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A Simple Way of Obtaining Optimized Patterns Using the Woodward-Lawson Method

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Abstract

This paper describes a new way of applying the Woodward-Lawson method in order to avoid its limitations concerning pattern performance. The proposed method obtains the Woodward-Lawson beam coefficients by sampling a previously synthesized optimal pattern using the Orchard-Elliott method, which provides the advantages of both synthesis methods. This work also discusses new insights about the multiplicity of solutions in the Woodward-Lawson method.

Keywords: Antenna arrays; optimization methods; antenna radiation patterns; multibeam antennas; shaped beam antennas

1. Introduction

Multi-beam antenna arrays have applications in the fields of electronic countermeasures, in satellite communications, and in adaptive nulling [1]. For these systems, the most natural method for synthesizing multiple beams is the Woodward-Lawson method, which can be implemented using lossless orthogonal beam-forming networks such as the Butler matrix [2]. An example is the Rotman lens, which is inherently broadband, where each beam port may simply be weighted and summed to generate a shaped beam pattern [3]. The major objections to the Woodward-Lawson method have been its inability to control either the amplitude of the ripple in the shaped region of the pattern, or the heights of sidelobes in the unshaped region [2, 4]. Another telling objection is that the pattern zeros occur in pairs. This means that there are only half as many adjustments available as there could be for a given number of elements in an array. The practical result is that in the shaped-pattern region, the number of ripples is half what it could be, and thus the ripples tend to be larger [3]. These weaknesses are ultimately due in part to the fact that the Woodward-Lawson method constrains all the beams with which it synthesizes the desired pattern to have the same phase, so that the far-field pattern is real. They are also in part due to the method's zero tolerance of deviation from the desired field at the points at which it samples this field. Cid et al. [5] showed that the performance of the patterns obtained can be dramatically improved by perturbing the amplitudes and/or phases of the beams. However, this perturbation was performed using the simulated-annealing technique, a numerical method that may require a lot of iterations for arrays with a large number of elements.

In this letter, we present a method that obtains the beam coefficients by sampling a previously synthesized pattern using the

Orchard-Elliott method [6], which overcomes the limitations of the original Woodward-Lawson method. Besides this, the Orchard-Elliott technique is computer efficient, inexpensive, and rapidly convergent.

2. Description of the Method

Briefly, the Woodward-Lawson method for a linear array of N antenna elements spaced a distance d apart consists in first sampling a desired field pattern, $F(\theta)$ in $2M+1$ directions, θ_i, located at $\cos^{-1}(i\lambda/Nd)$ (where θ is measured from end-fire, M is such that the visible region is just covered, and i is an integer). $F(\theta)$ is then approximated by [7]

$$WL(\theta) = \sum_{i=-M}^{M} F(\theta_i) \frac{\sin(Nu_i)}{N\sin(u_i)}, \quad (1a)$$

where

$$u_i = \frac{\pi d}{\lambda}\left[\cos(\theta) - \cos(\theta_i)\right]. \quad (1b)$$

Each of the component quasi-sinc beams, $\sin(Nu_i)/\sin(u_i)$, is realized by an excitation distribution with uniform amplitude and uniform phase differences between the elements. $WL(\theta)$ is produced by the weighted sum of these distributions, with weights $F(\theta_i)/N$. $WL(\theta)$ reproduces $F(\theta)$ exactly at each sample point, because all the component quasi-sinc beams except the ith are zero at θ_i.

As noted above, the original Woodward-Lawson method assumes that all the $F(\theta_i)$ are in phase. However, we have found that the phase information of the beam coefficients is critical to obtaining good-quality patterns. For example, it is possible to double the number of ripples in shaped region just by introducing a phase in the Woodward-Lawson coefficients. This yields a pattern with a narrower shaped region and with a lower ripple level. We have also found that the Woodward-Lawson method always provides a multiplicity of solutions for the current distributions: once we have calculated the excitations provided by the Woodward-Lawson method, it is possible to factorize the polynomial associated with the array factor, and to represent the roots obtained in a Schelkunoff circle. As it is well known, with Q roots off the unit circle, there are 2^Q current distributions producing the same normalized power pattern (although the phase pattern is different). One of these will be the easiest to realize in an actual situation that takes mutual coupling into account. This multiplicity of solutions does exist even if the roots are grouped in pairs, as is usual in the original Woodward-Lawson method. In this case, there are $3^{Q/2}$ different solutions, since these pairs can be ungrouped to compose double roots. This is also applicable to any pattern-synthesis method that synthesises real patterns, such as [8].

In this work, we obtain the Woodward-Lawson beam coefficients by sampling (keeping both the amplitude and phase) of a previously synthesised pattern. Although this pattern could be obtained using any optimal pattern-synthesis technique available in the previous literature – such as the method described by Bucci et al. [9] – we have chosen the Orchard-Elliott method [6], a powerful technique that uses the physical principles of adjusting zeros in the array pattern polynomial.

The proposed method combines the Orchard-Elliott procedure and the Woodward-Lawson method. First, given a power pattern with a desired ripple and sidelobe topography, the Orchard-Elliott method is used to synthesise this pattern. As is well known, this method allows excellent control in both the shaped and the sidelobe regions of the obtained pattern (which, in the case of shaped beams, is complex, unless we group the filled roots in pairs). After that, we apply the Woodward-Lawson method, selecting as beam coefficients the samples of both the amplitude and phase patterns obtained. The pattern so obtained with these samples is exactly the same as the previously synthesized optimal pattern.

