A Simplified Lattice Factorization for Linear-Phase Paraunitary Filter Banks With Pairwise Mirror Image Frequency Responses

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***Abstract*—In this paper, by extending our previous work on general linear-phase paraunitary filter banks even-channel (LPPUFBs), we develop a new structure for LPPUFBs with the pairwise mirror image (PMI) frequency responses, which is a simplified version of the lattice proposed by Nguyen *et al.* Our simplification is achieved through trivial matrix manipulations and the cosine–sine (C–S) decomposition of a general orthogonal matrix. The resulting new structure covers the same class of PMI-LPPUFBs as the original lattice, while substantially reducing the number of free parameters involved in the nonlinear optimization. A design example is presented to demonstrate the effectiveness of the new structure.**

***Index Terms*—Cosine–sine (C–S) decomposition, linear-phase, pairwise mirror image (PMI) frequency responses, parameterization, paraunitary.**

I. Introduction

LINEAR-PHASE paraunitary filter banks (LPPUFBs) have been extensively used in the application of image processing. Over the past decade, many works have exploited the design of M-channel LPPUFBs through the lattice factorization [1]–[10]. In [3], Soman *et al.* first developed a complete and minimal factorization for even-channel LPPUFBs. An alternative, but equivalent form of this structure with slightly fewer parameters was presented in [5]. Later, extension to perfect reconstruction systems was provided in [6]. Simplified structures of [3], [5], and [6] were recently reported in [7] and [8], which results in a considerable reduction on the number of free parameters, while retaining the generality of the factorizations in [3] and [5]. This facilitates both the design and the implementation of LPPUFBs.

Even after the simplification, the number of free parameters can be still quite large with the increase of the number of channels and the filter length. In this paper, we aim to further reduce the design complexity by adding the pairwise mirror image (PMI) property to LPPUFBs, in which the frequency responses of each pair of filters are symmetric with respect to $\pi/2$. Several works on this topic have been reported by other researchers in the past. A factorizaton for even-channel system was presented in [2]–[4], while odd-channel factorizations can be found in [9] and [10]. However, in these factorizations, the number of free parameters is nearly equal to that of simplified factorizations for general LPPUFBs in [7] and [8]. Intuitively, by imposing the PMI property, fewer parameters are required than those of the general LPPUFBs. Therefore, the method introduced in [8] is extended in this paper to simplify the factorization of PMI-LP-PUFBs. For simplicity, we only consider *even*-channel systems and the simplification is based on the structure in [2]. After our simplification, the degrees of design freedom are reduced by a large margin, while the design space is not affected at all. This can significantly simplify the design complexity.

Notations: For a real number x, $\lceil x \rceil$, and $\lfloor x \rfloor$ represent the ceiling and the floor of x, respectively. Vectors and matrices are indicated in bold-faced letters. Subscripts will be provided only if their sizes are not clear from the context. Superscript T stands for transposition. Special matrices used extensively throughout this paper are: the identity matrix $\mathbf{I}$, the reversal matrix $\mathbf{J}$, and the null matrix $\mathbf{0}$. Besides, $\mathbf{W}$ and $\hat{\mathbf{W}}$ are $2m \times 2m$ butterfly matrices as follows:

$$\mathbf{W} = \begin{bmatrix} \mathbf{I}_m & \mathbf{I}_m \\ \mathbf{I}_m & -\mathbf{I}_m \end{bmatrix} \text{ and } \hat{\mathbf{W}} = \begin{bmatrix} \mathbf{I}_m & \mathbf{J}_m \\ \mathbf{I}_m & -\mathbf{J}_m \end{bmatrix}.$$

Moreover, $\Lambda(z)$ is an $2m \times 2m$ diagonal matrix with the form of $\Lambda(z) = \mathrm{diag}(\mathbf{I}_m, z^{-1}\mathbf{I}_m)$.

