Unconventional Network Theory

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THE CLASSIC AGE of network theory has ended. This period stretches from about 1924, the date of publication of Foster's Reactance Theorem [1] to perhaps 1954, the date of publication of the complete synthesis of reciprocal [2] and nonreciprocal [3] lumped passive n ports in terms of the scattering matrix by Belevitch, and Oono, and Yasuura. We also have had a preclassic or medieval period in which Kirchhoff [4], Campbell [5], Heaviside [6], and Pupin [7] figured as key contributors, the latter two in the domain of transmission line circuits. Included within the three decades of the age of classicism are the contributions that have made the names of Carson [8], Cauer [9], Bayard [10], Guillemin [11], Bode [12], Brune [13], Gewertz [14], Darlington [15], Bott-Duffin [16], and Tellegen [17] into household words (that is in the households of network theorists). These men and others of the same generation have been primarily concerned with the theory of positive element *RLC* networks, which, of course continues to be important in the engineering design of electronic circuits. However, in the last decade, as pointed out earlier [18], there are signs that a new era may be upon us, and that a network approach to problems outside the now conventional domain of lumped passive circuits will play a significant role in the newly developing electronic technologies that surround us.

What is an electric network? It seems to me that if for a given structure you can, by one means or another, constitute a set of ports across which electromagnetic energy may flow, and at which voltage and current are defined and measurable, then you have a network. Impedance and scattering functions of a frequency variable can then be discussed. Network theory is the study of such physical structures at their accessible ports. This study, which includes in its domain an immense variety of physical configurations such as control systems, multiport microwave cavities, integrated and microminiaturized circuits, parametric amplifiers, quantum electronic devices, Cerenkov radiators, you name it, demands at the very outset a consideration of the problem of physical realizability. A neatly packaged set of elementary building blocks (*e.g.*, R, L, C) is not available for these generalized systems and components, but in 1954 Raisbeck [19] had the interesting idea of defining the realizability of a network in terms of its satisfaction of various physical postulates, for example, causality, linearity, passivity, time invariance, in the time domain. It is then necessary to determine the associated properties of network functions in the complex frequency domain; and indeed, it is even important to decide under which physical postulates a frequency response exists. This idea of network realizability was treated with considerable rigor in a paper by Youla, Castriota, and Carlin [20], and a new generalized definition of a bounded real scattering matrix and a positive real impedance matrix in the frequency domain was deduced which is necessary and sufficient for the satisfaction of a set of physical postulates in the time domain. Network theorists may take particular pleasure from a result derived by network methods in that paper (but first reported in 1957), to the effect that causality is a consequence of linearity and passivity, because it has been quoted by scientists in fields far removed from circuits [21]–[23]. Much remains to be done in this area of general realizability. A physical postulational approach to nonlinear circuits, or time-varying linear systems, or for that matter linear, *active* time invariant networks has still not been formulated. It has however, become evident that a good many interesting realizability criteria are forthcoming, even in the lumped domain, if we relax

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the passivity requirement and carry linearity with rational frequency functions as far as possible. Thus workers such as Kinariwalla [24] and Sandberg [25] have exploited negative circuit elements and idealized linear amplifiers in the synthesis of linear active circuits; and if linearity is extended to the limit you arrive at the nullator (an open and short circuit at the same time) and norator (a nonreciprocal one-port) as elements required for the synthesis of the most general linear system [26], [27].

In the context of broad utilization of network concepts it is of prime importance to be able to construct a network model of the most diverse physical systems. An early example of this is the book by Brillouin [28] on the representation of the properties of crystal lattices in terms of periodic networks. Many more recent examples are also available on how to unearth a network representation for a physical system, which to the untutored eye has no resemblance whatsoever to a circuit. Thus Cerenkov radiation from a moving electron above dielectric medium can be represented in terms of a set of transmission lines and sources, constituting a circuit whose cutoff frequencies establish the nature of the radiation [29]. The use of transmission line circuits and coupling elements can also provide network models for electromagnetic-plasma sheath interaction problems [30], and even VLF ionospheric propagation can be analyzed by network-waveguide methods [31]. An interesting recent extension of network methods in electromagnetics uses a transmission line circuit representation with parametric elements, and generalizes the Brillouin diagram of periodic time-invariant networks to describe propagation phenomena in time and space varying periodic media [32]. The general technique used in most of these studies is to establish a network model for the propagating system and then draw on the broad class of network results available to conveniently obtain answers relevant to the physical problem.

