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Benders decomposition

applying Benders decomposition to power systems

IT IS APPARENT THAT POWER system restructuring provides a major forum for the application of decomposition techniques—including the Benders decomposition algorithm—to coordinate the optimization of various objectives among self-interested entities. These entities include power generators (GENCOs), transmission providers (TRANSCOs), and distribution companies (DISCOs).

Consider a decomposition example in which an individual GENCO optimizes its annual generating-unit maintenance schedule based on local constraints such as available fuel, emissions, and crew and the seasonal load profile. The GENCO's optimization intends to maximize its payoff in a competitive environment. Individual GENCOs submit their maintenance schedules to the ISO, which examines the proposed schedules to minimize the loss of load expectation while maintaining the transmission security based on the available transfer capacity and the forced and scheduled outages of power system components. The ISO could return the proposed schedules to the designated GENCOs if the operating constraints would be violated. The ISO's rejection of a proposed schedule could include a suggestion (Benders cut) for revising the proposed maintenance schedule that would satisfy the GENCOs' and the ISO's constraints.

The full text of this techtorial is available at http://motor.ece.iit.edu/ms/benders.pdf.

In the 1960s and 1970s, many of the decomposition techniques were motivated by the inability to solve large-scale centralized problems with the available computing power of that time. The dramatic improvement in computing technology since then allowed power engineers to solve very large problems easily. Consequently, the interest in decomposition techniques dropped dramatically. Now, however, there is an increasingly important class of optimization problems in restructured power systems for which decomposition techniques are becoming most relevant.

In principle, one may consider the optimization of a system of independent entities by constructing a large-scale mathematical program and solving it centrally (e.g., through the ISO), using currently available computing power and solution techniques. In practice, however, this is often impossible. In order to solve a problem centrally, one needs complete information on local objective functions and constraints. As these entities are separated geographically and functionally, this information may be unattainable or prohibitively expensive to retrieve. More importantly, independent entities may be unwilling to share or report on their propriety information because it is not "incentive compatible" to do so; i.e., these entities may have an incentive to misrepresent their true preferences.

In order to optimize certain objectives in restructured power systems, one must turn to decomposition's coordination aspects. Specifically, with limited information, one must coordinate entities to reach an optimal solution. The goal will be to coordinate the entities by optimizing a certain objective (such as finding the equilibrium resource price) while satisfying local and system constraints.

One commonly used decomposition technique in power systems is Benders decomposition. J.F. Benders introduced the Benders decomposition algorithm for solving large-scale, mixed-integer programming (MIP) problems. Benders decomposition has been successfully applied to take advantage of underlying problem structures for various optimization problems, such as restructured power systems operation and planning, electronic packaging and network design, transportation, logistics, manufacturing, military applications, and warfare strategies.

In applying Benders decomposition, the original problem will be decomposed into a master problem and several subproblems. Generally, the master program is an integer problem and subproblems are the linear programs. The lower-bound solution of the master problem may involve fewer constraints. The subproblems will examine the solution of the master problem to see if the solution satisfies the remaining constraints. If the subproblems are feasible, the upper-bound solution of the original problem will be calculated while forming a new objective function for the further optimization of the master-problem solution. If any of the subproblems is infeasible, an infeasibility cut represent-

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ing the least satisfying constraint will be introduced to the master problem. Then, a new lower-bound solution of the original problem will be obtained by recalculating the master problem with more constraints. The final solution based on the Benders decomposition algorithm may require iterations between the master problem and subproblems. When the upper bound and the lower bound are sufficiently close, the optimal solution of the original problem will be achieved.

Figure 1 depicts the hierarchy for calculating a security-constrained unit commitment (SCUC), which is based on the existing set up (GENCOs and TRANSCOs as separate entities) in restructured power systems. The hierarchy utilizes a Benders decomposition which decouples the SCUC into a master problem (optimal generation scheduling) and network security check subproblems. The output of the master problem is the on/off state of units, which are examined in the subproblem for satisfying the network constraints. The network violations are formulated in the form of Benders cuts, which are added to the optimal generation scheduling formulation for recalculating the original unit commitment solution.

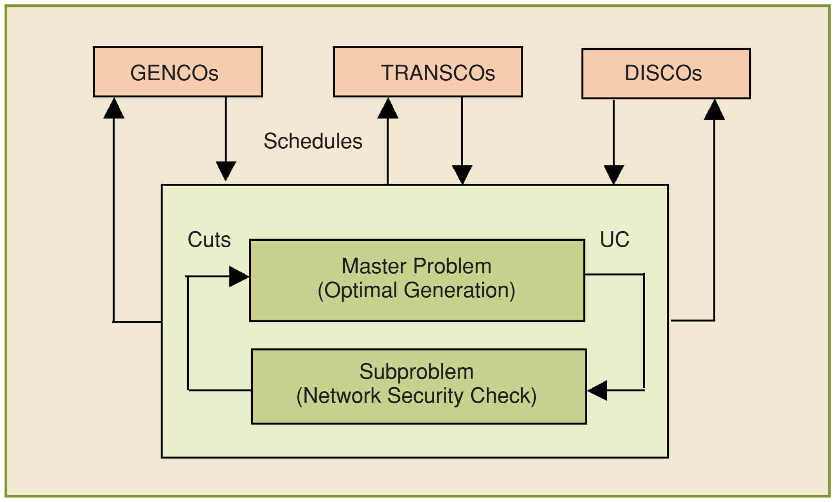

figure 1. ISO and market participants.

Other applications of Benders decomposition to security-constrained power systems include:

- ✔ generating-unit planning
- ✔ transmission planning
- ✔ optimal generation bidding and valuation
- ✔ reactive power planning
- ✔ optimal power flow
- ✔ hydrothermal scheduling
- ✔ generation maintenance scheduling
- ✔ transmission maintenance scheduling
- ✔ long-term fuel budgeting and scheduling
- ✔ long-term generating-unit scheduling and valuation. p&e