theorem in the paper[1] gives a necessary condition for the existence of coefficients for nm rectangular pulse functions that constitute the control vector, so that the state of the system is driven to zero in a finite given time. It is shown in this note that rank adj$(A - s_iI)$, where A has n distinct eigenvalues s_i, $i = 1, 2, \cdots, n$, is one. As a result, there are only n linearly independent equations in Farlow's condition for the nm coefficients and as such only n of the coefficients are determined uniquely. The other $n(m - 1)$ coefficients can be chosen to achieve other desirable characteristics of the system response.

I. Rank of the Adjoint Matrix

Farlow's theorem in the paper[1] gives a necessary condition for the coefficients of nm rectangular pulse functions, so that a control can be determined to drive the state of the system to zero on a finite time interval. The condition is based on the selection of nm linearly independent equations from

$$(A-s_iI)^*BE(s_i)C=-(A-s_iI)^*x_0, \qquad i=1, 2, \cdots, n \qquad (1)^2$$

where M^* denotes the adjoint matrix of a matrix M and s_i, $i = 1, 2, \cdots, n$ are the n distinct eigenvalues of A.

This is based on the argument that the rank $(A - s_iI)^*BE(s_i)$ is m, the minimum of the rank of the three matrices in the product. There are n systems of equations of this type (extensions are also made to complex and repeated eigenvalues).

The fact is that this rank cannot be greater than one, since the rank of the matrix $(A - s_iI)^* \leq 1$, for $i = 1, 2, 3, \cdots, n$.

To show this, let T be a similarity transformation which diagonalizes A, that is

$$T^{-1}AT=\text{diag}\ (s_i) \triangleq D, \qquad i=1, 2, \cdots, n. \qquad (2)$$

Then,

$$\text{rank}\ (A-s_iI)^*=\text{rank}\ [T(D-s_iI)T^{-1}]^*$$
$$\leq \min\ (\text{rank}\ T,\ \text{rank}\ (D-s_iI)^*,\ \text{rank}\ T^{-1})$$

but,

$$\text{rank}\ [\text{diag}\ (s_1-s_i, s_2-s_i, \cdots, s_{i-1}-s_i, 0, s_{i+1}-s_i, \cdots, s_n-s_i)]^* \leq 1 \quad (3)$$

because all matrices A_{jk} obtained by deleting the jth row and the kth column have at least one row (column) of zeros, and thus all cofactors are zero, except det $A_{ii} = \pi_{j\neq i}(s_j - s_i)$. Thus, rank $(D - s_iI)^*$ is one and it follows that the rank of the matrix $(A - s_iI)^* \leq 1$.

II. An Undetermined System

Following Farlow's approach we note that nm equations are selected out of a set of nn equations. What we have just shown is that only n equations of this set are linearly independent and the system of equations should have infinitely many solutions. Any of these solutions would yield a control that drives the system to the desired state. This confirms the fact that Farlow's theorem is indeed only a necessary condition, as stated in the paper.[1] It follows that the nm coefficients in Farlow's paper[1] can be determined in many ways. For example, n linearly independent equations can be selected and used as constraints to a quadratic optimization problem; one possible cost function is

$$J=(1/2)C'\left(\int_0^T U'(t)RU(t)\,dt\right)C$$

where $U(t)$ is the $n \times nm$ matrix of rectangular pulse functions given by Farlow, C is the nm coefficient vector, R is an $n \times n$ weighting matrix, and T is the final time.

[1] S. J. Farlow, *IEEE Trans. Automat. Contr.*, vol. AC-30, pp. 825–827, Aug. 1985.

[2] This equation is written incorrectly in the paper.[1]

Comments on "On the Computational Aspect of the Matrix Exponentials and Their Use in Robot Kinematics"

RANGASWAMY MUKUNDAN AND A. K. PRADEEP

In the paper[1] Salam and Yoon suggest that the exponential matrix representation has potential computational use and may have geometric appeal in robot kinematics. In this regard the following comments are in order.

1) For an n-axis robotic manipulator it can be easily shown that the well-known homogeneous transformation representation leads to $64(n - 1)$ multiplications and $48(n - 1)$ additions while the approach of Salam and Yoon[1] requires $66(n - 1)$ multiplications and $52(n - 1)$ additions. It is evident from this that not too much is gained (or lost) computationally. It is hoped that the rationale for choosing the exponential representation must be due to other significant factors.

2) The 4×4 homogeneous transformation matrix is usually *nonorthogonal* due to any translational component in the robotic manipulator configuration. Consequently, it cannot be represented by an exponential matrix. Hence, concluding remark 3) by Salam and Yoon in the paper[1] is unachievable.

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[1] F. M. A. Salam and C. S. Yoon, *IEEE Trans. Automat. Contr.*, vol. AC-31, pp. 376–378, Apr. 1986.

Correction to "Potentially Global Stabilizability"

A. BACCIOTTI

The conclusion of Theorem 3, as stated in [1], is false.

A counterexample is provided by the system

$$\begin{cases} \dot{x}=y-xy \\ \dot{y}=u \end{cases}$$

for which the line $x = 1$ is an invariant set independent of the choice of u. The system has a controllable linear part and, hence, it is locally stabilizable by a linear feedback. However, the region of attraction of the closed-loop system includes no points with $x \geqslant 1$.

The conclusion of Theorem 3 of the paper[1] becomes true under additional assumptions. For instance, if B is nonsingular, one can choose $u = cB^{-1}x$ for a suitable constant c. Then, no change of coordinates is needed and the remaining part of the proof works with minor variations.

This remark is of interest only if we limit ourselves to the use of linear feedbacks. Indeed, if B is nonsingular and nonlinear feedbacks are allowed, then it is obviously possible to suppress the nonlinear terms of the system and to obtain global stability.

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[1] A. Bacciotti, *IEEE Trans. Automat. Contr.*, vol. AC-31, pp. 974–976, 1986.

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