II. Review of Existing Factorizations

A. General LPPUFBs

Consider an M-channel $(M = 2m, m > 1)$ LPPUFB with all filters of the same length $L = KM$ each. Let $\mathbf{E}(z)$ be the corresponding polyphase matrix. It was first proved in [3] and [5] that $\mathbf{E}(z)$ can be always factorized into

$$\mathbf{E}(z) = \mathbf{G}_{K-1}(z)\mathbf{G}_{K-2}(z)\cdots\mathbf{G}_1(z)\mathbf{E}_0 \tag{1}$$

where each propagation matrix $\mathbf{G}_k(z)$ (for $k = 1, \ldots, K-1$) and the initial matrix $\mathbf{E}_0$ can be expressed as

$$\mathbf{G}_k(z) = \frac{1}{2}\mathrm{diag}(\mathbf{U}_k, \mathbf{V}_k)\mathbf{W}\Lambda(z)\mathbf{W} \tag{2}$$

$$\mathbf{E}_0 = \frac{1}{\sqrt{2}}\mathrm{diag}(\mathbf{U}_0, \mathbf{V}_0)\hat{\mathbf{W}} \tag{3}$$

in which $\mathbf{U}_k$ and $\mathbf{V}_k$ (for $k = 0, \ldots, K-1$) are $m \times m$ general orthogonal matrices.

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In (1), each propagation matrix $\mathbf{G}_k(z)$ contains two $m \times m$ arbitrary orthogonal matrices $\mathbf{U}_k$ and $\mathbf{V}_k$. According to [8], the above structure can be simplified through the following trivial matrix manipulations. Note that the product $\mathbf{G}_{K-1}(z)\mathbf{G}_{K-2}(z)$ can be rewritten as

$$\begin{aligned}\mathbf{G}_{K-1}(z)\mathbf{G}_{K-2}(z) &= \frac{1}{2}\begin{bmatrix}\mathbf{U}_{K-1} & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_{K-1}\end{bmatrix}\begin{bmatrix}\mathbf{I}+z^{-1}\mathbf{I} & \mathbf{I}-z^{-1}\mathbf{I}\\ \mathbf{I}-z^{-1}\mathbf{I} & \mathbf{I}+z^{-1}\mathbf{I}\end{bmatrix}\\ &\times\frac{1}{2}\begin{bmatrix}\mathbf{U}_{K-2} & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_{K-2}\end{bmatrix}\begin{bmatrix}\mathbf{I}+z^{-1}\mathbf{I} & \mathbf{I}-z^{-1}\mathbf{I}\\ \mathbf{I}-z^{-1}\mathbf{I} & \mathbf{I}+z^{-1}\mathbf{I}\end{bmatrix}\\ &= \frac{1}{2}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_{K-1}\mathbf{U}_{K-1}^T\end{bmatrix}\begin{bmatrix}\mathbf{I}+z^{-1}\mathbf{I} & \mathbf{I}-z^{-1}\mathbf{I}\\ \mathbf{I}-z^{-1}\mathbf{I} & \mathbf{I}+z^{-1}\mathbf{I}\end{bmatrix}\\ &\times\frac{1}{2}\begin{bmatrix}\mathbf{U}_{K-1}\mathbf{U}_{K-2} & \mathbf{0}\\ \mathbf{0} & \mathbf{U}_{K-1}\mathbf{V}_{K-2}\end{bmatrix}\\ &\times\begin{bmatrix}\mathbf{I}+z^{-1}\mathbf{I} & \mathbf{I}-z^{-1}\mathbf{I}\\ \mathbf{I}-z^{-1}\mathbf{I} & \mathbf{I}+z^{-1}\mathbf{I}\end{bmatrix}. \end{aligned} \tag{4}$$

That is, the component $\mathrm{diag}(\mathbf{U}_{K-1}, \mathbf{U}_{K-1})$ can be moved across the delay chain and butterfly matrices to combine with $\mathrm{diag}(\mathbf{U}_{K-2}, \mathbf{V}_{K-2})$ in $\mathbf{G}_{K-2}(z)$. As $\mathbf{U}_{K-1}, \mathbf{U}_{K-2}$, and $\mathbf{V}_{K-2}$ are free orthogonal matrices, $\mathbf{U}_{K-1}\mathbf{U}_{K-2}$ and $\mathbf{U}_{K-1}\mathbf{V}_{K-2}$ are also free orthogonal matrices. Thus, $\mathbf{U}_{K-1}$ can be set to $\mathbf{I}$ without loss of generality. By iteratively applying the above manipulations to $\mathbf{G}_k(z)\mathbf{G}_{k-1}(z)$ (for $k = K-1, \ldots, 1$), all $\mathbf{U}_k$ (except $\mathbf{U}_0$) can be removed, and consequently, $\mathbf{G}_k(z)$ can be replaced by $\mathbf{G}'_k(z)$ as follows:

$$\mathbf{G}'_k(z) = \frac{1}{2}\mathrm{diag}(\mathbf{I}, \mathbf{V}_k)\mathbf{W}\Lambda(z)\mathbf{W}. \tag{5}$$

After such a modification, the design space remains the same, while the complexity for both design and implementation can be reduced by nearly 50%.

