Reviews of Current Literature

This section of each issue is devoted to a few rather extensive reviews of recent papers related to circuit theory and published in journals other than the PROCEEDINGS. Suggested titles and reprints of papers for review are invited by the committee.

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The Complete Specification of a Network by a Single Parameter—M. S. Corrington, T. Murakami, and R. W. Sonnenfeldt. (*RCA Rev.*, vol. 15, pp. 389–444; September, 1954.)

This paper is concerned with the transfer function and transient response of a linear, stable, two-terminal-pair network. The major points are: 1) transfer gain or phase alone does not determine the transfer function or the transient response; 2) the real or imaginary part of the transfer function is a much more nearly complete specification; 3) the real part of the transfer function, the imaginary part of the transfer function, and the step-function response are simply related; 4) graphical computation of one from another is convenient and acceptably accurate; 5) relatively simple laboratory equipment that will present real or imaginary part versus frequency, or real part versus imaginary part with frequency as a parameter, is useful in network development. This material, with the exception of item 5, is not wholly new, but it is nonetheless important, and the present article provides a valuable recapitulation of otherwise scattered developments and accompanies it with useful graphs, illustrative examples, and circuit diagrams. The restrictions on the network are stated in terms of its transient response, a convenience analytically in treating distributed networks. Fourier integral representations are used.

Under quite general restrictions a stable network possesses a well-behaved transfer function free from singularities in a half-plane. Under these conditions the real and imaginary parts are Hilbert transforms of one another. Computation of the pairs in which one member is a finite line segment (Bode's nomenclature) once for all permits approximation of other pairs by graphical addition. This analysis is essentially similar to that of Bayard, Weiner and Lee, and Bode. There is, as expected, an additive constant to be determined in going from the imaginary part to the real part.

If this constant and the imaginary part, or the real part, is known, the transfer function is completely specified. If, in addition, the transform of the input is specified, the problem of determining the output reduces to the calculation of an inverse transform. The authors compute and plot the step-function response of a system with a finite line segment for real or imaginary part of the transfer function, using logarithmic time and frequency scales to obtain universal curves. More complicated cases are handled by graphical combination of these curves. The reader may wish to refer to Floyd's work, reported in Brown and Campbell's *Servomechanisms*, for an analysis based on the simpler case of linear frequency scale and impulse excitation. While the choice of excitation influences primarily the difficulty of analysis of the elementary case, the choice of linear or logarithmic scales is of importance in application of the results, with the advantage apparently lying with the latter choice.

Quite complete circuit descriptions and schematics are given for a device using modulation techniques to obtain the real and imaginary parts of the transfer function on a swept-frequency basis. Illustrations of plots obtained with the equipment demonstrate its utility. Possible applications to teaching should not be overlooked.

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Extension de la Méthode du Diagramme de Phase Généralisé dans l'étude de la Stabilité des Systèmes Linéaires (Extension of the Method of the Generalized Phase Diagram in the Study of the Stability of Linear Systems)—P. Lefèvre. (*Rev. Générale d'Electr.* vol. 63, pp. 619–640; October, 1954.)

The article completes various results generalizing Nyquist's stability criterion according to a method described in a preceding paper.[1] We will summarize here the main contributions of both papers.

A single loop feedback system with transmittance (output/input ratio) $T(p)$ is considered, and the discussion is based on the complex zeros of the equation $T(p)-\Lambda=0$, where a positive or negative numerical parameter Λ is introduced in order to display more easily the effects of a flat change in the loop transmission. The rational fraction $T(p)$ is normalized in such a way that the leading coefficients of its numerator and denominator have the same sign; their respective degrees are denoted by r and s; μ is the order of a possible pole of T at $p=0$, σ the total number of poles of T (counted with their order) on the positive half of the imaginary axis (excluding 0 and infinity) and N and P are the numbers of zeroes and poles of $T-\Lambda$ in the inner right half plane.

The application of the theorem of logarithmic residues gives

$$m = N - P = 2n + \sigma + (r - s + \mu)/2 \qquad \text{if } r > s$$

$$m = N - P = 2n + \sigma + \mu/2 \qquad \text{if } r \leqq s$$

where n is the number of times that the locus of $T(j\omega)$ encircles the point $\Lambda+j0$ for ω running from 0 to ∞.

