Robert W. Lucky

# The Adaptive Equalizer

**EDITORS' INTRODUCTION**

Our guest in this issue is Robert W. Lucky. He was born on 9 January 1936 in Pittsburgh, Pennsylvania. He obtained his B.S. (1957), M.S. (1959), and Ph.D. (1961) degrees from Purdue University, all in electrical engineering. Dr. Lucky was an executive director with Bell Laboratories, New Jersey, (1961–1992) and a corporate vice president with Bellcore, later known as Telcordia Technologies (1992–2002). His early work focused on data communication and later on research management. He coauthored *Principles of Data Communication* (1968) and authored *Silicon Dreams* (1989) and *Lucky Strikes Again* (1962). Dr. Lucky received the IEEE Armstrong Award (1975), the IEEE Edison Medal (1995), and the IEEE Emberson Award (2002). He was awarded several honorary doctorate of philosophy degrees (1988, 1991, 1999, and 2000) and an exceptional civilian contribution medal from the U.S. Air Force (1990).

Dr. Lucky has had a very long collaboration ("forever," he tells us) with Jack Salz, with whom he continues to meet for lunch regularly and discuss communications theory. He appreciates foremost a "give and take" quality in his collaborators and believes that, although many people are all give or all take, one needs both. Despite receiving a job offer that he could not refuse from Bellcore in 1992, leaving Bell Labs was difficult. However, in retrospect he was … well, Lucky, to make that choice. His last name "draws a lot of witticism," and he has heard it all. Perhaps one of the most flattering ones that he recalls is that from Arno Penzias, who once told him that Napoleon only asked one thing of his generals: that they be Lucky. What are his hobbies? When he does not stay glued to his computer, Bob Lucky plays the piano and the violin, reads a lot, and goes on biking trips all over Europe. His favorite book is *The Magus* by John Fowles. A regular columnist for *IEEE Spectrum*, Dr. Lucky has become well known for his fluent and captivating writing and his interesting views on digital signal processing, research, and management topics. You will recognize his signature style in the story told next, which goes back on the trails of his adaptive equalization work.

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I was driving home on a fall evening in 1964 after an ordinary day at Bell Labs, with no premonition that this particular commute would change my life. I had stopped the car at a traffic light in the small, suburban New Jersey town of Red Bank. In the time that it took the light to change from red to green, I invented my automatic equalizer.

I suppose the idea had been incubating in my mind for an hour or so before that, but it burst forward into my consciousness in that instant. I was only a mile from my apartment, but I rushed home, fearful that the idea would drift away before I could write it down. I stayed up all night, and I remember watching the sun rise the next morning. It couldn't rise fast enough for me as I waited in a nervous and elated state for the earliest possible time that I could go to Bell Labs and tell other people about my idea. In my experience, nothing has been as exhilarating as an idea that you believe is world class. In my long career after this fateful day, I had only about a half-dozen such ideas and most of them came to nothing but each put me on a high that couldn't be touched by achievements or accolades like promotions and awards.

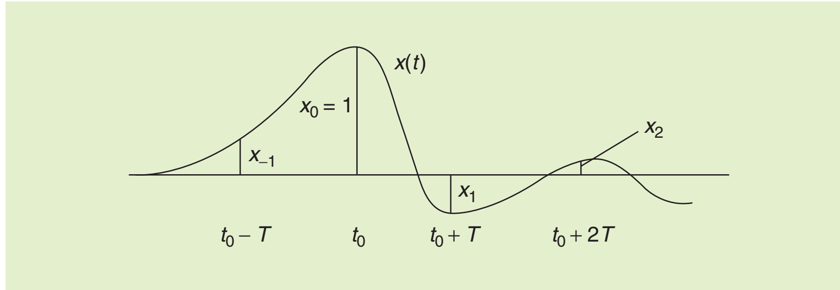
But I was nervous too. Ideas are fragile things, and often something that seems wonderful in the middle of the night cannot stand the light of day. I was confident that my idea was technically correct but paranoid about the thought that someone else had thought of the same thing long ago. In graduate school I had kidded with some of my fellow students that everything could be found in the voluminous writings of Pliny the Elder. Maybe he had the equalizer in there too.

That morning, I waited anxiously in the office of Floyd Becker, the supervisor of the group responsible for the development of the new high-speed data modem. A few weeks earlier I had been in this same office with a small group when he had drawn a block diagram of the modem on the blackboard. The highest speed modems available at that time transmitted at 2,400 b/s. This new modem was to transmit at 9,600 b/s. To achieve this speed, distortions in the telephone channel that caused intersymbol interference ("echoes" from preceding or succeeding symbols adding or subtracting from the symbol being detected) would have to be automatically corrected. In Floyd's block diagram there was a block for this automatic equalizer. In the block he had written "Bob Lucky." It would be my responsibility—and his great gift to me.