B. PMI-LPPUFBs

In this paper, we aim to extend the above simplification to LPPUFBs with the PMI property, where the analysis filters $H_i(z)$ (for $i = 0, \ldots, m-1$) satisfy [2]

$$H_{M-1-i}(z) = H_i(-z). \tag{6}$$

It was proved in [2] that this property can be obtained by restricting $\mathbf{U}_k$ in (2) to be

$$\mathbf{U}_k = \begin{cases}\mathbf{\Gamma}\mathbf{V}_k\mathbf{\Gamma}, & k = 0, \ldots, K-2\\ \mathbf{J}\mathbf{V}_k\mathbf{\Gamma}, & k = K-1\end{cases} \tag{7}$$

in which $\mathbf{\Gamma}$ is a diagonal matrix whose diagonal entries are $\mathbf{\Gamma}(l, l) = (-1)^l$, for $l = 0, \ldots, m-1$. With this restriction, each propagation matrix contains one $m \times m$ arbitrary orthogonal matrix $\mathbf{V}_k$. As we have said, only one free matrix is needed for each propagation building block in general LPPUFBs. By adding the PMI constraint, fewer parameters should be needed, which implies that the structure for PMI-LPPUFBs in (1) can be further simplified.

III. Simplified Structure for PMI-LPPUFBs

In this section, by slightly modifying the simplification for general LPPUFBs, we first arrive at a new representation of (1), where each order-one building block contains one special orthogonal matrix. The cosine–sine (C–S) decomposition [11] is then exploited to parameterize the special orthogonal matrix, which leads to significant parameter reduction.

A. New and Equivalent Factorization

In PMI systems, due to the relation of $\mathbf{U}_k$ and $\mathbf{V}_k$, we can not simply discard $\mathbf{U}_k$ as in general LPPUFBs. Thus, we need to slightly modify the manipulation in (4). Note that since $\mathbf{V}_{K-1}$ is an orthogonal matrix, we have

$$\mathbf{U}_{K-1}\mathbf{V}_{K-2} = \mathbf{U}_{K-1}\mathbf{V}_{K-1}^T \cdot \mathbf{V}_{K-1}\mathbf{V}_{K-2}. \tag{8}$$

Substituting (8) into (4), we can thus reformulate $\mathbf{G}_{K-1}(z)\mathbf{G}_{K-2}(z)$ into

$$\begin{aligned}\mathbf{G}_{K-1}(z)\mathbf{G}_{K-2}(z) &= \frac{1}{2}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_{K-1}\mathbf{U}_{K-1}^T\end{bmatrix}\begin{bmatrix}\mathbf{I}+z^{-1}\mathbf{I} & \mathbf{I}-z^{-1}\mathbf{I}\\ \mathbf{I}-z^{-1}\mathbf{I} & \mathbf{I}+z^{-1}\mathbf{I}\end{bmatrix}\\ &\times\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{U}_{K-1}\mathbf{V}_{K-1}^T\end{bmatrix}\\ &\times\frac{1}{2}\begin{bmatrix}\mathbf{U}_{K-1}\mathbf{U}_{K-2} & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_{K-1}\mathbf{V}_{K-2}\end{bmatrix}\\ &\times\begin{bmatrix}\mathbf{I}+z^{-1}\mathbf{I} & \mathbf{I}-z^{-1}\mathbf{I}\\ \mathbf{I}-z^{-1}\mathbf{I} & \mathbf{I}+z^{-1}\mathbf{I}\end{bmatrix}. \end{aligned} \tag{9}$$

Namely, the component $\mathrm{diag}(\mathbf{U}_{K-1}, \mathbf{V}_{K-1})$, instead of $\mathrm{diag}(\mathbf{U}_{K-1}, \mathbf{U}_{K-1})$, is merged with $\mathrm{diag}(\mathbf{U}_{K-2}, \mathbf{V}_{K-2})$. Since $\mathbf{U}_{K-1} = \mathbf{J}\mathbf{V}_{K-1}\mathbf{\Gamma}$ and $\mathbf{U}_{K-2} = \mathbf{\Gamma}\mathbf{V}_{K-2}\mathbf{\Gamma}$, one can derive that

$$\mathbf{U}_{K-1}\mathbf{U}_{K-2} = \mathbf{J}\mathbf{V}_{K-1}\Gamma \cdot \Gamma\mathbf{V}_{K-2}\Gamma = \mathbf{J}\mathbf{V}_{K-1}\mathbf{V}_{K-2}\Gamma. \tag{10}$$

