Comments and Corrections

Comments on "Spectrum Sensing in Cognitive Radio Using Goodness-of-Fit Testing"

Nhan Nguyen-Thanh, Thuc Kieu-Xuan, and Insoo Koo

***Abstract*—In this paper, we verify goodness-of-fit testing through the use of the Anderson-Darling (AD) test [1] for spectrum sensing in cognitive radio. In [1], it was shown that spectrum sensing based on the AD test outperforms the energy detection method. However, this positive result can only be obtained when the primary signal is assumed to be static during sensing interval, which is a very rare case in cognitive radio. This assumption reduces the generality of the proposed test in [1]. The verification results of the AD test with some more general and practical primary signals in this paper show that the application of the AD sensing scheme for spectrum sensing in cognitive radio is still a challenge and requires further research.**

***Index Terms*—Cognitive radio, spectrum sensing, Anderson-Darling test, energy detection.**

I. Introduction

Recently, research has dealt with solving the spectrum sensing problem, an important challenge in cognitive radio (CR), by using a goodness-of-fit testing approach called Anderson-Darling (AD) sensing [1]. In the proposed scheme, an AD method is used to test whether the observed samples are drawn independently from the noise distribution, and to further detect the presence of primary users. The general form of the AD statistic is as follows:

$$A_c^2 \triangleq n \int_{-\infty}^{\infty} \left(F_Y(y) - F_0(y)\right)^2 \phi\left(F_0(y)\right) dF_0(y) \quad (1)$$

where $\phi(t) = 1/\left(t(1-t)\right)$ and

$$F_Y(y) = \left|\{i : Y_i \le y, 1 \le i \le n\}\right| / n$$

denote the empirical distribution of the observation. The pragmatic formula of the AD statistic is

$$A_c^2 = -\frac{\sum_{i=1}^{n} (2i-1)\left(\ln Z_i - \ln\left(1 - Z_{n+1-i}\right)\right)}{n} - n \quad (2)$$

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where $Z_i = F_0(Y_i)$. The spectrum sensing problem in [1] can be re-described by:

$$\begin{cases} H_0: & Y_i = W_i \\ H_1: & Y_i = \sqrt{\rho} m + W_i \end{cases} \quad i = 1, 2, ..., n \quad (3)$$

where H_0 and H_1 represent the hypothesis of absence and presence of a primary signal, respectively. Furthermore, W_i is the Gaussian noise with zero mean and unit variance, m represents the transmitted signal, and ρ is the signal to noise ratio (SNR).

The model (3) can be utilized for RF, IF and baseband sensing. The details of the RF and IF sensing models can be found in [3]-[5]. The primary signal, which is pre-filtered through a band pass filter or down-converted to an IF frequency, will be sampled at least twice as fast as the highest frequency of the signal. For baseband sensing case, since it is possible to select IF frequency to be the same as the baseband frequency, the model (3) can be adopted for baseband sensing case [11].

The authors in [1] declared that there was no need to make any assumption about the transmitted signal since spectrum sensing is equivalent to testing the null hypothesis:

$$H_0: Y \text{ is an i.i.d. sequence drawn with distribution } F_0(y),$$

where $Y = \{Y_i\}_{i=1}^{n}$ denotes n observations, and $F_0(y)$ is the noise distribution function. Therefore, the previous study [1] assumed that the transmitted signal $m = 1$. A similar assumption can also be found in [2]. This assumption can be considered realistic only for some rare cases. The first is the case of the static signal which can be found in the event detection of a wireless sensor network, or in the demodulated reflection pulse of an AM radar system. The second is the case of the sensed signal that is sampled with very high frequency, i.e., n times faster than the RF, IF and baseband frequency when a certain levels of synchronization is provided. These conditions are very difficult to obtain in practice. Obviously, beside the disadvantage of the requirement of high speed ADC, the constraint of synchronization is big challenge to ensure that all samples are taken from the same level or symbol period. Further, in the communication systems, since the RF, IF and baseband signal are usually a random signal and is commonly a sum of multiple Non-Line-of-Sight (NLOS) signals, the primary signal m in (3) can approximates to a Gaussian random variable with zero mean due to central limit theorem [10]. Consequently, the assumption of $m = 1$ for generally modeling a primary signal is inappropriate.

In addition, from the detection probability given in (14) from [1]:

$$P_{d,a} \geq 1 - \frac{e^{-C\sqrt{n}}\mathbf{E}\left[e^{B_n}\right]}{e^{-\sqrt{t_0}}} \quad (4)$$

where

$$C = \sqrt{\int_{-\infty}^{\infty} \left(F_1\left(y\right) - F_0\left(y\right)\right)^2 \phi\left(F_0\left(y\right)\right) dF_0\left(y\right)}$$

and

$$B_n = \sqrt{n \int_{-\infty}^{\infty} \left(F_Y\left(y\right) - F_1\left(y\right)\right)^2 \phi\left(F_0\left(y\right)\right) dF_0\left(y\right)},$$

it is obvious that the spectrum sensing performance of the AD sensing method strictly depends on the distribution of observations under the H_1 hypothesis $F_1(y)$. Therefore, the AD sensing method applied for spectrum sensing in the CR network should be verified with other primary signals, which are more realistic and more general. For this purpose, in the following section, we consider two other cases of primary signals: one for practical example of RF/IF sensing and one for a genral case of all RF/IF and baseband sensing.