I was fortunate that the new high-speed modem was a vestigial sideband system, so that the automatic equalizer, whatever it was to be, operated at baseband after the signal had been demodulated by a recovered carrier. The transmitted signal was a sequence of amplitude-modulated pulses with 16 possible amplitudes, representing 4 b/pulse. This pulse train was simply multiplied by a carrier frequency, and then the upper sideband was filtered out at the transmitter. At the receiver, the carrier was recovered in a phase-locked loop and used to demodulate the pulse train. Then my yet-to-be-invented equalizer would correct for the inevitable distortion that would occur when the pulses passed through the telephone channel. The development of this new modem was proceeding rapidly under the assumption that I would invent an automatic equalizer in time to meet the schedule. As I look back on it now, I think I should have felt a certain pressure, but the truth was that I was simply enjoying being in the midst of a great research problem.

When coming out of the demodulator, an isolated pulse might look similar to that illustrated in Figure 1. At the sampling instants $(t_0 + nT)$, the nonzero amplitudes of the precursors (e.g., $x_{-1}$) and tails (e.g., $x_1, x_2$) of this pulse would interfere with neighboring pulses. Ideally, the pulse could be "equalized" or corrected to pass through zero at these sampling instants by suitable adjustment of some equalizing filter.

Now, almost a half-century later, it is difficult to recreate the state of knowledge and technology of those days that made the adaptive equalizer a difficult problem at the time. First, there needed to be a theoretical understanding of intersymbol interference. Next, an adjustable filter needed to be found to compensate for this interference. Finally, there had to be a simple algorithm that could be used to adjust this filter. Obviously, we didn't have microprocessors or any means of making even the simplest calculations in the equalizer itself. I didn't think that an algorithm calculation could be done with the technology that we had, but just thinking about this problem was probably the most fun I ever had as a researcher.

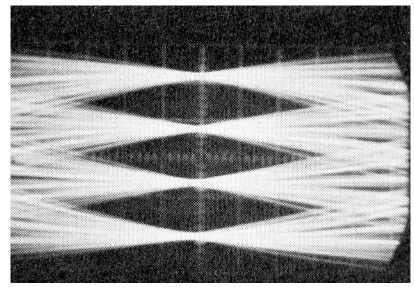


**[FIG1] An example impulse response.**

## UNDERSTANDING THE INTERSYMBOL INTERFERENCE

Intersymbol interference was a new problem in those days, one that hadn't been encountered because of the low speeds of modems. In all its glory, it was of daunting mathematical complexity. For decades to come, researchers would write papers about the optimum reception of a sequence of digital symbols in the presence of intersymbol interference. However, my problem was more constrained. What I wanted to do was adjust the gains on the taps of a transversal filter, so as to minimize the probability of error for a random data sequence. Unfortunately, that problem was mathematically intractable, so I looked at metrics that were surrogates for the probability of error. In this regard, I considered both a maximum distortion criterion and a mean square error criterion.

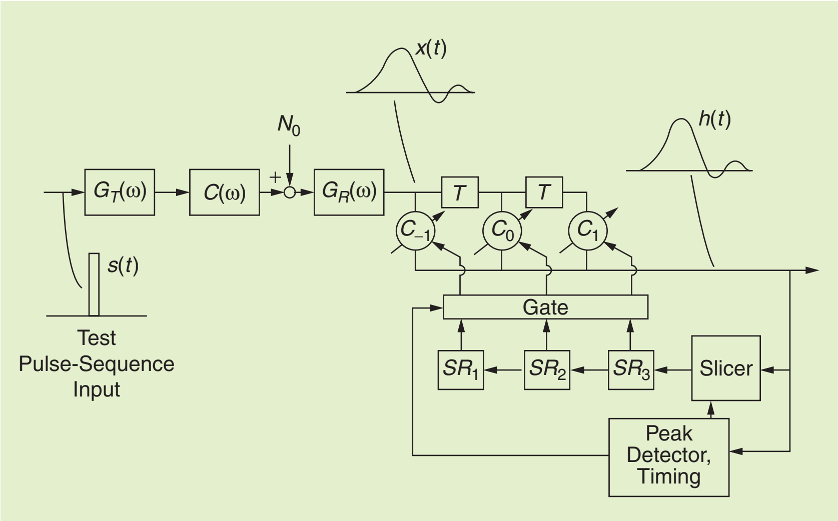
I was first drawn toward the maximum distortion criterion, because developers in those days judged the quality of modem transmission by the "eye opening," which was the display of the received signal on an oscilloscope synched to the pulse repetition rate. An example of such an eye pattern is shown in Figure 2. In this figure, the pulses have four possible amplitudes. It can be seen that, at the proper sampling time (in the center of the timeline shown in the picture), the signal passes almost exactly through one of these four amplitudes. When intersymbol interference is present, the "eye" on the oscilloscope closes because of the superposition of tails and precursors of surrounding pulses. At the sampling time, the various traces seen in the eye diagram will have different amplitudes depending on what surrounding symbols have been transmitted. If any of these traces crosses one of the decision thresholds, represented by horizontal lines in the center of each of the three eye openings shown, then an error will occur. The margin against noise is graphically seen as the relative opening of the eye on the scope. A metric that seeks the maximum eye opening is a minimax criterion—one that minimizes the maximum probability of error over all possible data sequences.