This implies that the relation of $\mathbf{U}_{K-1}$ and $\mathbf{V}_{K-1}$ is preserved in $\mathbf{U}_{K-1}\mathbf{U}_{K-2}$ and $\mathbf{V}_{K-1}\mathbf{V}_{K-2}$. Likewise, the component $\mathrm{diag}(\mathbf{U}_{K-1}\mathbf{U}_{K-2}, \mathbf{V}_{K-1}\mathbf{V}_{K-2})$ can be combined with $\mathrm{diag}(\mathbf{U}_{K-3}, \mathbf{V}_{K-3})$. Repeating this process for $K-1$ times, we can get a new and equivalent structure of (1) as follows:

$$\mathbf{E}(z) = \mathbf{G}'_{K-1}(z)\mathbf{G}'_{K-2}(z)\cdots\mathbf{G}'_1(z)\mathbf{E}'_0 \tag{11}$$

$$\mathbf{G}'_k(z) = \frac{1}{2}\mathrm{diag}(\mathbf{I}, \mathbf{X}_k)\mathbf{W}\Lambda(z)\mathbf{W}\mathrm{diag}\left(\mathbf{I}, \mathbf{X}_k^T\right) \tag{12}$$

$$\mathbf{E}'_0 = \frac{1}{\sqrt{2}}\mathrm{diag}(\mathbf{U}'_0, \mathbf{V}'_0)\hat{\mathbf{W}} \tag{13}$$

where each $\mathbf{X}_k$ (for $k = 1, \ldots, K-1$) in $\mathbf{G}'_k(z)$ is expressed as

$$\mathbf{X}_k = \mathbf{V}'_k\mathbf{U}'^T_k \tag{14}$$

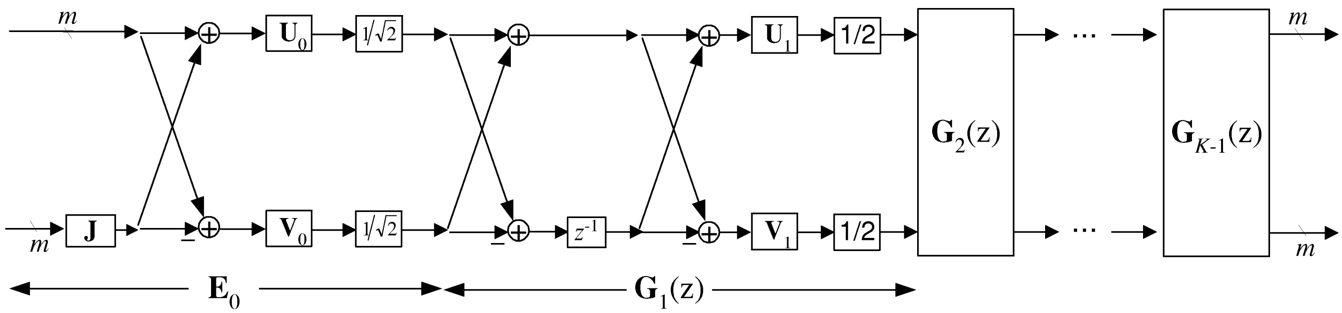


Fig. 1. Implementation of two equivalent structure for PMI-LPPUFBs. (a) Structure of (1). (b) Simplified structure in (11).

in which

$$\mathbf{U}'_k = \prod_{i=K-1}^{k} \mathbf{U}_i, \quad \mathbf{V}'_k = \prod_{i=K-1}^{k} \mathbf{V}_i, \quad k = 1, \ldots, K-1. \tag{15}$$

Besides, in $\mathbf{E}'_0, \mathbf{U}'_0$, and $\mathbf{V}'_0$ are

$$\mathbf{U}'_0 = \prod_{i=K-1}^{0} \mathbf{U}_i \qquad \mathbf{V}'_0 = \prod_{i=K-1}^{0} \mathbf{V}_i. \tag{16}$$