II. Verification of the AD Sensing Scheme

As mentioned above, besides the rare case of $m = 1$ considered in [1], for RF/IF sensing, we consider the AD sensing scheme with a very simple but practical example, i.e., the primary signal includes only one single carrier frequency f_c in the following sine waveform:

$$m\left(t\right) = \sqrt{2}\sin\left(2\pi f_c t + \varphi\right) \quad (5)$$

where φ is the arbitrary initial phase. The discrete version of $m(t)$ is as follows:

$$m_i = \sqrt{2}\sin\left(\frac{2\pi}{k}i + \varphi\right) \quad (6)$$

where $k = f_s/f_c$ is the ratio between the sampling frequency and the carrier frequency. In this simulation k is assumed to be 6.

Next, for a general case of primary signal that can be utilized for all of RF, IF and baseband sensing, the primary signal m is assumed to be a Gaussian variable with zero mean and variance σ_s^2 which can be found in many spectrum sensing literatures [6]-[10]. This assumption of primary signal is considered to general since the aforementioned fact that the received signal is usually a sum of multiple NLOS signals in practice, and the random characteristics of the received primary signal at RF/IF sensing as well as baseband down-converted sensing. In order to ensure the compatibility with the model in (3), the signal variance σ_s^2 is selected as 1, i.e., $m \sim N\left(0, 1\right)$.

The simulation is first conducted with similar sampling numbers of n = 14 and 28, with -2 dB SNR as in [1]. The performances of the AD and energy detection (ED) sensing schemes for three different primary signals m are shown in

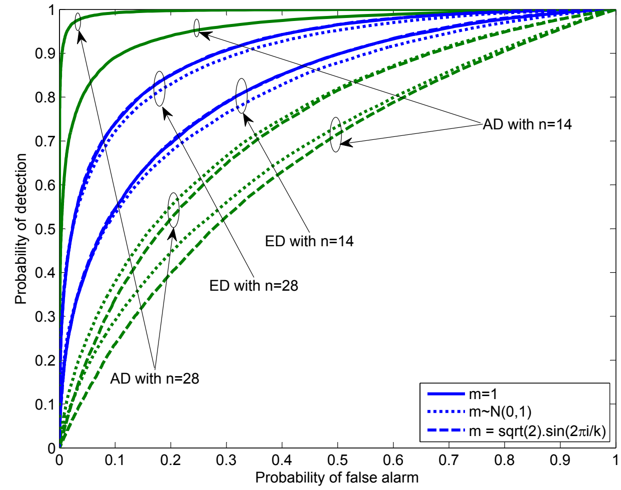

Fig. 1. ROC curves of AD and ED spectrum sensing schemes with different kinds of primary signals and the same SNR = -2 dB, when n = 14 and 28.

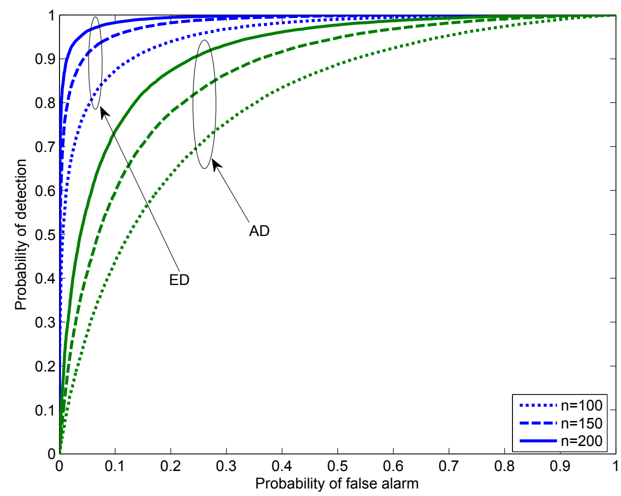

Fig. 2. ROC curves of AD and ED spectrum sensing schemes when n = 100, 150, and 200 with SNR = -4 dB.

Fig. 1. The performances of the AD and ED sensing schemes when $m = 1$ are the same as the result in [1], where the AD sensing scheme outperformed the ED sensing scheme. However, under the realistic condition where m is a single carrier frequency, or the general condition where m is a Gaussian variable, the AD sensing scheme performs poorly compared to the ED sensing scheme. For the same number of samples n, the sensing performances of the ED scheme are similar, regardless of the type of m. Therefore, it is obvious that the conclusion drawn in [1], which suggested that the AD sensing scheme outperforms the ED sensing scheme, is not always true.