**[FIG2] Eye diagram after equalization for a four-level signal.**

I toyed with the idea of using a minimum mean square error criterion, which would minimize the mean square deviation at the signal sampling points. I confess that I was somewhat bothered that there was no real mathematical justification for such a criterion, even though it was certainly intuitively appealing. Moreover, as I later tried minimization algorithms, the mean square error metric seemed to require multiplications, which at that time were difficult to implement. For those reasons, I began by trying to minimize the peak distortion, which was equivalent to maximally opening the eye and yielding the minimax probability of error.

## THE ADJUSTABLE FILTER

To "equalize" or correct the pulse in Figure 1 to pass through zero at the




**[FIG3] Block diagram of a three-tap equalizer. Notations are explained in Figure 4.**

sampling instants, I considered several filter candidates for the equalizer. The transversal filter (a tapped delay line with adjustable gains at the taps) was one of these candidates. A typical distorted pulse in those days might have three or four significant interfering precursor samples and perhaps six significant tail samples. Through simulations I learned that a transversal filter with at least a dozen taps would be required. These days, of course, a transversal filter is simply a shift register, but in those days it was a big clunky piece of hardware incorporating a series of all-pass filters implemented with inductors and capacitors. It used up a whole rack of equipment and was quite heavy. Nevertheless, it was relatively well known and had the flexibility that I needed, so I concentrated most of my study on this approach.

### THE ADJUSTABLE FILTER ALGORITHM

Thus I had a mathematical framework for optimization and an adjustable filter. But what settings were optimal, and how was the filter to be adjusted automatically to achieve those settings? The wonderful insight that I achieved at that traffic light in Red Bank was that the maximum distortion mentioned earlier was a convex function of the tap gains and had a single minimum that could be reached by an iterative steepest descent algorithm. Moreover, the incremental adjustments necessary for this optimization would be incredibly easy to make, requiring only knowledge of the polarities of samples of test pulses. Each tap gain would be incremented one unit in a direction opposite to the polarity of the test pulse at its position when the peak of the pulse was at the center of the filter. After a series of test pulses, all the tap gains would be "walked" down to the minimum distortion. The block diagram of the original zero-forcing equalizer is illustrated in Figure 3. The corresponding algorithm is described in Figure 4.

The simplicity of this equalizer, four decades ago a source of pride, seems now almost an embarrassment. It is impossible for an engineer of today to fully appreciate the environment of that long ago time when the world was analog and integrated circuits were unknown. I was in that world, and yet even I can barely recreate the thought processes of those quaint days.

### THE IMPLEMENTATION

I described this equalizer in Floyd Becker's office that morning long ago. I couldn't wait for his response, but when I finished, he simply said, laconically, "Yes, that will work." And it did. Eric Port designed and wired the modem with my equalizer. It was a six-foot-high rack of equipment, most of which was occupied by the equalizer. There were 13 taps on the delay line; the gain of each tap was adjusted by a resistor network, with seven relays for 128 possible gain settings. When the training pulses were sent, the relays would emit satisfying clicks as they "walked" down the valley of minimum distortion.

I was really happy about the equalizer but disliked the necessity of adjustment only during the training pulse period, so I immediately sought a way to adjust the taps continuously during normal data transmission. It was only a couple of weeks later that I conceived the idea (that, in retrospect, I thought was my best invention) to use a decision-directed approach to the tap setting. I had read a paper where someone had used data decisions to adjust timing recovery, and so the idea had been planted in my head. However, the whole thing seemed dangerous—using your own faulty decisions about data values as a source of reference instead of known training pulses. Would it actually work?