The implementations of (1) and (11) are shown in Fig. 1(a) and (b), respectively. It is clear that these two structures have the same implementation complexity. However, unlike $\mathbf{V}_k$ in $\mathbf{G}_k(z), \mathbf{X}_k$ in $\mathbf{G}'_k(z)$ is *not* an arbitrary orthogonal matrix. The relation between $\mathbf{U}_k$ and $\mathbf{V}_k$ in (7) imposes certain constraints on $\mathbf{X}_k$. Note that following the same approach as that in (10), we have

$$\begin{aligned}\mathbf{U}'_k &= \prod_{i=K-1}^{k} \mathbf{U}_i \\ &= \mathbf{J}\mathbf{V}_{K-1}\mathbf{\Gamma} \prod_{i=K-2}^{k} \mathbf{\Gamma}\mathbf{V}_i\mathbf{\Gamma} \\ &= \mathbf{J} \prod_{i=K-1}^{k} \mathbf{V}_i \cdot \mathbf{\Gamma} = \mathbf{J}\mathbf{V}'_k\mathbf{\Gamma}.\end{aligned} \tag{17}$$

Then, substituting (17) into (14) yields

$$\mathbf{X}_k = \mathbf{V}'_k\mathbf{U}'^T_k = \mathbf{V}'_k\mathbf{\Gamma}\mathbf{V}'^T_k\mathbf{J} = \mathbf{X}'_k\mathbf{J} \tag{18}$$

in which

$$\mathbf{X}'_k = \mathbf{V}'_k\mathbf{\Gamma}\mathbf{V}'^T_k. \tag{19}$$

One can verify that $\mathbf{X}'_k$ in $\mathbf{X}_k$ is a special symmetric orthogonal matrix, i.e., $\mathbf{X}'_k = \mathbf{X}'^T_k$. Obviously, $\mathbf{X}_k$ should contain fewer degrees of design freedom than the general orthogonal matrix $\mathbf{V}_k$. Section III-B will discuss the parameterization of $\mathbf{X}_k$.

B. Matrix Parameterization

Let $m_0 = \lceil m/2 \rceil$ and $m_1 = \lfloor m/2 \rfloor$. As $\mathbf{\Gamma}$ has m_0 1's and $m_1(-1)$'s, there exists a permutation matrix $\mathbf{P}$ such that $\mathbf{P}\mathbf{\Gamma}\mathbf{P}^T = \mathrm{diag}(\mathbf{I}_{m_0}, -\mathbf{I}_{m_1})$. Define a new orthogonal matrix $\hat{\mathbf{V}}_k = \mathbf{V}'_k\mathbf{P}^T$, i.e., $\mathbf{V}'_k = \hat{\mathbf{V}}_k\mathbf{P}$. From (18), $\mathbf{X}_k$ can be written into

$$\begin{aligned}\mathbf{X}_k &= \mathbf{V}'_k\mathbf{\Gamma}\mathbf{V}'^T_k\mathbf{J} \\ &= \hat{\mathbf{V}}_k\mathbf{P}\mathbf{\Gamma}\mathbf{P}^T\hat{\mathbf{V}}^T_k\mathbf{J} \\ &= \hat{\mathbf{V}}_k\mathrm{diag}\left(\mathbf{I}_{m_0}, -\mathbf{I}_{m_1}\right)\hat{\mathbf{V}}^T_k\mathbf{J}.\end{aligned} \tag{20}$$

With this new form of $\mathbf{X}_k$, the parameterization can be obtained by using the C-S decomposition of $\hat{\mathbf{V}}_k$ [11]. Recall that in the C-S decomposition [11], $\hat{\mathbf{V}}_k$ can be completely characterized as

$$\hat{\mathbf{V}}_k = \mathrm{diag}(\mathbf{Y}_{k,0}, \mathbf{Y}_{k,1})\Sigma_k\mathrm{diag}\left(\mathbf{Z}^T_{k,0}, \mathbf{Z}^T_{k,1}\right) \tag{21}$$

where $\mathbf{Y}_{k,0}$ and $\mathbf{Z}_{k,0}$ are $m_0 \times m_0$ general orthogonal matrices, while $\mathbf{Y}_{k,1}$ and $\mathbf{Z}_{k,1}$ are $m_1 \times m_1$ general orthogonal matrices and Σ_k is an $m \times m$ special orthogonal matrix defined as

$$\Sigma_k = \begin{cases}\begin{bmatrix}\mathbf{C}_k & -\mathbf{S}_k \\ \mathbf{S}_k & \mathbf{C}_k\end{bmatrix} & m \text{ even} \\ \begin{bmatrix}1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_k & -\mathbf{S}_k \\ \mathbf{0} & \mathbf{S}_k & \mathbf{C}_k\end{bmatrix} & m \text{ odd}\end{cases} \tag{22}$$

