9% for all numbers of nodes compared with the vector-based positioning method EHLS.

4) Network Size: Network size is an another important factor to evaluate the performance of the positioning method. Fig. 9 illustrates the positioning error of the positioning method as a function of the network size. To increase the network size while maintaining node density, both the number of nodes and the distribution area of nodes vary simultaneously. The number of nodes is changed to $[100, 144, 196, 256]$, while the distribution area of nodes is changed to $[20^2, 24^2, 28^2, 32^2]$ (m^2). Similar to the other simulation results, it is observed that the proposed positioning method exhibits better positioning performance than the existing positioning methods even with changes in the network size. Meanwhile, in larger network environments, the predicted position of a single node can significantly impact the positioning of its neighboring nodes. The accumulation of such effects can be considerable. In such cases, the conventional vector-based positioning method EHLS that uses vector combination is significantly affected by these error accumulations, causing the positioning error to increase compared with the conventional distance-based positioning method MDS-MAP(C,R) when the network size exceeds a certain threshold (28^2 m^2). When the network size is set to 20^2 m^2, the proposed positioning method exhibits 31% and 17% improvement in positioning performance results compared with the conventional positioning methods MDS-MAP(C,R) and EHLS, respectively.

5) Number of Iterations and Refinement Range: We also evaluate the positioning performance of the positioning method in terms of computational complexity, including the number of iterations and the refinement range h_{ref}. Since the proposed

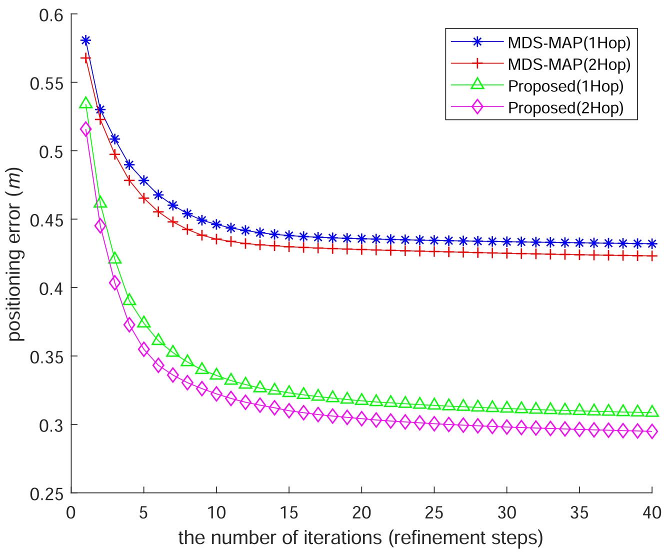

Fig. 10. Effect of calculation parameters (i.e., the number of iterations, refinement range) in an environment where each node can measure distance and angle information from one's neighboring nodes ($N = 100$, nf $= 0.1$, $\sigma_\phi = 5°$).

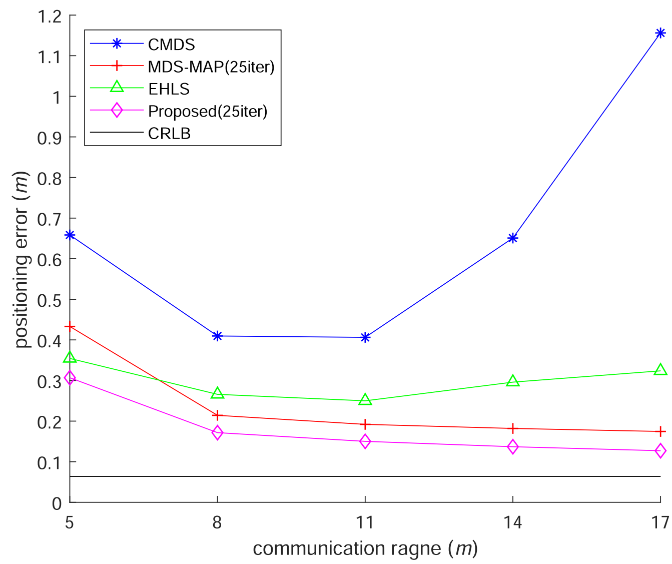

Fig. 11. Effect of communication range in an environment where each node can measure distance and angle information from one's neighboring nodes ($N = 100$, nf $= 0.1$, $\sigma_\phi = 5°$).