This new equalizer approach did not require test pulses for training. The tap increment polarities were obtained by correlating the assumed analog "errors," the difference voltages between the actual samples and the decision thresholds, with past and previous symbols. I did simulations of the algorithm, which I now thought of as an "adaptive" equalizer, rather than an "automatic" equalizer. In those days, I did simulations on the central time-shared machine using punched cards with Fortran statements. The simulations showed that the decision-directed adaptation worked well as long as the "eye" had some opening before the adaption began. In other words, the adaptation worked well if the error rate wasn't too high to start with.

The new adaptive equalizer was quickly implemented, and I loved listening to the relays as they tried to minimize interference. At first, there would be sporadic clicks as the algorithm didn't have much sense about where it was going. Then there would be a rush of clicks as the relays got a real sense of direction and the error rate (and consequently, the accuracy

In Fig. 3, a test pulse s(t) is sent through a transmitting filter GT(w), a distorting channel C(w) and a receiving filter GR (w). The resultingpulse x(t) has nonzero values at sampling instants. This pulse passes through the transversal filter with cj the coefficients of the filter, aka tap gains, and T the tap delays, whose output pulse is denoted by h(t). At the sampling instants, the values of h(t) are

$$h_n = S_j c_j x_{n-j}, \qquad (1)$$

where the index j is summed over the tap coefficients, which in this example are -1, 0, and 1. The slicer is a binary quantizer that passes the polarity of the sample through the shift registers SR1, SR2, and SR3.

The most vulnerable data sequence that might be transmitted, i.e., the one corresponding to the innermost trace in the eye-pattern and hence the one that has the highest probability of error, has binary "ones" where h(t) is positive, and "zeros" where h(t) is negative, so that interference adds coherently. In this case, the intersymbol interference is the sum of the absolute values of the nonpeak values of the impulse response. This is the maximum amount (distortion D) by which a symbol value can be deviated at the sampling point.

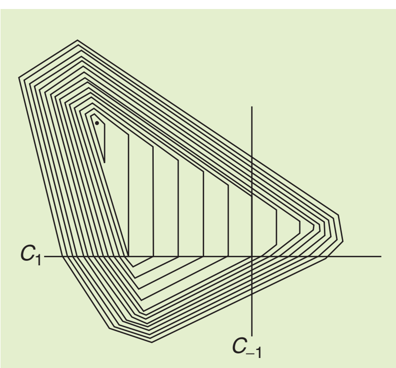
$$D = 1/h_0 \; S_{n\pi 0} \; |h_n|, \qquad (2)$$

where h0 is the pulse height. Then, the output pulse distortion (with h0 constrained to unity) becomes

$$D = S_{n\pi 0} \; | S_j c_j (x_{n-j} - x_n x_{-j}) + x_n |. \qquad (3)$$

Equal distortion contours as functions of c1 and c-1 (with c0 constrained to normalize pulse height h0 ) are shown in Figure 5. Note that distortion is a convex, piecewise-linear function of the tap gains and has a single minimum. This minimum can be reached bya steepest descent algorithm. The downward direction in this space in my original equalizer was determined by simply passing back toeach tap gain the polarity of the impulse response at its corresponding sampling point. It can be shown that, to minimize distortion, theoptimum setting of an (N+1)-tap transversal filter results in N zeros at off-peak sampling times in the output pulse response. In otherwords, for this example the minimum distortion is achieved when h1and h-1 are zero. Thus, the equalizer came to be known as a zero-forcing equalizer.

**[FIG4] The algorithm for the original equalizer in Figure 3.**




**[FIG5] Equal distortion contours for the three-tap equalizer.**

of measurement) improved. Finally, the relays would settle into random clicks as settling occurred. They sounded much like making popcorn, although we didn't have microwave ovens then.

## PAVING THE WAY

I think what I accomplished with the adaptive equalizer was like an existence proof. In retrospect it wasn't that hard a problem, and as soon as people saw the dramatic results a burst of innovation on equalization followed. A little belatedly, I did design a mean-square equalizer and became aware of Bernie Widrow's beautiful work on his least-mean squares (LMS) algorithm. Other associates quickly designed and experimented with adaptive echo cancellers, passband equalizers, and many variations and applications for adaptive filtering.

Innovation in adaptive equalization still continues today. I never would have dreamed 40 years ago that equalizers would become so sophisticated, yet would only occupy a tiny sliver of silicon, cost almost nothing, and be ubiquitous in ordinary households around the world.

Sometimes when I drive through Red Bank, New Jersey, and pause at that pregnant traffic light, I remember the invention I made there. Alas, I have had no further inspiration from that particular spot.

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