in which $\mathbf{C}_k$ and $\mathbf{S}_k$ are $m_1 \times m_1$ diagonal matrices with diagonal entries $\mathbf{C}_k(l,l) = \cos\alpha_{k,l}$ and $\mathbf{S}_k(l,l) = \sin\alpha_{k,l}$ (for $l = 0, \ldots, m_1 - 1$ and $0 \le \alpha_{k,l} \le \pi/2$). Substituting (21) into (20) produces

$$\begin{aligned}\mathbf{X}_k = &\,\mathrm{diag}(\mathbf{Y}_{k,0}, \mathbf{Y}_{k,1})\Sigma_k\mathrm{diag}\left(\mathbf{Z}^T_{k,0}, \mathbf{Z}^T_{k,1}\right) \\ &\times \mathrm{diag}\left(\mathbf{I}_{m_0}, -\mathbf{I}_{m_1}\right)\mathrm{diag}(\mathbf{Z}_{k,0}, \mathbf{Z}_{k,1})\Sigma^T_k \\ &\times \mathrm{diag}\left(\mathbf{Y}^T_{k,0}, \mathbf{Y}^T_{k,1}\right)\mathbf{J}.\end{aligned} \tag{23}$$

Then, taking advantage of the fact that

$$\begin{aligned}&\mathrm{diag}\left(\mathbf{Z}^T_{k,0}, \mathbf{Z}^T_{k,1}\right)\mathrm{diag}\left(\mathbf{I}_{m_0}, -\mathbf{I}_{m_1}\right)\mathrm{diag}(\mathbf{Z}_{k,0}, \mathbf{Z}_{k,1}) \\ &\qquad = \mathrm{diag}\left(\mathbf{I}_{m_0}, -\mathbf{I}_{m_1}\right)\end{aligned} \tag{24}$$

we can further reduce $\mathbf{X}_k$ to

$$\begin{aligned}\mathbf{X}_k = &\,\mathrm{diag}(\mathbf{Y}_{k,0}, \mathbf{Y}_{k,1})\Sigma_k\mathrm{diag}\left(\mathbf{I}_{m_0}, -\mathbf{I}_{m_1}\right)\Sigma^T_k \\ &\times \mathrm{diag}\left(\mathbf{Y}^T_{k,0}, \mathbf{Y}^T_{k,1}\right)\mathbf{J} \\ = &\,\mathrm{diag}(\mathbf{Y}_{k,0}, \mathbf{Y}_{k,1})\Sigma'_k\mathrm{diag}\left(\mathbf{Y}^T_{k,0}, \mathbf{Y}^T_{k,1}\right)\mathbf{J}\end{aligned} \tag{25}$$

TABLE I
NUMBER OF FREE PARAMETERS IN AN M-CHANNEL ($M = 2m$) PMI-LPPUFB WITH FILTER LENGTH OF $L = KM$

Parameters	Rotation angles		Signs
	m even	m odd	
Original structure of (1)	$\frac{Km(m-1)}{2}$	$\frac{Km(m-1)}{2}$	Km
Our structure of (11)	$\frac{(K-1)m^2}{4} + \frac{m(m-1)}{2}$	$\frac{(K-1)(m^2-1)}{4} + \frac{m(m-1)}{2}$	m
Difference	$\frac{(K-1)(m^2-2m)}{4}$	$\frac{(K-1)(m^2-m)}{4}$	$(K-1)m$

where $\Sigma'_k = \Sigma_k \mathrm{diag}(\mathbf{I}_{m_0}, -\mathbf{I}_{m_1})\Sigma_k^T$ is given by

$$\Sigma'_k = \begin{cases} \begin{bmatrix} \mathbf{C}'_k & \mathbf{S}'_k \\ \mathbf{S}'_k & -\mathbf{C}'_k \end{bmatrix} & m \text{ even} \\ \begin{bmatrix} 1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}'_k & \mathbf{S}'_k \\ \mathbf{0} & \mathbf{S}'_k & -\mathbf{C}'_k \end{bmatrix} & m \text{ odd} \end{cases} \tag{26}$$

in which $\mathbf{C}'_k$ and $\mathbf{S}'_k$ are two diagonal matrices whose entries are $\mathbf{C}'_k(l,l) = \cos\alpha'_{k,l}$ and $\mathbf{S}'_k(l,l) = \sin\alpha'_{k,l}$, respectively, with $\alpha'_{k,l} = 2\alpha_{k,l}(0 \le \alpha'_{k,l} \le \pi)$. As a result, we arrive at the following Theorem.

