Correspondence

A Generalized Capon Estimator for Localization of Multiple Spread Sources

Aboulnasr Hassanien, Shahram Shahbazpanahi, and Alex B. Gershman

***Abstract*—In this correspondence, we develop a generalized Capon spatial spectrum estimator for localization of multiple incoherently distributed (spread) sources in sensor arrays. The proposed generalized Capon technique estimates the source central angles and angular spreads by means of a two-dimensional (2-D) parameter search. Simulation results show that the proposed method has a substantially improved performance compared with several popular spread source localization methods.**

***Index Terms*—Array processing, generalized Capon estimator, incoherently distributed sources.**

I. Introduction

Most conventional direction-finding techniques are based on the assumption that the source energy is concentrated at discrete angles that are referred to as the source directions-of-arrival (DOAs). However, in several applications such as sonar, radar, and wireless communications, such a point source assumption can be irrelevant because signal scattering phenomena may result in angular spreading of the source energy [1]–[15]. In such cases, a *distributed source model* is more realistic than the point source one.

In wireless communication systems with antenna arrays at base stations, one of the central problems is the fast fading due to a local scattering in the vicinity of the mobile [3]–[5]. In the presence of such a fading, the source can no longer be modeled using the point assumption. In particular, depending on the environment of the mobile, the base-mobile distance and the base station height, angular spreads up to 10° can be commonly observed in practice [3], [4] . Depending on the relationship between the channel coherency time and the observation period, the sources can be viewed either as coherently distributed (CD) or incoherently distributed (ID). A source is called CD if the signal components arriving from different directions are replicas of the same signal, whereas in the ID source case, all signals coming from different directions are assumed to be uncorrelated [1], [14], [15]. Indeed, if the channel coherency time is much smaller than the observation period, then the ID model is relevant [14]. In the opposite case, the CD model or a partially coherent model can be used [6].

Furthermore, source localization in presence of angular spreading (or, equivalently, multiplicative noise) is one of the main problems in synthetic aperture radar (SAR) interferometry (see [7]–[9] and references therein).

Recently, ID source localization has been a focus of intensive research. Many techniques have been developed for scenarios with a single source, e.g., [6]–[12]. Several other techniques have been presented to estimate the angular parameters of multiple ID sources [1], [2], [13]–[16]. Unfortunately, all techniques developed for multiple source localization are based on certain approximations of the array covariance matrix, and hence, the resulting parameter estimates are biased.

In the present paper, we develop a new algorithm for ID source localization that does not use any approximation of the covariance matrix. The popular Capon estimator [17]–[19] is generalized to the case of multiple ID sources. The proposed technique is shown to substantially outperform the popular DISPARE algorithm [2] as well as the root-MUSIC based estimator for spread sources [13], [14].

II. Problem Formulation

Assume that the signals of q narrowband stationary sources impinge on an array of p sensors. The complex envelope of the array output can be written as

$$\mathbf{x}(t) = \sum_{i=1}^{q} \mathbf{s}_i(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{x}(t)$ is the $p \times 1$ array snapshot vector, $\mathbf{s}_i(t)$ is the $p \times 1$ vector that describes the contribution of the ith signal source to the array output, and $\mathbf{n}(t)$ is the $p \times 1$ vector of sensor noise.

In point source modeling, the baseband signal of the ith source is modeled as

$$\mathbf{s}_i(t) = s_i(t)\mathbf{a}(\theta_i) \tag{2}$$

where $s_i(t)$ is the complex envelope of the ith source, θ_i is its DOA, and $\mathbf{a}(\theta_i)$ is the corresponding steering vector.

In distributed source modeling, the source energy is considered to be spread over some angular volume. Hence, $\mathbf{s}_i(t)$ is written as [1], [14]

$$\mathbf{s}_i(t) = \int_{\theta\in\Theta} \tilde{s}_i(\theta, \boldsymbol{\psi}_i, t)\mathbf{a}(\theta)d\theta \tag{3}$$

where $\tilde{s}_i(\theta, \boldsymbol{\psi}_i, t)$ is the angular signal density of the ith source, $\boldsymbol{\psi}_i$ is the vector of its location parameters, and Θ is the angular field of view. Examples of the parameter vector $\boldsymbol{\psi}_i$ are the two angular bounds of a uniformly distributed source or the mean and standard deviation of a source with Gaussian angular distribution [14].

