Correction and Comment on "An Adaptive Resolution Asynchronous ADC Architecture for Data Compression in Energy Constrained Sensing Applications"

Weidong Kuang

***Abstract*—This comment points out an error in a well accepted fundamental signal-to-noise ratio (SNR) equation for asynchronous ADCs, and presents a correct SNR equation. Meanwhile, a wrong concept of calculating mean-related quantities for non-uniform sampling analysis is corrected. The result is validated by MATLAB simulation.**

***Index Terms*—Level-crossing analog-to-digital converter (ADC), signal-to-noise ratio (SNR).**

I. Introduction

Recently, asynchronous ADCs, also referred to as level-crossing or continuous-time ADCs, have attracted much attention due to their potential applications with burst-like signals, such as ECG/EEG monitoring, implantable biomedical devices and distributed sensors. In contrast to conventional ADCs where input signal is sampled at a uniform rate and the amplitude is approximated by a quantizer, asynchronous ADCs sample input signal only when the signal crosses predefined threshold levels. Therefore, the quantization error for asynchronous ADCs is theoretically equal to zero. However, the time information for sampled points is required for signal reconstruction from the sampled data. For example, a counter with clock resolution Tclk is typically employed to measure the time between two consecutive samples. Thus the finite resolution of the clock introduces quantization in time. This time quantization noise can be transformed to an equivalent amplitude quantization noise. In [1] and [2], both authors derived the equivalent signal-to-noise ratio (SNR) when a sinusoidal input signal is applied to asynchronous ADCs. They obtained the same erroneous result. This comment points out the mistake, and gives a correct SNR equation which is verified by MATLAB simulation.

II. Error and Correction

The SNR of ADCs output can be calculated by transforming the quantization error in time to an error in amplitude. This is illustrated in Fig. 1, where an input signal crosses an amplitude threshold level Vj. Quantization of the time axis with resolution $\mathrm{T_{clk}}$ results in a time error equal to $\delta\ \mathrm{t_i}$, and as a result, an output sample pair $(\mathrm{t_i}, \mathrm{y_i})$ is generated, i.e., the amplitude $\mathrm{y_i} = \mathrm{Vj}$ is considered as the value at time $\mathrm{t_i} = \mathrm{iT_{clk}}$ (i is an integer). Thus an amplitude error $\delta\ \mathrm{y_i}$ at sample pair $(\mathrm{t_i}, \mathrm{y_i})$ is introduced due to $\delta\ \mathrm{t_i}$.

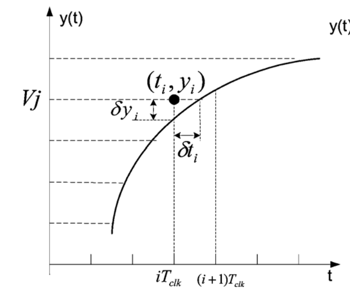

Fig. 1. Error due to quantization of time.

A prevalent, but wrong, equation for Signal (y(ti))-to-Noise $(\delta\mathrm{y_i})$ Ratio was derived in [1], [2] as

$$\mathrm{SNR} = 20\log R - 11.2\ (\mathrm{dB}) \tag{1}$$

where R is the counter resolution ratio, which is equal to the ratio of the equivalent counter frequency $(1/\mathrm{T_{clk}})$ to the input signal frequency, assuming input signal is a sinusoidal signal.

In this comment, a correct equation is given as

$$\mathrm{SNR} = 20\log R - 14.2\ (\mathrm{dB}) \tag{2}$$

The 3 dB difference between (1) and (2) stems from a wrong concept of calculating mean for non-uniform sampled signals. In order to validate the SNR equation (2), we define the SNR of non-uniformly sampled data first, then derive the theoretical SNR for sampling frequency approaching infinite, and finally simulate the level-crossing sampling using MATLAB.