Theorem 1: Any $\mathbf{E}(z)$ in (1) can be always written as in (11), where each order-one building block $\mathbf{G}'_k(z)$ and the initial matrix $\mathbf{E}'_0$ are shown in (12) and (13), respectively. In $\mathbf{G}'_k(z)$, each $m \times m$ orthogonal matrix $\mathbf{X}_k$ can be represented as in (25). While in $\mathbf{E}'_0, \mathbf{V}'_0$ is an $m \times m$ arbitrary orthogonal matrix, and $\mathbf{U}'_0$ can be expressed as $\mathbf{U}'_0 = \mathbf{J}\mathbf{V}'_0\mathbf{\Gamma}$.

Now, let us compare the degrees of design freedom of (1) and (11). It can be readily seen that $\mathbf{E}_0$ and $\mathbf{E}'_0$ hold the same design freedom since they have one free orthogonal matrix $\mathbf{V}_0$ and $\mathbf{V}'_0$, respectively. The difference lies in $\mathbf{G}_k(z)$ and $\mathbf{G}'_k(z)$. In $\mathbf{G}_k(z)$ of (1), each $\mathbf{V}_k$ requires $n = \binom{m}{2} = m(m-1)/2$ rotation angles and m sign parameters for a complete parameterization. On the other hand, in $\mathbf{G}'_k(z)$, (25) implies that $\mathbf{X}_k$ can be fully characterized by two general orthogonal matrices $\mathbf{Y}_{k,i}$ $(i = 0, 1)$ and one special orthogonal matrix Σ'_k. As $\mathbf{Y}_{k,0}, \mathbf{Y}_{k,1}$, and Σ'_k contain $\binom{m_0}{2}, \binom{m_1}{2}$, and m_1 rotation angles, respectively, the total number of free rotation angles in $\mathbf{X}_k$ is $n' = \binom{m_0}{2} + \binom{m_1}{2} + m_1$. One can calculate that $n' = m^2/4$ for even m and $n' = (m^2-1)/4$ for odd m. Besides, in (25), if each $\alpha'_{k,l}$ is allowed to take arbitrary values in parameterization, the m sign parameters in $\mathbf{Y}_{k,i}(i = 0, 1)$ can be discarded. Based on the above analysis, the number of free parameters in each structure is listed in Table I.

From Table I, one can get the following conclusions.

- When $K = 1$, same number of parameters is required in each structure.
- When $K > 1$ and $m = 2$, both structures have the same number of rotation angles, but our structure contains fewer sign parameters than the original structure.
- When $K > 1$ and $m > 2$, our structure gains a reduction in both rotation angles and sign parameters. For a fixed M (thus, fixed m), the differences in both rotation angles and sign parameters are linear to $(K-1)$. For a fixed K, the difference in rotation angles is a quadratic function of m, while that of sign parameters is linear to m.

Summarizing, in our new structure of (11), the design complexity is much less than that in (1), especially for large number of channels and long filters.

IV. DESIGN EXAMPLE

This section presents a design example of PMI-LPPUFB with $M = 8$ and $K = 5$. The chosen criterion is a weighted combination of the coding gain, the dc leakage and the stopband attenuation as follows:

$$C = 0.1C_{\text{coding gain}} + 0.2C_{\text{DC}} + 0.7C_{\text{stop}}. \tag{27}$$

To be more specific, $C_{\text{coding gain}}$ represents the coding gain, which is given by [12]

$$C_{\text{coding gain}} = 10\log_{10} \frac{\sigma_s^2}{\left(\prod_{i=0}^{M-1} \sigma_{v_i}^2\right)^{1/M}} \tag{28}$$

where σ_s^2 and $\sigma_{v_k}^2$ are the variances of the input signal and the i-th subband signal, respectively. Following the convention, the signal in use is an AR(1) process with a correlation coefficient $\rho = 0.95$. The dc leakage C_{dc} is defined as

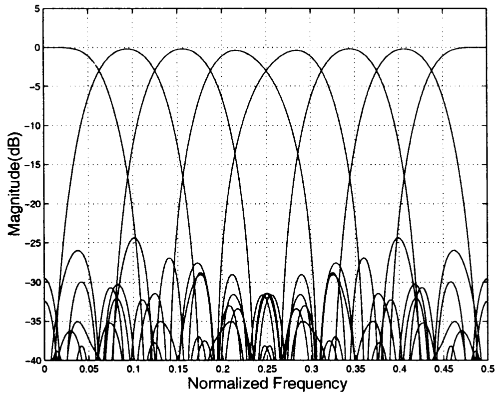
$$C_{\text{dc}} = \sum_{i=1}^{M/2-1} |H_{2i}(1)|. \tag{29}$$