Throughout the paper, we will consider the ID source model.[1] For the ith ID source, we have [14]

$$E\{\tilde{s}_i(\theta, \boldsymbol{\psi}_i, t)\tilde{s}_i^*(\theta', \boldsymbol{\psi}_i, t)\} = \sigma_i^2\rho(\theta, \boldsymbol{\psi}_i)\,\delta(\theta - \theta') \tag{4}$$

where $E\{\cdot\}$ denotes the statistical expectation, $\delta(\theta - \theta')$ is the Dirac delta-function, σ_i^2 is the power of the ith source, and $\rho(\theta, \boldsymbol{\psi}_i)$ is its *normalized angular power density* $\left(\int \rho(\theta, \boldsymbol{\psi}_i)d\theta = 1\right)$. We assume that

[1]The assumption of ID sources has been theoretically and experimentally shown to be relevant in wireless communications in the case of rural and suburban environments with a high base station [3]–[5].

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different sources have the same (known) shape of the angular power density but different (unknown) vectors of location parameters.

III. Conventional Capon DOA Estimator

In application to point source modeling, the Capon spatial spectrum estimator has been developed in [17]. This estimator can be considered as a spatial filter (beamformer) that passes the signal of a hypothetical point source arriving from the direction θ while maximally rejecting the signals coming from other directions. The $p \times 1$ beamformer coefficient vector $\mathbf{w}_{\rm opt}$ is obtained as the solution to the following optimization problem [18], [19]:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_x \mathbf{w} \text{ subject to } \mathbf{w}^H \mathbf{a}(\theta) = 1 \tag{5}$$

where $\mathbf{R}_x \triangleq E\{\mathbf{x}(n)\mathbf{x}^H(n)\}$ is the array covariance matrix. In adaptive beamforming, (5) is commonly referred to as the minimum variance distortionless response (MVDR) beamforming problem; see [19]. The solution to (5) is given by

$$\mathbf{w}_{\rm opt}(\theta) = \frac{\mathbf{R}_x^{-1}\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\mathbf{R}_x^{-1}\mathbf{a}(\theta)}. \tag{6}$$

For any direction θ, the Capon pseudo-spectrum is defined through the output power of the MVDR beamformer as [18], [19]

$$P_C(\theta) \triangleq \mathbf{w}_{\rm opt}^H(\theta)\mathbf{R}_x\mathbf{w}_{\rm opt}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{R}_x^{-1}\mathbf{a}(\theta)}. \tag{7}$$

It is clear that if the energy of the ith point source impinges from a direction θ_i, then $P_C(\theta)$ is expected to have a separate peak at $\theta = \theta_i$. Hence, the point source DOAs can be estimated from the q highest maxima of (7), which can be obtained by means of a one-dimensional spectral search.

IV. Generalized Capon Parameter Estimator

To estimate the parameters of ID sources, (5) can be generalized as

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_x \mathbf{w} \text{ subject to } \mathbf{w}^H \mathbf{R}_s(\boldsymbol{\psi})\mathbf{w} = 1 \tag{8}$$

where

$$\mathbf{R}_s(\boldsymbol{\psi}) = \int_{\theta\in\Theta} \rho(\theta,\boldsymbol{\psi})\mathbf{a}(\theta)\mathbf{a}^H(\theta)d\theta \tag{9}$$

is the normalized covariance matrix of the ID source with the parameter vector $\boldsymbol{\psi}$ [14]. According to (8), the generalized Capon spatial filter maintains distortionless spatial response to a hypothetical source with the vector parameter $\boldsymbol{\psi}$ while maximally rejecting the contribution of any other sources. Such a response is now represented by means of the covariance matrix $\mathbf{R}_s(\boldsymbol{\psi})$ (which can be full rank in the general case) rather than the steering vector $\mathbf{a}(\theta)$. In other words, in contrast to (5), the distortionless response is maintained in (8) in the mean power sense rather than in the deterministic sense. The solution to (8) can be found by means of minimization of the Lagrangian function

$$L(\mathbf{w},\lambda) = \mathbf{w}^H\mathbf{R}_x\mathbf{w} + \lambda(1 - \mathbf{w}^H\mathbf{R}_s(\boldsymbol{\psi})\mathbf{w}) \tag{10}$$

where λ is the Lagrange multiplier. Taking the gradient of (10) and equating it to zero, we obtain that the solution to (8) is given by the following generalized eigenvalue problem:

$$\mathbf{R}_x\mathbf{w} = \lambda\mathbf{R}_s(\boldsymbol{\psi})\mathbf{w} \tag{11}$$

where the Lagrange multiplier λ plays the role of the corresponding generalized eigenvalue of the matrix pencil $\{\mathbf{R}_x, \mathbf{R}_s(\boldsymbol{\psi})\}$. Note that the matrices $\mathbf{R}_x$ and $\mathbf{R}_s(\boldsymbol{\psi})$ are both positive semidefinite, and therefore, all generalized eigenvalues of the matrix pencil $\{\mathbf{R}_x, \mathbf{R}_s(\boldsymbol{\psi})\}$ are non-negative real numbers.