The non-uniformly sampled data $\{\mathrm{y_i}\}, i = 0, 1 \ldots \mathrm{N}-1$ includes two components: signal $\mathrm{y(t_i)}$ and error $\delta\mathrm{y_i}$. i.e.,

$$y_i = y(t_i) + \delta y_i \tag{3}$$

Thus the signal energy can be defined as

$$E_{\mathrm{signal}} = \sum_{i=0}^{N-1} |y(t_i)|^2 \tag{4}$$

The energy of amplitude error $\{\delta \mathrm{yi}\} i = 0, 1 \ldots \mathrm{N}-1$ can be calculated by

$$E_{\mathrm{noise}} = \sum_{i=0}^{N-1} |\delta y_i|^2 \tag{5}$$

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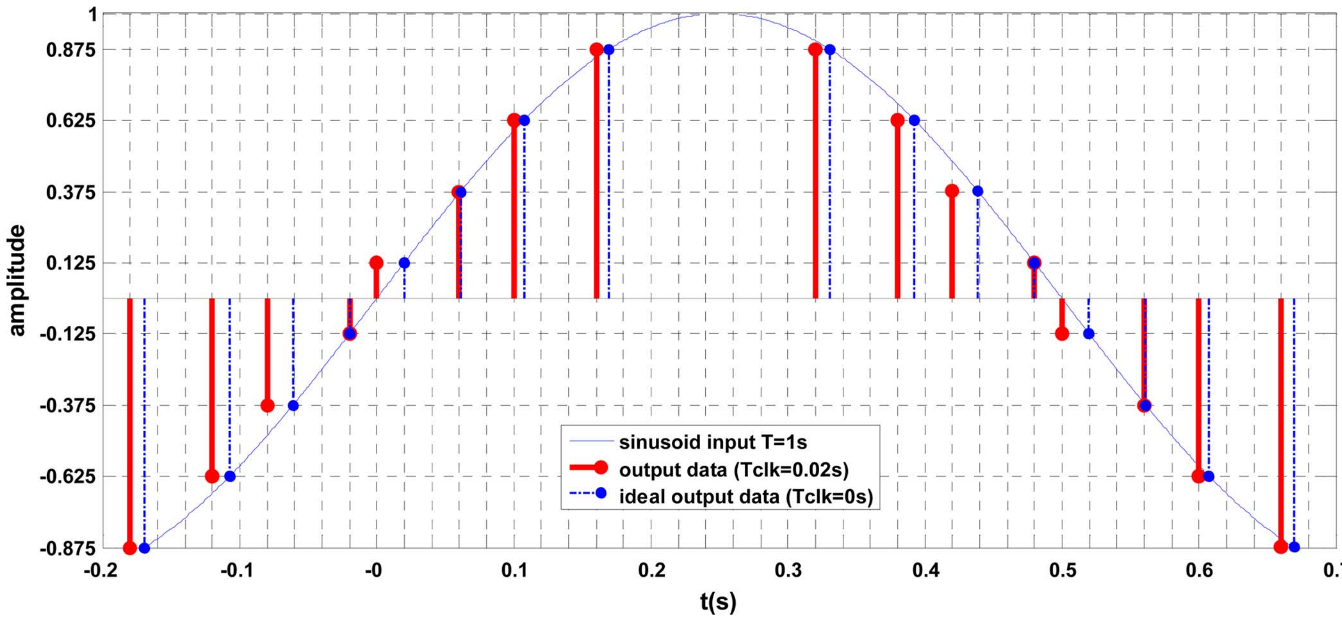

Fig. 2. The output of a 3-bit asynchronous ADC for a sinusoidal signal (MATLAB simulation).

