III. Computational Requirements

Next, let us investigate the advantages and disadvantages of computing the DFT using the results in the last section. First, let us assume that N is odd and we are interested in computing (1). Using (5) and (9), we have

$$F = P_1^T L_0 B_2 (L_0^T P_1 f) \tag{21}$$

where P_1 and P_1^T are permutation matrices. Combining $(1/\sqrt{2})$ in L_0 and L_0^T results in $\frac{1}{2}$, which simply corresponds to bit-shifting. Also, L_0 and L_0^T contain one $\sqrt{2}$, and the rest of the entries are 0's and 1's. Therefore, the total number of multiplications required in executing (21) is approximately

$$T_1 = \left(\frac{N+1}{2}\right)^2 + \left(\frac{N-1}{2}\right)^2. \tag{22}$$

These are real multiplications when f is real. The number of additions is approximately equal to $2N + T_1$ where N are complex additions and the remaining are real when f is real. The number in (22) can be further reduced by using the results in Section II-B. Now consider the computation of

$$Cx = y \tag{23}$$

where x is real. From (16) and (17), it follows that (23) can be written as

$$2(G_1 G_1^T)x - x = y. \tag{24}$$

Without losing any generality, we can assume that G_1 can be written as

$$G_1 = \begin{matrix} (k) \\ (m-k) \end{matrix} \begin{bmatrix} G_{11} \\ G_{21} \end{bmatrix}$$

where G_{11} is a lower triangular matrix. The computation of (24) requires $k(2m - k + 1)$ number of real multiplications and $k(2m - k + 1) + m$ real additions. Assuming

$$k \cong \frac{m}{2} \cong \frac{N}{4}$$

it follows that the total number of computations required by this method is approximately equal to

$$\frac{3}{8} N^2 \tag{25}$$

which is unfortunately higher than by the method given by Rader [10]. However, it should be pointed out that the number of computations given in (25) corresponds to one set of orthogonal eigenvectors of DFT matrices. Noting the nonuniqueness of orthogonal eigenvectors of DFT matrices, other eigenvectors may be selected which may reduce the number of computations. This needs further research.

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Addendum to Review of *Digital Signal Processing: Theory, Design and Implementation*

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In my review of *Digital Signal Processing: Theory, Design and Implementation* by Peled and Liu,[1] I neglected to comment on the suitability of the book as a course text.

This question of "why" was asked of me on several occasions. Sheer oversight. As an introductory text I recommend the book without reservation (as long as the chapter 2 material is supplemented).

For the practicing engineer who encounters digital signal processing through an intensive short course, or an evening extension course, or an in-plant industrial course, I feel that this book is *the* text.

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[1] S. A. White, *IEEE Trans. Acoust., Speech, Signal Processing* (Book Review), vol. ASSP-25, pp. 206–207, Apr. 1977.

Corrections to "Short Term Spectral Analysis, Synthesis, and Modification by Discrete Fourier Transform"

JONT B. ALLEN

In the above paper,[1] equation (8) should be:

$$X_{nm} = \sum_{k=nD-T/2}^{nD+T/2-1} w(nD-k)\, x(k)\, e^{j2\pi km/T}. \tag{8}$$

Equation (10) and the line following it should be:

$$F^{-1}\{X_{nm}\} = \begin{cases} x(k)\, w(nD-k) & -T/2 \leqslant k - nD \leqslant T/2 - 1 \\ 0 & k \text{ otherwise} \end{cases} \tag{10}$$

where in the nonzero range of k

Equation (17) should be dropped since it is redundant.

The vertical scale for the lower panel of Fig. 2 is in decibels.

I would very much like to thank Dan E. Dudgeon of Bolt, Beranek and Newman, Inc., for pointing out the errors in (8) and (10).

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[1] J. B. Allen, *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-25, pp. 235–238, June 1977.