Since $H_{2i+1}(z)$ are antisymmetric, their frequency responses are equal to zeros at $z = 1$. Thus, only even-numbered filters are considered in (29). C_{stop} is the sum of stopband energy of $H_i(z)$ (for $i = 0, \ldots, M/2-1$) as follows:

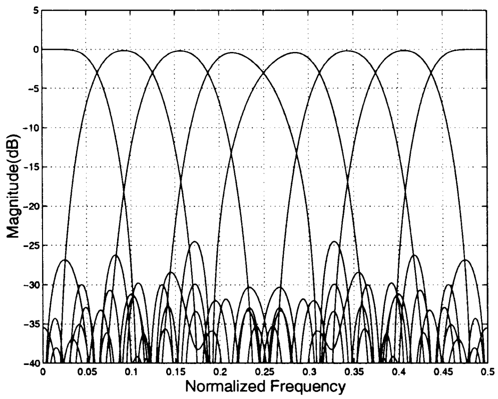
$$C_{\text{stop}} = 2\sum_{i=0}^{M/2-1} \int_{\omega\in\Omega_i} |H_i(e^{j\omega})|^2 d\omega \tag{30}$$

where $\Omega_i = [0, \omega_{i,L}] \cup [\omega_{i,H}, \pi]$ denotes $H_i(z)$'s stopband, with $\omega_{i,L}$ and $\omega_{i,H}$ given by $\omega_{i,L} = \max(0, (i-0.6)\pi/M)$ and $\omega_{i,H} = \min(\pi, (i+1.6)\pi/M)$, respectively. Note that due to the PMI property, only the first half filters are included in (30).

The design was carried out for both (1) and (11) through the Matlab function *fminunc* in the *optimization* toolbox. From Table I, it can be calculated that the number of free rotation angles in (1) and (11) are 30 and 22, respectively. For simplicity, all the sign parameters are set to be ones. To have a fair comparison of both structures, the initial values are chosen so that the optimizations start from $\mathbf{E}(z) = z^{-2}\mathbf{WHT}$, in which $\mathbf{WHT}$



(a)



(b)

Fig. 2. Optimized PMI-LPPUFBs with $M = 8$ and $K = 5$. (a) Results of (1). (b) Results of (11).

denotes the 8×8 Walsh–Hadamard transform. To be more specific, both $\mathbf{E}_0$ and $\mathbf{E}'_0$ are chosen as $\mathbf{E}_0 = \mathbf{E}'_0 = \mathbf{WHT}$. For the order-one propagation matrix, the free matrix $\mathbf{V}_k$ in $\mathbf{G}_k(z)$ (for $k = 1, \ldots, 4$) are set to be $\mathbf{V}_k = \mathbf{J}$. While in $\mathbf{G}'_k(z), \mathbf{X}_k = (-1)^k\mathbf{I}$. These $\mathbf{X}_k$ can be obtained by setting $\mathbf{Y}_{k,0} = \mathbf{I}, \mathbf{Y}_{k,1} = \mathbf{J}$, and $\alpha'_{k,l} = (-1)^k(\pi/2)$.

On a Pentium IV, 1.9-GHz computer, it turns out that the optimization time for (1) and (11) are 23 and 16 s, respectively. Fig. 2(a) and (b) show the frequency responses of the optimized PMI-LPPUFBs based on (1) and (11), respectively. Table II documents the numerical results. One can see that the PMI-LPPUFB based on our structure outperforms that of the original lattice in all accounts. Thus, by discarding redundant parameters, our lattice structure can not only speed up the optimization, but also more effectively avoid being trapped in a local minimum.

TABLE II
COMPARISON OF TWO OPTIMIZED PMI LPPPUFBs

Filter Banks	Results of (1)	Results of (11)
Coding Gain	9.3587dB	9.3856 dB
DC Attenuation	-29.515 dB	-35.4457dB
Stopband Attenuation	-24.3415dB	-24.4712 dB

V. Conclusion

This paper proposes a new structure for PMI-LPPUFBs with even-channel, which is a simplified version of the lattice in [2]. The new structure spans the same class of PMI-LPPUFBs as the original lattice, while the number of free parameters is significantly reduced. Through this way, better results with faster convergence in the optimization can be achieved. A design example is presented to demonstrate the efficiency of the proposed lattice structure.

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