Table 1. Arctan expressions versus octant location.	
Octant	**Arctan approximation**
first or eighth	$\theta' = \frac{IQ}{I^2 + 0.28125Q^2}$
second or third	$\theta' = \pi/2 - \frac{IQ}{Q^2 + 0.28125I^2}$
fourth or fifth	$\theta' = \pi + \frac{IQ}{I^2 + 0.28125Q^2}$
sixth or seventh	$\theta' = -\pi/2 - \frac{IQ}{Q^2 + 0.28125I^2}$

number residing in any octant. We do this by using the rotational symmetry properties of the arctangent

$$\tan^{-1}(-Q/I) = -\tan^{-1}(Q/I) \quad (3)$$

$$\tan^{-1}(Q/I) = \pi/2 - \tan^{-1}(Q/I) \quad (3')$$

Those properties allow us to create Table 1, listing the appropriate arctan approximation based on the octant location of complex x.

So we have to check the signs of Q and I, and see if $|Q| > |I|$, to determine the octant location and then use the appropriate approximation in Table 1. The maximum angle approximation error is 0.26° for all octants.

When θ is in the fifth octant, the above algorithm will yield a θ' that's more positive than $+\pi$ radians. If we need to keep the θ' estimate in the range of $-\pi$ to $+\pi$, we can rotate any θ residing in the fifth quadrant $+\pi/4$ rad (45°) by multiplying $(I + jQ)$ by $(1 + j)$, placing it in the sixth octant. That multiplication yields new real and imaginary parts defined as

$$I' = (I - Q) \text{ and } Q' = j(I + Q). \quad (4)$$

The fifth octant θ' is then estimated using I' and Q' with

$$\theta'_{\text{5th oct.}} = -3\pi/4 - \frac{I'Q'}{Q'^2 + 0.28125I'^2}. \quad (5)$$

Concluding Remarks

This arctangent algorithm may be useful in a digital receiver application where I^2 and Q^2 have been previously computed in conjunction with an amplitude modulation demodulation process or envelope detection associated with automatic gain control.

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An Update to the Sliding DFT

Eric Jacobsen and **Richard Lyons**

Because of our continued investigation of the sliding DFT (SDFT), and the interest the March 2003 article [1] generated among our DSP brethren, we provide this update to our readers:

▲ 1) Referring to [1], while the typical Goertzel algorithm description in the literature specifies the frequency resonance variable k in (2) and Figure 1 to be an integer (making the Goertzel filter's output equivalent to an N-point DFT bin output), k can in fact be any value between 0 and $N-1$ giving us full flexibility in specifying a Goertzel filter's resonance frequency.

▲ 2) Since we wrote the article, we've been made aware of several other versions of the SDFT expression, $S_k(n)$ in (4). While (4) in [1] provides the correct DFT magnitude results for real-time spectrum analysis, its $S_k(n)$ phase contains a fixed offset requiring correction if DFT phase results are required. A better expression for the SDFT is

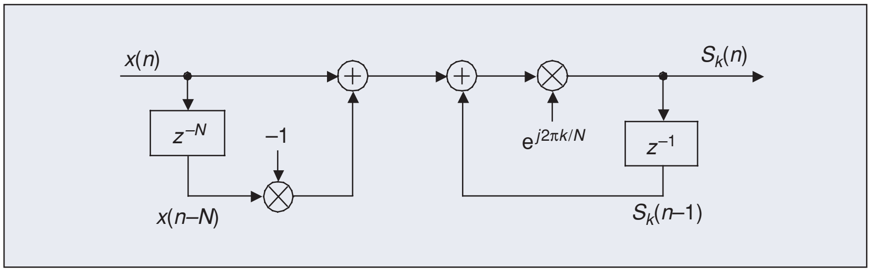
$$S_k(n) = e^{j2\pi k/N}[S_k(n-1) + x(n) - x(n-N)]. \quad (1')$$

Equation (1′), implemented with a comb filter followed by a complex resonator, as shown in Figure 1, provides both correct DFT magnitude and phase results.