There are, of course, many other network applications that are not especially concerned with conventional building blocks. One of these is gain-bandwidth theory, in which general results for optimal equalizers are deduced from an understanding of a given device, but without *a priori* knowledge of the detailed components of the equalizer. Thus the Bode-Fano [8], [33] gain-bandwidth theory has been extended to negative resistance circuits, and indeed a complete delineation of the gain-bandwidth properties of tunnel diode amplifiers has been obtained [34]. Some similar results have also been derived for parametric varactor amplifiers in the high-gain case [35] and more recently with this restriction removed [36]. An interesting extension of gain-bandwidth ideas has been given for a maser amplifier by using negative inductance and capacitance in a network model for a ruby loaded cavity [37].

Another useful area for network exploitation is the theory of invariance. In 1954, Mason [38] gave an invariant, directly related to insertion loss, for a nonreciprocal dissipative two-port network imbedded in an arbitrary lossless reciprocal structure. This idea has found several interesting applications. For example, it has been used to determine optimum performance of microwave ferrite devices [39], and has also been extended [40] to determine minimal loss properties of multiterminal semiconductor Hall plate nonreciprocal circuits.

A characteristic of most of the investigations so far described is the attempt to deduce network properties of an n port (either distributed or lumped) by resorting to general physical principles without regard to the detailed structure of the elements inside. Impedance concepts are deduced from Poynting's theorem and modal concepts [41], generalized statements of Foster's reactance theorem from variational properties of energy integrals [41], positive reality from linearity and energy conservation [20], properties of structures containing gyromagnetic media from appropriate statements of the Lorentz reciprocity law and Onsager's principle of irreversibility [42] and noise in linear amplifier circuits from the uncertainty principle and quantum mechanical considerations [43]. On the other hand, an organized building block synthesis approach to distributed structures has only been successful for certain classes of transmission line structures by applications of Richards' transformation [44], and the most recent trend in this direction is to include lump-loaded transmission line circuits by developing a network theory involving functions of two complex-frequency variables [45], [46].

The preceding discussion is only illustrative of the fecundity of network ideas, and their penetration into the most out-of-way reaches of science and technology. It is true that the "good ole days" of rational network function synthesis seem numbered, but it is clear that network theory is still a vigorous art. We may therefore expect our long and honorable tradition to be reflected in a trend towards hypermodernism, but a trend in which our classic preoccupation with the terminal or port performance of physical structures will remain as the dominant motivation.

REFERENCES

[1] R. M. Foster, "A reactance theorem," *Bell Sys. Tech. J.*, vol. 3, pp. 259–267; April, 1924.
[2] V. Belevitch, "Synthesis of a passive electric network with *N* pairs of terminals from a prescribed scattering matrix," *Ann. des Télécommunications*, no. 11, pp. 302–312; 1951.
[3] Y. Oono and K. Yasuura, "Synthesis of finite passive 2n-terminal networks with prescribed scattering matrices," *Memoirs Faculty of Engrg.*, vol. 14, no. 2, pp. 125–177; 1954.
[4] G. Kirchhoff, "Über der Auflösuing der Gleichurgen auf Welchen man bei der Untersuchunig der Linearen Verteilunig Galvanisher Ströme Gefuhrt Wird," *Poggendorff's Annalen der Physik*, band 72, Ser. III-12, p. 497; 1847.
[5] G. A. Campbell, "Cisoidal oscillations," *Trans. AIEE*, vol. XXX, pt. II, pp. 873–909; 1911.
[6] O. Heaviside, "Electromagnetic Theory V 1, 2, 3," The Electrician Printing and Publ. Co., Great Britain; 1893, 1899, and 1912.
[7] M. I. Pupin, "Wave propagation over non-uniform cables and long distance lines," *Trans. AIEE*, vol. XVII, pp. 445–507; 1900.
[8] J. R. Carson, "Electromagnetic theory and the foundations of electric circuit theory," *Bell Sys. Tech. J.*, vol. 6, pp. 1–17; January, 1927.