Multiplying (11) by $\mathbf{w}^H$ from right and using the constraint $\mathbf{w}^H\mathbf{R}_s(\boldsymbol{\psi})\mathbf{w} = 1$, we obtain that $\lambda = \mathbf{w}^H\mathbf{R}_x\mathbf{w}$. Therefore, the minimal value of the objective function $\mathbf{w}^H\mathbf{R}_x\mathbf{w}$ is equal to smallest generalized eigenvalue of the matrix pencil $\{\mathbf{R}_x, \mathbf{R}_s(\boldsymbol{\psi})\}$. Mathematically, this means that if $\mathbf{w}^H\mathbf{R}_s(\boldsymbol{\psi})\mathbf{w} = 1$, then

$$\min_{\mathbf{w}} \mathbf{w}^H\mathbf{R}_x\mathbf{w} = \lambda_{\min}\{\mathbf{R}_x, \mathbf{R}_s(\boldsymbol{\psi})\} \tag{12}$$

where $\lambda_{\min}\{\cdot,\cdot\}$ denotes the minimal generalized eigenvalue of a matrix pencil.

Similar to the point source case, we define the generalized Capon pseudo-spectrum as the beamformer output power when the beamformer is "steered" to an ID source with the parameter vector $\boldsymbol{\psi}$. Hence, using (12), the generalized Capon (GC) estimator can be written as

$$P_{GC}(\boldsymbol{\psi}) = \lambda_{\;min}\{\mathbf{R}_x, \mathbf{R}_s(\boldsymbol{\psi})\}. \tag{13}$$

Using (11), it is easy to show that (13) can be rewritten as

$$P_{GC}(\boldsymbol{\psi}) = \frac{1}{\sigma_{\max}\{\mathbf{R}_x^{-1}\mathbf{R}_s(\boldsymbol{\psi})\}} \tag{14}$$

where $\sigma_{\max}\{\cdot\}$ stands for the maximal eigenvalue of a matrix. The parameter vector estimates $\hat{\boldsymbol{\psi}}_i$ $(i = 1, 2, \ldots, q)$ can be obtained from the q main maxima of (14). Generally, a d-dimensional search, where d is the length of the vector $\boldsymbol{\psi}$, is required. However, it is common to characterize spread sources by two parameters only (the central angle and the angular spread; see [1], [2], and [14]). In this case, a two-dimensional (2-D) search is required in (14) to estimate the source parameters.

It is worth noting that the conventional Capon estimator (7) is *nonparametric*, whereas the generalized Capon technique (14) is a *parametric* (model-based) estimator.

In a point source case, the vector $\boldsymbol{\psi}$ reduces to the scalar θ and $\mathbf{R}_s(\boldsymbol{\psi})$ transforms to $\mathbf{a}(\theta)\mathbf{a}^H(\theta)$. In this case, it can be readily verified that

$$\sigma_{\max}\{\mathbf{R}_x^{-1}\mathbf{a}(\theta)\mathbf{a}(\theta)^H\} = \mathbf{a}(\theta)^H\mathbf{R}_x^{-1}\mathbf{a}(\theta). \tag{15}$$

Therefore, in the point source case, (14) simplifies to the conventional Capon estimator (7).

V. Simulation Results

In our simulation examples, we consider a uniform linear array (ULA) of ten sensors with the half-wavelength interelement spacing. We used 500 snapshots to estimate the array covariance matrix $\mathbf{R}_x$. Each simulated point is obtained as an average of 100 independent simulation runs. Furthermore, each simulated point is averaged over the sources. The performance of the proposed method is compared with that of the DISPARE method [2] and the root-MUSIC-based technique [13] with additional improvements introduced in [14].