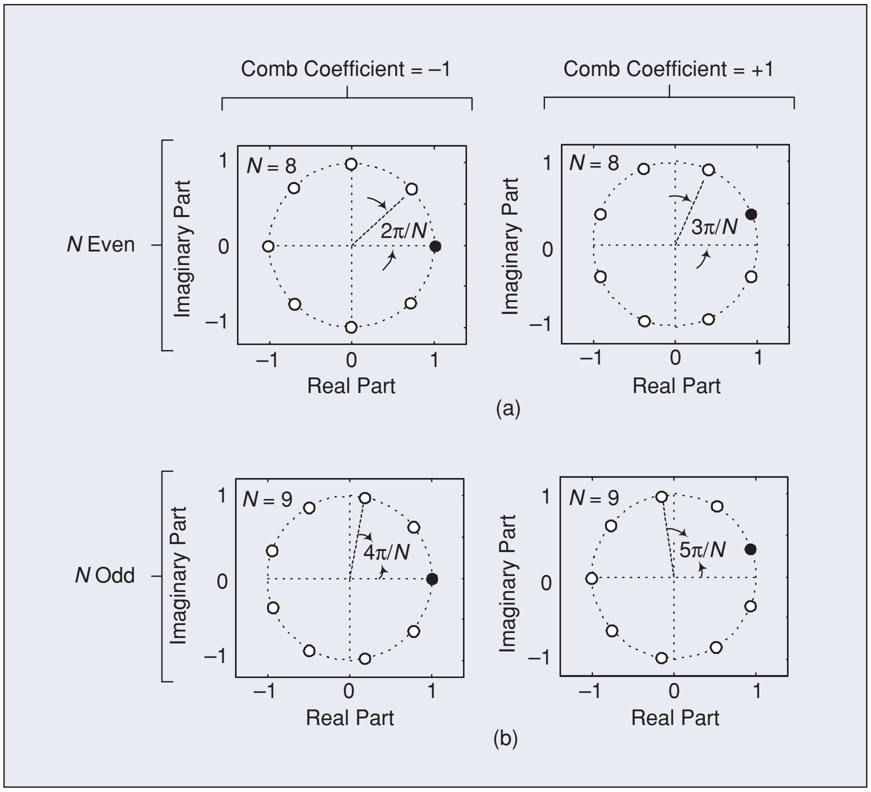
▲ 3) We've discovered a useful property of the SDFT that's not widely known but is important. If we change the SDFT's comb filter feedforward coefficient from −1 to +1, the comb's zeros will be rotated counterclockwise around the unit circle by an angle of π/N radians. Tans. This situation, for $N = 8$, is shown on the right side of Figure 2(a). The zeros are located at angles of $2\pi(k+1/2)/N$ radians. The $k = 0$ zeros are shown as solid dots. Figure 2(b) shows the zeros locations for an $N = 9$ SDFT under the two conditions of the comb filter's feedforward coefficient being -1 and $+1$.

This alternate situation is useful, and we can now expand our set of spectrum analysis center frequencies to more than just N angular frequency points around the unit circle. The analysis frequencies can be $2\pi k/N$ or $2\pi(k+1/2)/N$, where integer k is in the range $0 \le k \le N-1$. Thus we can build an SDFT analyzer that resonates at any one of $2N$ frequencies between 0 and f_s Hz. Of course, if the comb filter's feedfoward coefficient is set to $+1$, the resonator's feedforward coefficient must be $e^{j2\pi(k+1/2)/N}$ to achieve pole/zero cancellation.

▲ 4) To correct typographical errors in Table 1 of [1], the column headings should be a_1, a_2, and a_3 (not α_1, α_2, and α_3). For the Hanning window in Table 1, coefficient $a_1 = 0.5$ (not 0.25).



▲ *1. Improved SDFT structure.*



▲ *2. Four possible orientations of comb filter zeros on the unit circle.*

References

[1] E. Jacobsen and R. Lyons, "The sliding DFT," *IEEE Signal Processing Mag.*, vol. 20, no. 2, pp. 74–80, Mar. 2003.