[9] W. Cauer, "Ein Resktanztheorem," *Preuss. Akad d Wissen schaften Phys.-Math. Kl, Sitzbar.*, nos. 30–32, pp. 673–681; November, 1931.

[10] M. Bayard, "Relations entre les parties reelles et imaginaries des impedances, et determination des impedances en fonction de l'une des parties," *Rev. Gen. de l'Electricte*, vol. 37, pp. 659–664; May 23, 1935.

[11] E. A. Guillemin, "Communication Networks V 2," John Wiley and Sons, Inc., New York, N. Y.; 1935.

[12] H. W. Bode, "Network Analysis and Feedback Amplifier Design," D. Van Nostrand Company, Inc., Princeton, N. J.; 1945.

[13] O. Brune, "Synthesis of a finite 2-terminal network whose driving-point impedance is a prescribed function of frequency," *J. Math. Phys.*, vol. 10, pp. 191–236; August, 1931.

[14] C. M. son Gewertz, "Synthesis of a finite four-terminal network from its prescribed driving point functions and transfer function," *J. Math. Phys.*, vol. 12, pp. 1–257; 1933.

[15] S. Darlington, "Synthesis of reactance 4-poles which produce prescribed insertion loss characteristics," *J. Math. Phys.*, vol. 28, pp. 257–353; September, 1939.

[16] R. Bott and R. J. Duffin, "Impedance synthesis without use of transformers," *J. Appl. Phys.*, vol. 20, p. 816; August, 1949.

[17] B. D. H. Tellegen, "The gyrator, a new electric network element," *Phillips Res. Repts.*, vol. 3, pp. 81–101; April, 1948.

[18] H. J. Carlin, "Pandora's black boxes," IRE Trans. on Circuit Theory, vol. CT-4, pp. 60–61; September, 1957.

[19] G. Raisbeck, "A definition of passive linear networks in terms of time and energy," *J. Appl. Phys.*, vol. 25, pp. 1510–1514; December, 1954.

[20] D. C. Youla, L. J. Castriota and H. J. Carlin, "Bounded real scattering matrices and the foundations of linear passive network theory," IRE Trans. on Circuit Theory, vol. CT-6, pp. 102–124; March, 1959.

[21] J. Meixner and H. Konig, "Zur theories der linearen dissipativen systems," *Rheologica Acta 2/3*, vol. 1, pp. 190–193; 1958.

[22] R. Groot and P. Mazur, "Non Equilibrium Thermodynamics," Interscience Publishers, Inc., New York, N. Y., ch. 8; 1962.

[23] C. L. Doph, "Positive real resolvents and linear passive Hilbest systems," *Ann. Acad. Scient. Fennicae*, ser. A, 336/9; 1963.

[24] B. K. Kinariwala, "Synthesis of active *RC* networks," *Bell Sys. Tech. J.*, vol. 38, pp. 1269–1316; September, 1959.

[25] I. W. Sandberg, "The realization of transformerless multiport networks containing positive resistance and positive and negative reactive elements," IRE Trans. on Circuit Theory, vol. CT-9, pp. 377–384; December, 1962.

[26] Y. Cono, "Formal realizability of linear networks," *Proc. Symp. on Active Networks and Feedback Systems,* Microwave Research Institute Symposia Ser., vol. 10, pp. 475–486; 1961.

[27] H. J. Carlin, "Singular network elements," IRE Trans. on Circuit Theory, vol. CT-11, pp. 67–72; March, 1964.