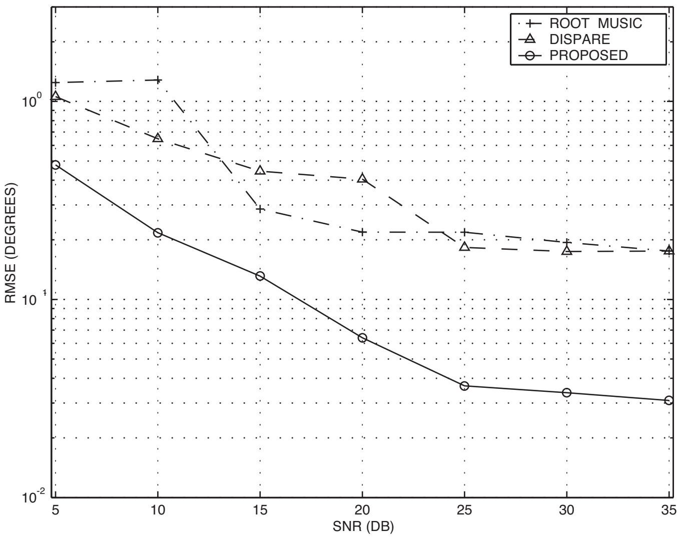

Fig. 1. RMSEs versus SNR for the central angle estimates; first example.

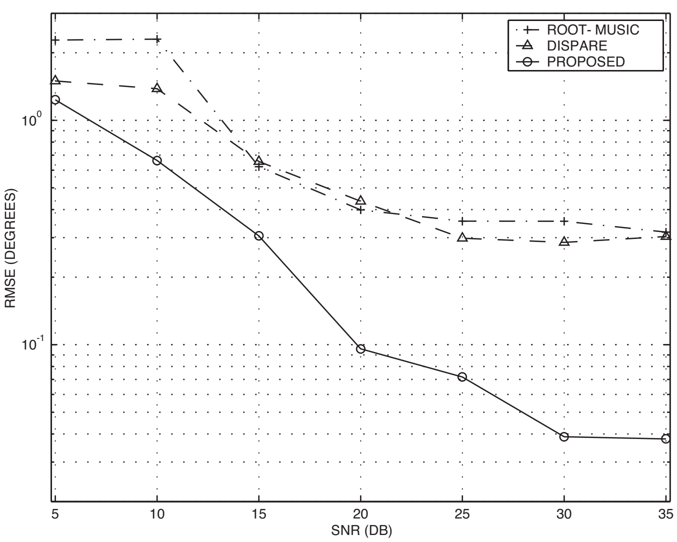

Fig. 2. RMSEs versus SNR for the angular spread estimates; first example.

In the first example, we consider two equipower uniform ID sources with the central angles 0° and 10° and the corresponding angular spreads 4° and 5°, respectively. The angular spread of a uniform source is defined as the total width (support interval) of its angular power density. The root-mean-square errors (RMSEs) of the central angle estimates are shown versus the source signal-to-noise ratio (SNR) in Fig. 1. Fig. 2 shows the RMSEs of the angular spread estimates versus the SNR. As it can be seen from these figures, the proposed generalized Capon estimator has a substantially better estimation performance as compared with the other two methods tested.

In our second example, we assume two equipower Gaussian ID sources with central angles 0° and 15° and the angular spreads (standard deviations) 2° and 3°, respectively. Figs. 3 and 4 show the RMSEs of the central angle and angular spread estimates, respectively, versus the SNR. Similar to the previous example, the proposed generalized Capon estimator can be seen to outperform substantially the other two techniques tested.

VI. Conclusions

In this correspondence, we have developed a new method for estimating the angular parameters of ID sources. The proposed technique

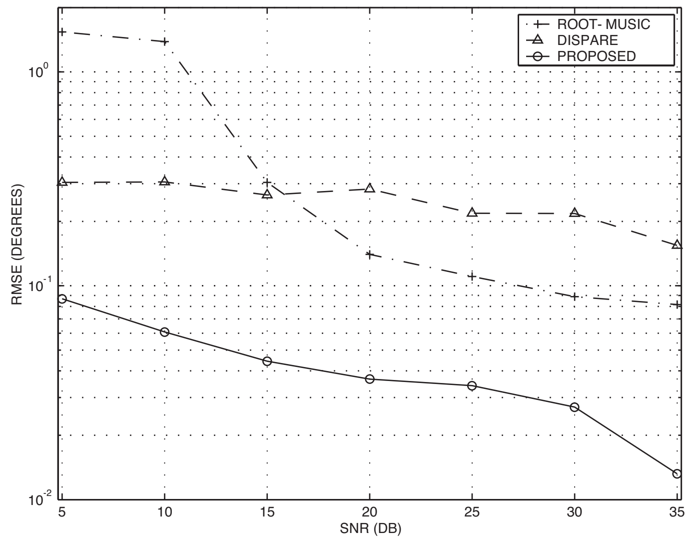

Fig. 3. RMSEs versus SNR for the central angle estimates; second example.