The advantages of this method are not only the quality of the resulting patterns, but also the multiplicity of the provided solutions. We stated above that Woodward-Lawson always has a multiplicity of solutions for the current distribution. Besides this, in the proposed method there is also a multiplicity of solutions for the beam coefficients: if the pattern has Q filled nulls, the Orchard-Elliott method produces 2^Q independent sets of excitations that generate the same (normalized) amplitude patterns, but different phase patterns. Therefore, we also have 2^Q sets of samples to be used in the Woodward-Lawson method. Among these sets, we can choose that one that leads to an easier physical realization of the array, using, for example, a Rotman lens-fed array.

The method is also applicable in those antenna applications that require an equiphase pattern, corresponding to a symmetric amplitude distribution. In that case, we apply the Orchard-Elliott method, grouping the filled roots of the pattern in pairs [8]. Obviously, the samples obtained for the Woodward-Lawson method are now real (with phase values of 0 or π radians), but the resulting pattern is again equal to the original.

Table 1. The positions and values of the samples of the cosecant-squared pattern synthesized by the Orchard-Elliott technique, to be used in the Woodward-Lawson method.

i	$w_i = \cos(\theta_i)$	Pattern samples $F(\theta_i)$	
		Amplitude	Phase (deg)
–8	–1.00	0.01	135.94
–7	–0.88	0.49	19.37
–6	–0.75	0.89	–168.40
–5	–0.63	0.90	–5.95
–4	–0.50	6.29	118.92
–3	–0.38	8.15	–6.30
–2	–0.25	10.93	–125.59
–1	–0.13	15.83	–176.43
0	0.00	24.88	–141.52
1	0.13	7.71	–135.17
2	0.25	0.30	43.39
3	0.38	0.26	43.67
4	0.50	0.57	–141.46
5	0.63	2.00	36.79
6	0.75	1.16	–146.32
7	0.88	0.54	29.65
8	1.00	0.01	–44.06

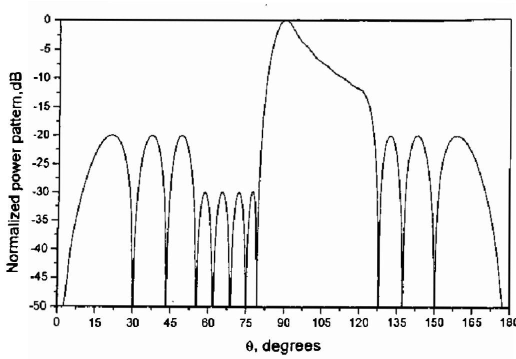

Figure 1. The cosecant-squared pattern obtained using the Woodward-Lawson method with the beam coefficients of Table 1.

3. Example

Suppose that we require a 16-element ($\lambda/2$)-spaced linear array to produce a $\text{cosec}^2(\theta)\cos(\theta)$ pattern with a ripple requirement of ±0.1 dB, four −30 dB sidelobes to the left of the cosecant beam, and all other sidelobes at −20 dB, as in the example of [6, Figure 4d]. This pattern can be easily synthesised by using the Orchard-Elliott method. After that, the pattern is sampled in order to apply the Woodward-Lawson technique. Table 1 lists the positions and values of the complex samples to be used as beam coefficients. Figure 1 shows the resulting pattern after applying the Woodward-Lawson method, which is the same as that previously synthesized.

Since the synthesised pattern has four filled nulls, there are 16 different sets of excitations, each producing the same (normalized) amplitude pattern but a different phase pattern, and hence a different set of samples for the Woodward-Lawson method. As stated in the previous section, this gives the possibility of choosing the solution that simplifies the ability to physically realize the antenna without modifying the power pattern.

4. Conclusions

The proposed method obtains the Woodward-Lawson beam coefficients by sampling a previously synthesized pattern using the Orchard-Elliott method. The combination of both synthesis methods will provide the advantages of each: on the one hand, power patterns with a controlled ripple and sidelobe topography, and also a set of coefficients of the orthogonal beams that can be realized in real life with an array feed consisting of a Butler matrix.

In the case of linear distributions, it is possible to use as Woodward-Lawson beam coefficients the samples of the shaped-beam patterns (such as those described in [10-12]), produced by these distributions. This allows obtaining the excitations of an equivalent equi-spaced linear array with a pattern that fits the optimal patterns produced by the linear distribution very well.

This method may be extended to direct-broadcast satellite phased arrays by combining it with the sampling-function method developed by Richie and Kritikos [13, 14]. In this case, the pattern to be sampled can be synthesized by using a recent quasi-analytical method that shapes the desired footprint as a composition of several ϕ-symmetric circular Taylor patterns exhibiting flat-topped beams [15].

5. Acknowledgment

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Ideas for Antenna Designer's Notebook

Ideas are needed for future issues of the Antenna Designer's Notebook. Please send your suggestions to Tom Milligan and they will be considered for publication as quickly as possible. Topics can include antenna design tips, equations, nomographs, or shortcuts, as well as ideas to improve or facilitate measurements.

Hidden Word

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R	D	E	C	O	N	V	O	L	U	T	I	O	N	P
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B	F	V	I	H	E	E	P	L	E	R	E	M	E	O
M	L	L	E	U	L	E	G	O	A	R	A	L	P	F
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H	C	N	A	R	B	L	N	S	N	T	I	T	S	L
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Anechoic, Arid
Beppu, Blarney, Bragg, Branch
Calibrate, Chamber, Cobbler
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Helix, Hurry
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Lease, Like, Loft, Loral
Magnitude, Mesoscope
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