positioning method is based on iterative LSM-based calculations, higher iterations in the positioning method guarantee better positioning performance, but also result in increased computational complexity. In addition, the refinement range h_{ref} used in the positioning method is directly related to the algorithm's complexity. Fig. 10 presents the positioning error while increasing the number of computational iterations of the proposed positioning method with h_{ref} set to 1 and 2. The proposed positioning method with h_{ref} set to 2 shows better positioning performance than the method with h_{ref} set to 1. The proposed positioning method with h_{ref} set to 2 gives 24% improvements in positioning performance when the number of computational iterations is 5 and 30% improvements when the number of computational iterations is 25.

6) Communication Range: Apparently, the communication range of the nodes must have a huge impact on the performance of the positioning method. Fig. 11 depicts the positioning error of the existing positioning methods and the proposed positioning method as the communication range of the nodes is increased to $[5, 8, 11, 14, 17]$ (m). As the communication range of nodes increases, the positioning error decreases in both the MDS-MAP(C,R) and the proposed positioning method. However, under the same conditions, the positioning error may increase in the CMDS and EHLS positioning methods due to the distance measurement model with multiplicative noise. This implies that as the actual distance between nodes increases, the distance measurement error also increases, and these positioning methods are significantly impacted by the accumulation of such errors. In addition, it is observed that the positioning error results of the proposed positioning method are closer to the CRLB than the error results of the existing positioning methods. Furthermore, if the communication range of nodes is increased, the number of hops between far-away nodes in network will decrease, the difference in positioning performance between the theoretical CRLB and proposed positioning method can be confirmed to decrease.

7) Proportion of Vector Nodes: The simulation results presented thus far assume that all nodes in the network are vector nodes that have the capability of measuring both distance and angle information from neighboring nodes. The conventional EHLS only works in environments where all nodes are vector nodes. However, the proposed positioning method can also operate in networks where only some nodes are vector nodes, and others are distance nodes. To evaluate the performance of the positioning methods in networks with mixed node types, we conducted simulations for several positioning methods, including CMDS, MDS-MAP(C,R), and the proposed positioning method.

Fig. 12 depicts the evaluation of the positioning performance with respect to the distance noise factor nf in an environment where vector nodes are set to 50% and 100%, respectively. Since the conventional positioning methods, such as CMDS and MDS-MAP(C,R), use only distance information, the proportion of vector nodes does not affect the positioning result. In contrast, since the proposed positioning method utilizes vector information, the positioning error is evaluated in two environments, where the proportion of vector nodes in the total nodes is 50% and 100%. In addition, results from the CRLB are also obtained in both environments. The proposed positioning method exhibits better positioning performance than the conventional positioning methods for all distance noise factors nf in environments where vector nodes account for 50% of total nodes. The improvement in the positioning performance of the proposed positioning method increases as the proportion of vector nodes in the network increases. For instance, in an environment where vector nodes account for 50% of the total nodes and the distance noise factor nf is 0.1, the proposed positioning method demonstrates a 17%

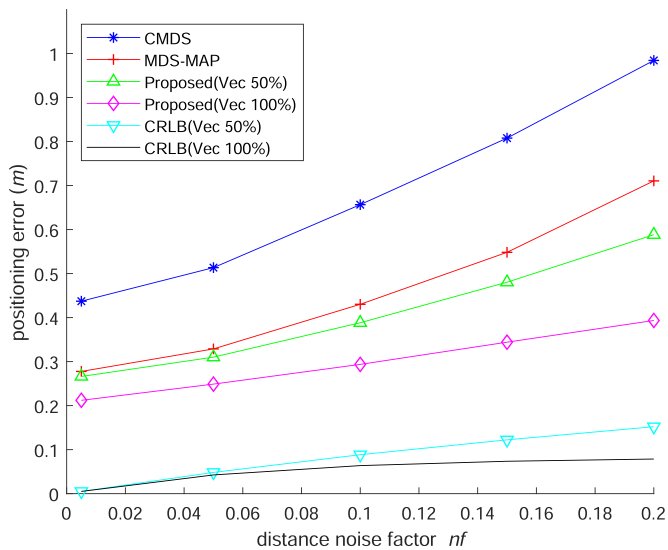

Fig. 12. Effect of the proportion of vector nodes in an environment where vector nodes and distance nodes are mixed in the network ($N = 100$, $\sigma_\phi = 5°$).