The system is stable for $N=0$. To translate this into a condition for n the knowledge of P is necessary, unless $P=0$ as in the Nyquist case, but this can be dispensed with if only the effects of varying Λ are studied. The locus of $T(j\omega)$ is split into simple arcs starting from and terminating on the positive real axis for $\Lambda>0$, or on the negative real axis for $\Lambda<0$. To every half-turn of such an arc around $\Lambda+j0$ will

[1] M. Demontvignier, P. Lefèvre, "Une nouvelle méthode harmonique d'étude de la stabilité des systèmes linéaires," *Rev. Gén. Electr.*, vol. 58, pp. 263–279; July, 1949.

correspond a variation of one unit in $N-P$; the phase of $T(j\omega)$ at the end points of these arcs is $2k\pi$ for $\Lambda>0$ and $(2k+1)\pi$ for $\Lambda<0$. Denoting by ω_k the critical frequencies at which this occurs, a classification of all possible shapes for the simple arcs leads to the following result: a variation Δm of $m=N-P$ may only occur at ω_k if $|\Lambda|<|T(j\omega_k)|$; this variation is an increase or a decrease by two units, depending on whether the slope of the phase of $T(j\omega)$ at ω_k is positive or negative; if ω_k is 0 or infinite, the variation Δm is only one unit in the same conditions. Finally $m=N-P$ is obtained by summing Δm at all ω_k where the loop phase is $2k\pi$ or $(2k+1)\pi$.

The result is a function $m(\Lambda)$ and will only change when $|\Lambda|$ crosses one of the critical amplitudes $|T(j\omega_k)|$. The poles P of $T(p)-\Lambda$ are poles of T and their number P is independent from Λ, so that $N(\Lambda)$ is determined for all values of Λ if it is known for one value. In particular it is easy to determine the interval of Λ in which the system is stable, and to find the stability margin: if a decrement α is required for the natural modes, the system $T(p-\alpha)$ must still be stable; the interval of Λ for which this modified system is stable is a function $\Lambda(\alpha)$ and the maximum stability margin is that value of α for which $\Lambda(\alpha)$ is reduced to zero.

Some systems are essentially unstable; this occurs when $m(\Lambda)$ and P are both even or both odd for all values of Λ, since then $N(\Lambda)=m(\Lambda)-P$ can never be zero. The occurrence of essential instability can be checked from the following properties:

(a) P is even if the leading coefficient and the constant term of the denominator of $T(p)$ have the same sign; P is odd in the opposite case.

(b) $m(\Lambda)$ is even or odd independently from Λ only in the following cases:

$$\Lambda < 0,\ T(0) \geqq 0; \qquad \text{then } m(\Lambda) \text{ is even}$$

$$\Lambda > 0,\ T(0) \leqq 0; \qquad \text{then } m(\Lambda) \text{ is} \begin{cases} \text{even if } r < s \\ \text{odd if } r > s. \end{cases}$$

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A New Method of Synthesis of Reactance Networks—A. Talbot. (IEE Monograph no. 77; October, 1953.)

Network analysis is the process whereby we proceed from the component values of some given network to a specification of its external behaviour in terms of equations relating the voltages and currents at its accessible terminals. The traversing of this path in the reverse direction, so as to discover a network with prescribed equations governing its external behaviour, is usually described as network synthesis, and it is in this sense that the author has used the phrase. Whereas the operation of analysis is unique and, theoretically, straightforward, that of synthesis is not so; in this respect they may be compared, for example, with the processes of differentiation and integration. The fact that synthesis is not unique is easily seen from our knowledge of the existence of equivalent circuits.

The theoretical aspects of network synthesis are concerned with showing what are the necessary and sufficient conditions to be satisfied by the prescribed external behaviour, in order that it may describe a physically-realizable network and, given some such external properties, of showing how one network at least may be found which possesses them. Classic publications in this field are those by Brune and Gewertz, who dealt respectively with one- and two-terminal-pair networks.