Therefore, the signal-to-noise (SNR) can be calculated, based on discrete samples, as

$$\mathrm{SNR} = 10\log\left(\frac{E_{\mathrm{signal}}}{E_{\mathrm{noise}}}\right) = 10\log\left(\frac{\sum_{i=0}^{N-1}|y(t_i)|^2}{\sum_{i=0}^{N-1}|\delta y_i|^2}\right) \tag{6}$$

To derive (2), the following assumptions are made: 1) input signal is sinusoidal signal $y(t) = A\cdot\sin(2\pi f_s t)$, 2) the amplitude of input signal is within the amplitude quantization range, and 3) time quantization error $\delta \mathrm{t_i}$ is uniformly distributed in the interval $(0, \mathrm{T_{clk}}]$ [3]. When $\mathrm{T_{clk}}$ is small enough, the equivalent error can be approximated as

$$\delta y_i \approx s\cdot\delta t_i \tag{7}$$

where s is the slope (derivative of the input signal), and $s = A\cdot 2\pi\cdot f_s\cdot\cos(2\pi f_s t)$.

The average power of $\delta \mathrm{t_i}$ with uniform distribution in $(0, \mathrm{T_{clk}}]$ is given by

$$E[(\delta t_i)^2] = \frac{T_{\mathrm{clk}}^2}{3} \tag{8}$$

By assuming that $\delta \mathrm{t_i}$ is independent of signal slope s, the average power of quantization error $\delta \mathrm{y_i}$ can be calculated as

$$P_{\mathrm{noise}} = E[(\delta y_i)^2] = E[s^2]\cdot E[(\delta t_i)^2] = E[s^2]\cdot\frac{T_{\mathrm{clk}}^2}{3} \tag{9}$$

To address the continuous counterpart of a discrete sum or average, an integral is usually applied to replace the discrete sum operation. For traditional uniform sampling, the integral is performed along with time axis.

However, for level-crossing sampling, the variable of integral has to be amplitude, by which the signal is uniformly sampled. Therefore, the mean of slope square $\mathrm{E[s^2]}$ should be calculated by integral with respect to amplitude as

$$\begin{aligned} E[s^2] &= \frac{1}{2A}\cdot\int_{-A}^{A}[s(x)]^2dx \\ &= \frac{1}{2A}\cdot\int_{-A}^{A}\left[A\cdot 2\pi f_s\cdot\cos\left(\arcsin\left(\frac{x}{A}\right)\right)\right]^2dx \\ &= \frac{8}{3}\cdot\pi^2 f_s^2 A^2 \end{aligned} \tag{10}$$

For the same reason, the average power of signal y(t) is calculated as

$$\begin{aligned} P_{\mathrm{signal}} &= E[y^2] \\ &= \frac{1}{2A}\cdot\int_{-A}^{A}\left[A\cdot\sin\left(\arcsin\left(\frac{x}{A}\right)\right)\right]^2dx \\ &= \frac{1}{2A}\cdot\int_{-A}^{A}x^2dx = \frac{A^2}{3} \end{aligned} \tag{11}$$

It is interesting to note that the average power of a sinusoidal signal is equal to $(A^2)/(3)$, not $(A^2)/(2)$ which is a well-known result for uniform sampling with sample frequency approaching infinite. This smaller power $(A^2/3 < A^2/2)$ can be visually explained by the fact that smaller values are sampled more frequently for sinusoidal signal, illustrated in Fig. 2. The authors in [1], [2] cited erroneous equations for $\mathrm{E[s^2]}$ and $\mathrm{P_{signal}}$ from [3] where the integrals were performed along with time axis without noticing the effect of non-uniform sampling.

By combining (9), (10), (11), the signal-to-noise ratio (SNR) can be written as

$$\begin{aligned} \mathrm{SNR} &= 10\log\left(\frac{P_{\mathrm{signal}}}{P_{\mathrm{noise}}}\right) = 10\log\left(\frac{\frac{A^2}{3}}{\frac{8}{3}\pi^2 f_s^2 A^2\cdot\frac{T_{\mathrm{clk}}^2}{3}}\right) \\ &= 10\log\left(\frac{R^2}{\frac{8}{3}\pi^2}\right) = 20\log(R) - 14.2\ (\mathrm{dB}) \end{aligned} \tag{12}$$

where $R = (1/T_{\mathrm{clk}})/(f_s)$.