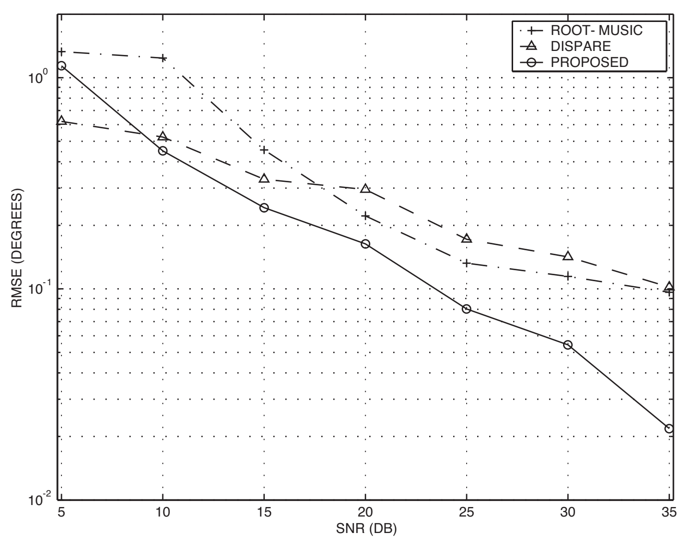

Fig. 4. RMSEs versus SNR for the angular spread estimates; second example.

is based on the generalization of the well-known Capon estimator. Our method involves a 2-D search over the parameter space and shows a substantially improved performance relative to several popular spread source localization techniques.

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Direction Finding for an Extended Target With Possibly Non-Symmetric Spatial Spectrum

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***Abstract*—We consider the problem of estimating the direction of arrival (DOA) of an extended target in radar array processing. Two algorithms are proposed that do not assume that the power azimuthal distribution of the scatterers is symmetric with respect to the mass center of the target. The first one is based on spectral moments which are easily related to the target's DOA. The second method stems from a previous paper by the present authors and consists of a least-squares fit on the elements of the covariance matrix. Both methods are simple and are shown to provide accurate estimates. Furthermore, they extend the range of unambiguous DOAs that can be estimated, compared with the same previous paper.**

***Index Terms*—Covariance matrix, direction-of-arrival estimation, extended target, spectral moments.**

I. Introduction and Motivation of the Work

The problem of estimating the direction of arrival (DOA) of an extended target is an important issue in radar array processing. Briefly speaking, a target can be considered as "extended" as soon as its physical dimensions are of the same order as the array beamwidth (although the signal to noise ratio has also to be accounted for in order to define an extended target). In such a case, the energy backscattered by the target seems to no longer emanate from a point source but from multiple, closely spaced scatterers [2]. This in turn implies that the signal received on the array does not result in a rank-one correlation matrix. In fact, the distribution of the eigenvalues correlation matrix (and in particular the value of the second eigenvalue of the correlation matrix compared to the noise floor) serves as an indicator for defining a target as extended; see, e.g., [3] for a related discussion. Interestingly enough, a similar problem has been recently evidenced in the area of wireless communications. Some campaigns of measurements [4] have shown that local scattering in the vicinity of a mobile is a non-negligible phenomenon. Owing to the presence of local scatterers around the mobile, the source appears to be spatially dispersed, as seen from a base station antenna array. This has a potential impact on the performance of any array processing algorithm and, thus, should be taken into account. Finally, note that in underwater acoustics, a nonhomogeneous propagation medium gives rise to coherence loss along the array [5], [6] and, therefore, to a full-rank correlation matrix.

Briefly stated, the signal received on an array of sensors from a spread source can be described by the following model:

$$\boldsymbol{y}(t) = \boldsymbol{x}(t) \odot \boldsymbol{a}(\theta_0)s(t) \tag{1}$$

where $\boldsymbol{x}(t)$ describes the random multiplicative effect due to local scattering, $s(t)$ is the emitted signal that is independent of $\boldsymbol{x}(t)$, and $\boldsymbol{a}(\theta_0)$ is the so-called steering vector. In the previous equation, $\odot$ denotes the Schur–Hadamard (i.e. element-wise) product. In the case of a point

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