An optimal solution is

$$u(0) = 0, \quad u(1) = -u(2) = 1.$$

It is impossible to find a nonzero vector (p^1, p^2) for which the Hamiltonian

$$H = p^1 u(i) + p^2 |u(i)|$$

is either locally maximum or stationary for that optimal solution.

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Correction to and Extension of "On the a priori Information in Sequential Estimation Problems"

CORRECTION

Dr. W. G. Tuel, Jr., of the IBM Research Division, San Jose, Calif., has brought to the author's attention a typographical error in equation (22) of reference [1]. The inverse sign should be omitted there. Thus the correct form should read:

$$E_{co}(k+1) = (\Phi - K_o M) \cdot [E_{co}(k) - E_{co}(k) M^T \overline{P_c(k)}^{-1} M E_{co}(k)] \cdot (\Phi - K_o M)^T. \quad (22)$$

Naturally, subsequent equations are not affected by this misprint.

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EXTENSION

When the process is discrete, the observation noise can be treated as components of the state vector x, as was done in reference [2], as well as in [1]. However, if the observation noise v is white and uncorrelated with the process noise w, a reduction of the order of matrices and a considerable simplification of computations become possible by implementing v in the observation equation. Thus the process and observation equations are described:

$$x(k+1) = \Phi(k)x(k) + G(k)w(k) \quad (1)$$

$$y(k) = M(k)x(k) + v(k) \quad (2')$$

where

$$\langle v(k) \rangle = 0 \quad (A\text{-}1)$$

$$\langle v(j)v(k)^T \rangle = R(k) \quad \text{for } j = k$$
$$= 0 \quad \text{for } j \neq k \quad (A\text{-}2)$$

$$\langle v(j)w(k)^T \rangle = 0. \quad (A\text{-}3)$$

The same symbols as those in reference [1] are employed for the other quantities such as x, y, w, Φ, G, and M, keeping in mind that their dimensions are reduced accordingly by the dimension of the vector v.

Then (13) is modified as

$$P_c(k+1) = (\Phi - K_c M) P_c(k) (\Phi - K_c M)^T + K_c R K_c^T + GQG^T \quad (13')$$

where

$$K_c(k) = \Phi P_c(k) M^T (M P_c(k) M^T + R(k))^{-1}. \quad (14')$$

Also (17) becomes

$$P_a(k+1) = (\Phi - K_c M) P_a(k) (\Phi - K_c M)^T + K_c R K_c^T + GQG^T. \quad (17')$$

Therefore (18) remains in the identical form

$$E_{ca}(k+1) = P_c(k+1) - P_a(k+1) = (\Phi - K_c M) E_{ca}(k) (\Phi - K_c M)^T. \quad (18')$$

However, the subsequent derivations are to be modified, replacing $\bar{P}_o$ by S_o and $\bar{P}_c$ by S_c where

$$S_o(k) = M P_o(k) M^T + R(k) \quad (A\text{-}4)$$

$$S_c(k) = M P_c(k) M^T + R(k). \quad (A\text{-}5)$$

Then (22) becomes

$$E_{co}(k+1) = (\Phi - K_o M) \cdot [E_{co}(k) - E_{co}(k) M^T S_c(k)^{-1} M E_{co}(k)] \cdot (\Phi - K_o M)^T \quad (22')$$

where the content of the middle bracket can be rewritten as

$$E_{co}(k) - E_{co}(k) M^T S_c(k)^{-1} M E_{co}(k) = E_{co}(k) M^T [\overline{E}_{co}(k) + \overline{E}_{co}(k) S_o(k)^{-1} \overline{E}_{co}(k)]^{-1} M E_{co}(k) + [I - E_{co}(k) M^T \overline{E}_{co}^{-1}(k) M] \cdot E_{co}(k) [I - E_{co}(k) M^T \overline{E}_{co}^{-1}(k) M]^T \quad (29')$$

assuming $S_o(k)$ and $S_c(k)$ are positive definite.

The same recurrence formulas hold for P_o and K_o as (13') and (14') if P_c and K_c are replaced by P_o and K_o there.

Finally $E_{ao}(k+1)$ of (25) can be reduced to the following form:

$$E_{ao}(k+1) = (\Phi - K_c M) E_{ao}(k) (\Phi - K_c M)^T + (\Phi - K_o M) \cdot [E_{co}(k) M^T S_c(k)^{-1} S_o(k) S_c(k)^{-1} M E_{co}(k)] \cdot (\Phi - K_o M)^T. \quad (25')$$

In view of (18'), (22'), (29'), and (25'), we can observe that four theorems in reference [1] remain valid.

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