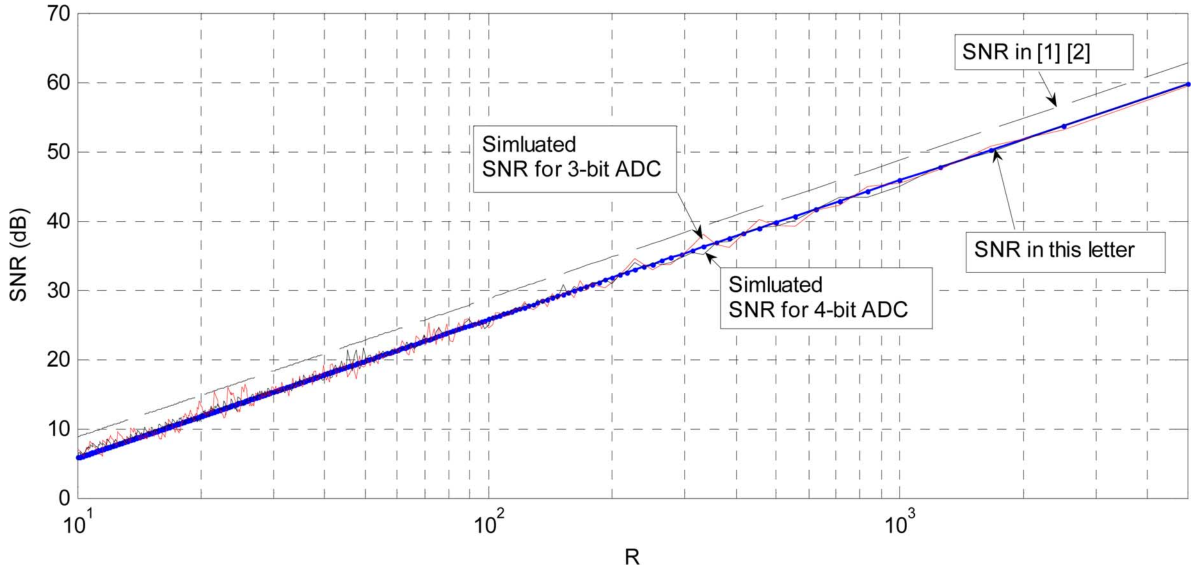

Fig. 3. Comparison of theoretical SNR and simluated SNR.

To validate (2) (or (12)), the level-crossing sampling process has been simulated using MATLAB. Fig. 2 visualizes the output sampled data for a 1 Hz sinusoidal input signal applied to a 3-bit asynchronous ADC with a timer resolution $\mathrm{T_{clk}} = 0.02$ s. Based on the waveforms in Fig. 2, the SNR can be calculated using (6). As a result, the SNRs are plotted versus R in Fig. 3 by changing $\mathrm{T_{clk}}$. The theoretical SNR (2) matches the simulated SNR for both 3-bit ADC and 4-bit ADC while the SNR equation (1) from [1], [2] over-estimates the SNR by 3 dB.

When R is smaller, a coarse measurement of the counter leads to a larger random variation from the theoretical value defined by (12). The 4-bit DAC delivers more data samples than 3-bit DAC, thus resulting in more data for average operations. The SNR curve of 4-bit DAC is closer to the theoretical SNR than 3-bit DAC, but this difference is much less significant than the effect of R.

III. Conclusion

This comment corrects an error in a prevalent fundamental equation for calculating quantization SNR in level-crossing ADCs. It is equally important to note that the classical method of mean calculation, such as average power or average slope of a continuous signal, through integrals with time, can not be applied to non-uniform sampling analysis. For level-crossing sampling, the related calculation can be achieved through integrals over amplitude since the signal is uniformly sampled in amplitude axis. The erroneous SNR equation in [1], [2] originated from a wrong way in [3] of calculating average slope and average power of a continuous signal when non-uniformly sampled, and this erroneous equation will continue to propagate [4] unless it is corrected.

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