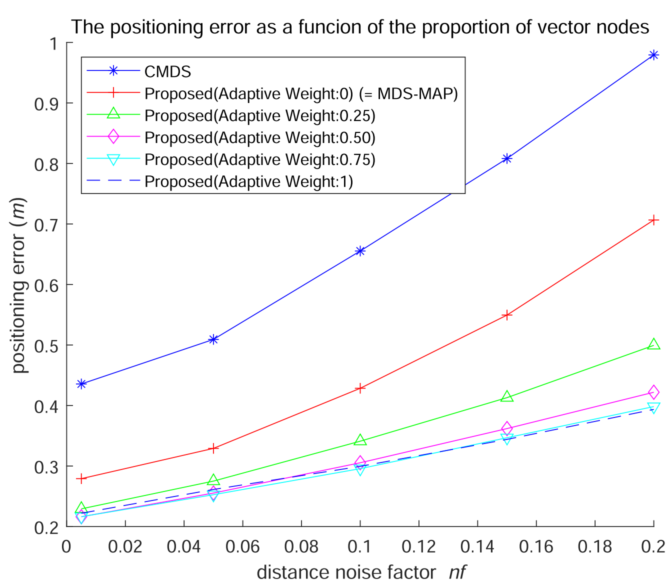

Fig. 13. Effect of adaptive weight in an environment where each node can measure distance and angle information from one's neighboring nodes ($N = 100$, $\sigma_\phi = 5°$).

improvement in positioning performance compared with the conventional positioning method MDS-MAP(C,R). However, in an environment where vector nodes account for 100% of the total nodes and nf is 0.1, the proposed positioning method shows a 31% improvement in positioning performance compared with the conventional positioning method.

8) Adaptive Weight: Finally, we conducted an experiment to investigate how the adaptive weight value affects the positioning accuracy. Fig. 13 shows the positioning errors of the proposed positioning method when the adaptive weight was set to $[0, 0.25, 0.5, 0.75, 1]$, respectively. It demonstrates that the proposed positioning method with adaptive weight always produces better positioning performance than the conventional positioning methods, such as CMDS, MDS-MAP(C,R). We confirmed that the adaptive weight value of 1 does not always yield good results in environments with low angle errors. For example, setting the adaptive weight to 0.75 in environments where nf is 0.1 or 0.15, and setting the adaptive weight to 1 in environments where nf is 0.2 showed the best positioning results among the adaptive weight values tested. Therefore, to obtain the best positioning performance, it is important to model the desired network environment and obtain the optimal adaptive weight value through multiple simulations.

VI. Conclusion

This article proposes a novel iterative vector-based localization method. The proposed positioning method can estimate the positions of all nodes in a large heterogeneous network where only some nodes can measure both distance and angle information to neighboring nodes. The proposed positioning method is capable of being applied flexibly to node localization, irrespective of the ratio of nodes that can measure distance and angle information in the network. The positioning method computes the vectors between nodes in a reference coordinate system and then calculates the positions of the nodes using LSM-based iterative optimization techniques. The proposed positioning method showed better positioning performance than conventional methods in all experimental environments. Especially in an environment with low angle error ($\sigma_\phi = 5°$), the proposed positioning method improves the positioning performance up to 44% compared with the conventional distance-based positioning method MDS-MAP(C,R), and up to 24% compared with the vector-based positioning method EHLS. Furthermore, the proposed positioning method shows robust positioning performance even in environments where the number of neighboring nodes per node is small, especially in a network with 1-edge connectivity.

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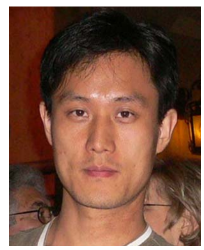

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