The practical aspects of network synthesis are, on the other hand, more properly a matter of network design and are concerned with obtaining a network that has not only the desired external properties but also a form of circuit which can be reproduced accurately with physical components. Achieving this usually involves accepting some further restrictions on the generality of the available characteristics.

The paper by Dr. Talbot is a theoretical treatment of a problem proper to this second category. The problem is that of determining the element values in a two-terminal-pair reactance network from a knowledge of its chain or cascade matrix (or equally well its impedance or admittance matrix). The construction of such networks is usually simplified by having the network arranged as a tandem connection of simple sections each of which is responsible for producing one or more of the poles of loss. This fact was recognised many years ago and several writers have shown how any physically—realizable reactance network can be expressed as a tandem connection of the simplest possible sections—what Darlington has called the "canonical tandem sections."

As the chain matrix of a tandem connection of networks is the product of the chain matrices of the individual networks, then the most natural way of tackling the problem is to take the given chain matrix and attempt to factorize it. If the factors can be found and can be suitably restricted, then each may be realized as one of the tandem sections.

How this factorization could be carried out was shown independently by both Piloty and Cocci. The factorization gave at once the simplest possible factors, and the corresponding network sections required pairs of coupled coils with unit coefficient of coupling. In practical applications, by imposing certain initial restrictions, it is usually possible to ease the requirement of tight coupling, often to the extent of dispensing with it completely.

Dr. Talbot's contribution has been to analyze this general problem of factorization and to show how factors of arbitrary degree, and not necessarily the smallest degree, may be split off from the chain matrix in one operation. The whole process is somewhat analogous to the numerical factorization of a polynomial—normally one would do it by removing linear or quadratic factors, one at a time, but there is no reason why one should not remove cubic or quartic or quintic factors, for example, in one operation.

At the outset it is necessary to decide the respective sizes of the two factors into which the given matrix is to be split. Then by pre- or post-multiplication of the given matrix by the inverse of one of the required factors, it is possible to obtain four polynomial equations of the form

$$FP - HQ = SV,$$

where the letters denote odd or even polynomials in the complex frequency variable p (or λ) $=i2\pi f$. Three of these polynomials, F, H, and V, say, will be known as parts of the original matrix, while the other three, P, Q, and S, belonging to the factors, are to be found. The main part of the paper is taken up with a most detailed study of this type of equation and its solution under the restrictive conditions set by the problem, namely that P, Q, and S shall belong to a physically realizable matrix. The difficulty of solution lies in arranging that these conditions shall be met; once these arrangements have been made, by a careful choice of the degrees of the polynomials, the actual solution can proceed numerically. The author suggests that the unknown coefficients can conveniently be found by solving the linear simultaneous equations obtained by equating corresponding powers of p. He also refers in passing to a method involving continued factions, but promises to publish this later.

Apart from the academic interest of the method of solution itself, which may be considerable, it is difficult to see what the paper offers that is new and valuable in circuit theory as distinct from mathematics. Considering again the polynomial factorization analogy, it is normally much more convenient, and certainly easier, to break up the polynomial into its primitive linear and quadratic factors than into factors of higher degree. This is also true of the chain matrix, as the author admits. He suggests that by doing this, however, we may have lost something because the probability of being able to realize a given factor matrix without mutual inductance increases with the degree of the matrix. He illustrates this by an example of a matrix of degree 5 (with 5 poles of loss, one at $p=\infty$ and two each at $p=\pm i/\sqrt{2}$) which can be realized in one lump as a kind of bridged-T section without mutual coupling, but which always requires coupled coils when realized as a tandem connection of simpler sections. This, of course, is a well known phenomenon.

What the author overlooks is that if we have split our given matrix into its simplest factors to start with, it is quite easy to multiply any two or three adjacent factors together and arrive at the larger factors that may have the desirable non-coupled properties in their network forms. Moreover, all the different combinations can be tried out fairly quickly by taking different sets of factors. This could not be done unless one had all the primitive factors available. In any practical design one would not hesitate to find all the factors, even at some increase of computing, in order to be able to try every possibility of eliminating coupled coils with suitable combinations in this way.

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