[28] L. Brillouin, "Wave Propagation in Periodic Structures; Electric Filters and Crystal Lattices," McGraw-Hill Book Company, Inc., New York, N. Y.; 1946.

[29] L. Felsen and A. Hessel, "A network approach to the analysis of Cerenkov radiation problems," *Nuovo Cimento*, ser. 10, vol. 19, pp. 1065–1071; March, 1961.

[30] A. Ishimaru, "The Effect on the Radiation from a Plasma Sheath of a Unidirectional Surface Wave Along a Perfectly Conducting Plane," Dept. Elec. Engrg. Univ. of Washington, Seattle, Wash., Tech. Rept. No. 64; April, 1962.

[31] K. G. Budden, "On the propagation of a radio atmospheric," *Phil. Mag.*, vol. 43, pp. 1179–1200; November, 1952.

[32] E. S. Cassedy and A. A. Oliner, "Dispersion relations in time-space periodic media: part I—stable interactions," Proc. IEEE, vol. 51, pp. 1342–1359; October, 1963.

[33] R. M. Fano, "Theoretical limitations on the broadband matching of arbitrary impedances," *J. Franklin Inst.*, vol. 249, pp. 57–83, January, 1950; and vol. 249, pp. 139–154, February, 1950.

[34] D. C. Youla and L. Smilen, "Optimum Negative Resistance Amplifiers," Polytechnic Inst. of Brooklyn, N. Y. Microwave Research Institute Symposia Ser., vol. 10, pp. 241–318; 1961.

[35] E. S. Kuh and M. Fukada, "Optimum synthesis of wide-band parametric amplifiers and converters," IRE Trans. on Circuit Theory, vol. CT-8, pp. 410–415; December, 1961.

[36] W. Ku, "A broad-banding theory for varactor parametric amplifiers—Part I, Part II," IEEE Trans. on Circuit Theory, vol. CT-11, pp. 50–58, 59–66; March, 1964.

[37] R. L. Kyhl, R. A. McFarlane and M. W. P. Strandberg, "Negative *L* and *C* in solid state masers," Proc. IRE, vol. 50, pp. 1608–1623; July, 1962.

[38] S. J. Mason, "Power gain in feedback amplifiers," IRE Trans. on Circuit Theory, vol. CT-1, pp. 20–25; June, 1954.

[39] T. Schaug-Pettersen and A. Tonning, "On the optimum performance of variable and non-reciprocal networks," IRE Trans. on Circuit Theory, vol. CT-6, pp. 150–158; June, 1959.

[40] J. M. Garg, "Network Theory of Semiconductor Hall Plate Circuits," D.E.E. dissertation, Polytechnic Inst. of Brooklyn, N. Y.; June, 1961. See also H. J. Carlin and A. Giordano, "Network Theory," Prentice-Hall, Inc., New York, N. Y., ch. V; 1964.

[41] C. G. Montgomery, R. H. Dicke and E. M. Prucell, "Principles of Microwave Circuits," McGraw-Hill Book Company, Inc., New York, N. Y.; 1948.

[42] H. B. G. Casimir, "Reciprocity theorems and irreversible processes," Proc. IEEE, vol. 51, pp. 1570–1573; November, 1963.

[43] H. Heffner, "The fundamental noise limit of linear amplifiers," Proc. IEEE, vol. 50, pp. 1604–1608; July, 1962.

[44] P. I. Richards, "Resistor-transmission lines," Proc. IRE, vol. 36, pp. 217–219; February, 1948.

[45] H. Osaki and T. Kasami, "Positive real functions of several variables and their application to variable networks," IRE Trans. on Circuit Theory, vol. CT-7, pp. 251–260; September, 1960.

[46] H. G. Ansell, "On certain two-variable generalizations of circuit theory, with applications to networks of transmission lines and lumped reactances," IEEE Trans. on Circuit Theory, vol. CT-11, pp. 214–223; June, 1964.