Scattering by a Paraboloidal Reflector Antenna: A Retraction and an Explanation

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Since January, 2002, there have been seven notes and short papers in this *Magazine* dealing with equivalent circuits and the scattering properties of reflector antennas [1-7]. In a seventh paper, by David Pozar [7], the Editor of this *Magazine* added a reference list for the first six of these. The list was in reverse chronological order. In [4] (April, 2003), Bob Collin disagreed with my claim that the equivalent circuit described by me in [5] (October, 2002) accounted for all the power scattered by an aperture antenna. In turn, I responded in [2] (August, 2003) with an argument defending my position. This was immediately followed by a refutation by Bob Collin in [3] (also August, 2003). Obviously, one of us must be wrong.

The purpose of this note is to admit that I made a serious error in [2] (August, 2003) in arguing that forward scattering could not account for the shadow zone behind a reflector antenna. For this mistake, I apologize to all readers of this *Magazine*, and to Bob Collin, in particular. Through e-mail and letters, Bob tried to convince me of the error of my ways, but to no avail: I simply could not get the right mental picture of the scattering process into my head. I knew that forward scattering occurs for a conducting sphere of radius a and that its total scattering cross section, called the extinction cross section, is $2\pi a^2$, exactly twice the backscattering cross section [8]. I did not believe, however, that this fact necessarily applied to a reflector antenna.

This was troublesome, for I could not reconcile that fact – which Bob Collin assured me was also true for a circular paraboloidal reflector – with my own feeling of certainty that the reflector has a receiving cross section of πa^2 if it is assumed to have a 100% efficient feed [9]. It would therefore receive power $P = S\pi a^2$ from a normally incident plane wave having a flux of S W/m². I could not understand how a second power equal to P could come from the incident wave to create the forward-scattered field. I was not alone in this regard, for several others expressed similar concerns via e-mail notes.

Understanding finally came as the result of a *gedanken* (thought) experiment, as I sat thinking about the problem. With pen and paper, I made two sketches, as shown in (a) and (b) of Figure 1. In Figure 1a of this geometrical-optics picture, the incident rays are shown reflected from the concave surface to the focus on the left, where the power, $P = S\pi a^2$, is entirely absorbed by the

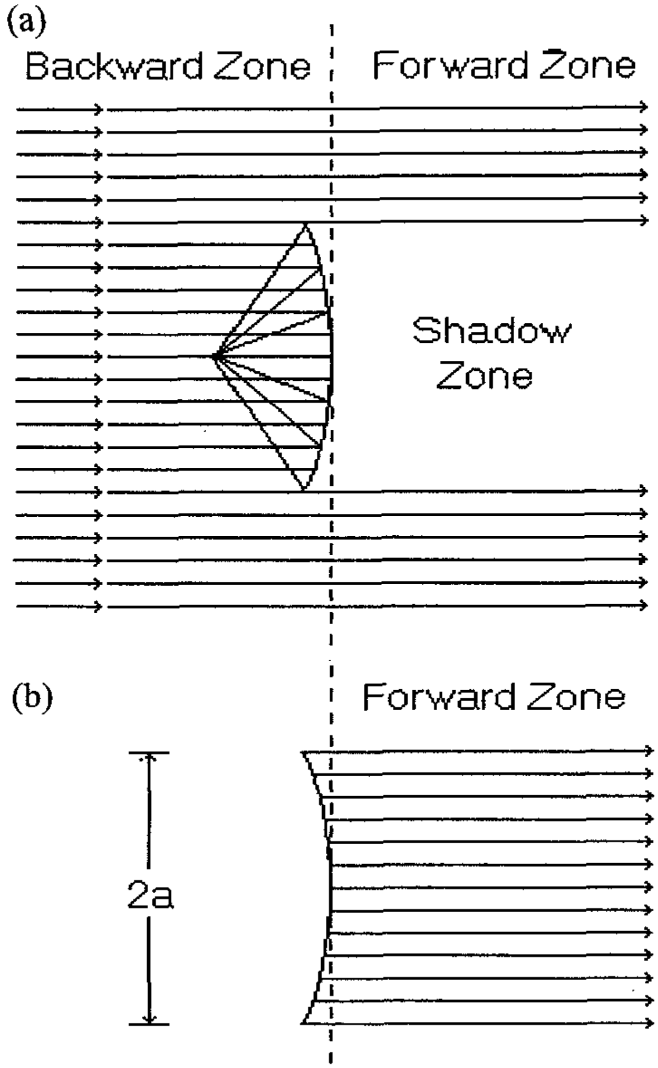

Figure 1. (a) A plane wave at normal incidence on a paraboloidal reflector antenna. (b) The case of (a), after superimposing a phantom field in anti-phase with the incident field. The flux is S Watts/meter².

feed. On the right is the clearly defined shadow zone, in which there are no rays and no field. Now, envision an imaginary field that is everywhere equal to but in anti-phase with the incident field, to be superimposed upon the latter. The result of adding this phantom field is shown in (b) of the figure. There is now no field anywhere except in the former shadow zone, and in this tunnel region the power level is clearly $P = S\pi a^2$. The phantom field must be a radiating field, and its properties are easy to deduce. It originates at a circular aperture, located in the plane indicated by the dashed line in (b) of Figure 1. Since that aperture is clearly uniformly illuminated in both phase and amplitude, it must have a radiation pattern in the distant Fraunhofer zone given by $2J_1(u)/u$, where $u = ka\sin\theta$ and θ is the angle from boresight. Furthermore, it will have a gain given by $G = (2\pi a/\lambda)^2$.

This pictorial representation is oversimplified, for it totally ignores the effects of diffraction, which will cause the parallel rays in Figure 1b to begin to diverge, and eventually to create the far-field radiation pattern described above by the Bessel function. Similarly, Figure 1a will be modified, presumably by the convergence of the outer rays downstream in the forward zone, in such a way as to fill in the shadow tunnel, eventually making it look like Figure 1b, but with a gap immediately behind the reflector. In this case, Figure 1a should also give rise to the same far-field Bessel-function pattern (except for sign), and the same gain, G, described above for Figure 1b. Since the flux is still equal to S, the power is given by $P = S\pi a^2$, and my problem of the missing power is solved. It comes from the forward-scattered field, and not from the field that is incident on the reflector. It appears that the forward-scattered field is entirely independent of the absorbed field, and also of any backscattered field, if the feed is not 100% efficient.

An interesting corollary is that the bistatic forward radar cross section of the paraboloid of Figure 1, defined by the product of the gain and the cross sectional area, is given by $4\pi^3\left(a^2/\lambda\right)^2$ [10]. Incidentally, the expression given by Equation (1) in [3] for the forward-scatter cross section is in error. It should be multiplied by the factor πa^2.

The argument presented above is conjectural, and I make no claim that it constitutes a proof. I do think it possesses a high degree of plausibility, and I hope it will shed some light on what has been, for me, a somewhat arcane subject. A question that remains unanswered here is, how far behind the reflector does the shadow zone extend? Some calculations I have made suggest that it is not very far (perhaps only a few aperture radii) before fringing fields appear at the boundary of the tunnel.

In addition to Professor Robert Collin, I am also indebted to the late Professor Chen-To Tai and to Professor P. J. B. Clarricoats for e-mail discussions on the topics of equivalent circuits and scattering theory.

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