Based on the analysis to find an applicable condition for AD sensing, it is found in [1] that $P_{a,d}$ goes to 1 at a speed of at least $O\left(e^{-C\sqrt{n}}\right)$ as $n \to \infty$. Therefore, AD and ED sensing schemes are verified in some large values of n. For observational convenience, only the spectrum sensing results for the single carrier frequency case with SNR = -4 dB are shown in Fig. 2. It is obvious that the performance of the AD sensing scheme is improved when the number of

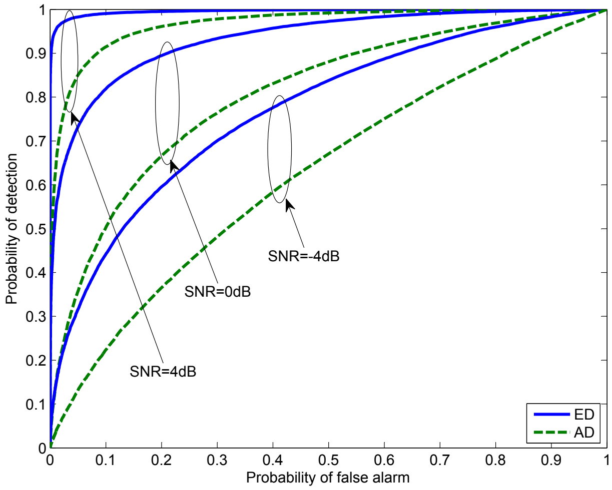

Fig. 3. ROC curves of AD and ED spectrum sensing schemes when n = 20 with SNR = -4, 0 and 4 dB.

samples n is increased. However, the AD sensing scheme cannot outperform the ED sensing scheme, even when the number of samples n is very large.

For further simulation, the number of samples is fixed at 20 and the AD and ED sensing schemes are tested with different SNR scenarios for the general case of primary signal $m \sim N(0, 1)$. The result is shown in Fig. 3. It is clear that the sensing performance is decreased when the SNR decreases. In all cases of SNR, however, the performances of the AD sensing scheme are lower than those of ED sensing scheme.

In addition to the verification using the three cases provided in this paper, the AD sensing scheme was also tested with different kinds of primary signal such as m-PSK, m-QAM and OFDM. Unfortunately, for all of the tested cases, the AD sensing scheme provided the same lower performance when compared to the energy detection method.

III. Conclusions

From the simulation results, it is obvious that the conclusion in [1] stating that AD sensing outperforms ED sensing is only valid when the primary signal is static during sensing interval, which is a very rare case in cognitive radio. Therefore, the application of the AD sensing scheme to spectrum sensing in CR is still a challenge and requires further research.

References

[1] H. Wang, E. Yang, Z. Zhao, and W. Zhang, "Spectrum sensing in cognitive radio using goodness of fit testing," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 5427–5430, 2009.

[2] L. Shen, H. Wang, W. Zhang, and Z. Zhao, "Blind spectrum sensing for cognitive radio channels with noise uncertainty," *IEEE Trans. Wireless Commun.*, pp. 1-4, accepted for publication.

[3] J. Reed, *Software Radio: A Modern Approach to Radio Engineering*. Prentice-Hall, 2002.

[4] K. J. Friedrich, "Software-defined radio: basics and evolution to cognitive radio," *EURASIP J. Wirel. Commun. Netw.*, vol. 2005, pp. 275–283.

[5] L. Luu and B. Daneshrad, "An adaptive weaver architecture radio with spectrum sensing capabilities to relax RF component requirements," *IEEE J. Sel. Areas Commun.*, vol. 25, pp. 538–545, 2007.

[6] L. Yirang, P. Juho, and S. Youngchul, "Upper bound for the loss of energy detection of signals in multipath fading channels," *IEEE Signal Process. Lett.*, vol. 16, pp. 949–952, 2009.

[7] N. Kundargi and A. Tewfik, "A performance study of novel sequential energy detection methods for spectrum sensing," in *Proc. 2010 IEEE International Conference on Acoustics Speech and Signal Processing*, pp. 3090–3093.

[8] A. Kortun, T. Ratnarajah, M. Sellathurai, and Z. Caijun, "On the performance of eigenvalue-based spectrum sensing for cognitive radio," in *Proc. 2010 IEEE Symposium on New Frontiers in Dynamic Spectrum*, pp. 1–6.

[9] J. Moragues, L. Vergara, J. Gosalbez, and I. Bosch, "An extended energy detector for non-Gaussian and non-independent noise," *Signal Process.*, vol. 89, pp. 656–661, 2009.

[10] J. Ma, G. Zhao, and Y. Li, "Soft combination and detection for cooperative spectrum sensing in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 11, pp. 4502–4507, 2008.

[11] S. J. Shellhammer, "Spectrum sensing in IEEE 802.22," in *Proc. 2008 CIP*.