Scattered and Absorbed Powers in Receiving Antennas

David Pozar

Electrical and Computer Engineering
University of Massachusetts at Amherst, Amherst, MA 01003 USA
E-mail: pozar@ecs.umass.edu

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Several articles have recently appeared in the *Magazine* concerning the question of how much power is scattered and absorbed by a receiving antenna and, in particular, whether an antenna can absorb *all* of the power incident upon it. While much has been written, it is not clear that this question has been answered and explained to the satisfaction of interested readers. Hopefully, this short note will help to clarify the situation by considering the absorbed and scattered power from dipole arrays in either free space, or over a ground plane. By defining a suitable "aperture efficiency" for the receiving case, we show that a dipole array without a ground plane can at best absorb half of the incident power (scattering the rest), while an array over a ground plane can absorb *all* of the incident power. We also show how aperture efficiency varies with load impedance, which is of practical interest for array designers.

Consider a square array of $N \times N$ center-fed parallel lossless dipoles of length L and radius c, with spacings a in both the E- and H-plane directions. Assume the dipoles lie in the $z = d$ plane, parallel to the x axis; a ground plane may be placed at $z = 0$. Applying the usual Moment Method procedure, with M piecewise sinusoidal (PWS) expansion modes per dipole, leads to the matrix equation

$$\{[Z]+[Z_L]\}[I]=[V], \tag{1}$$

where $[Z_L]$ is a diagonal matrix of the load impedances. For plane-wave excitation, the voltage vector has elements given as

$$V_i = \int_i F_i(x) E_x^{inc} dx, \tag{2}$$

where $F_i(x)$ is the PWS expansion mode function. For a broadside plane wave, the incident field for an array in free space is given as

$$E_x^{inc} = E_0, \text{ (free-space)}, \tag{3a}$$

but when a ground plane is spaced a distance d below the array, the incident field is given as

$$E_x^{inc} = 2jE_0 \sin kd \text{ (ground plane)}. \tag{3b}$$

The form of Equation (3b) is due to the fact that the excitation field of the Moment Method procedure is that field that is present when the expansion modes are set to zero, which, in this case, involves reflection of the incident plane wave from the ground plane.

The power delivered to the load (absorbed power) is given by

$$P_L = [I]^t [Z_L][I]^*. \tag{4}$$

If the array is in free space, the scattered power comes solely from the dipoles, and can be found as

$$P_s = [I]^t [Z][I]^* \text{ (free space)}. \tag{5}$$

If the array is over a ground plane the situation is more complicated, because there are components of scattered field from the dipoles and the ground plane (similar to what occurs in a reflectarray). The far fields from the dipoles can be found as

$$\bar{E}_{dip} = \sum_i I_i \bar{E}_i, \tag{6}$$

where $\bar{E}_i$ is the far-zone field from the ith dipole expansion mode. We must consider a finite-size ground plane to obtain spherical waves (and finite power) for the reflected field from the ground plane. A reasonable approach is to assume a ground plane of size $(Na)\times(Na)$, and apply Physical Optics to obtain the reflected field:

$$\bar{E}_{gp} = \frac{-jkE_0 N^2 a^2}{2\pi r} e^{-jkr}\left[\hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi\right] \operatorname{sinc}(kNau/2)\operatorname{sinc}(kNav/2), \tag{7}$$

where $\operatorname{sinc}(x) = \sin(x)/x$, $u = \sin\theta\cos\phi$, and $v = \sin\theta\sin\phi$. Then, the total power scattered from the array over a ground plane can be found by integration of the total field:

$$P_s = \frac{1}{\eta_0} \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \left|\bar{E}_{dip} + \bar{E}_{gp}\right|^2 r^2 \sin\theta \, d\theta \, d\phi. \tag{8}$$

There are at least two possible ways to define an efficiency metric for the power that is absorbed by the loads of the receiving array. One way is to define a *receiving efficiency* as the ratio of absorbed power to the sum of the absorbed and scattered powers (the total received power):

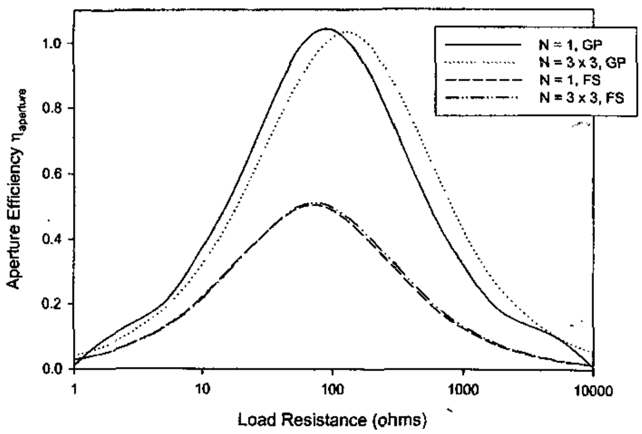

Figure 1. Receiving aperture efficiency for arrays of wire dipoles in free space and with a ground plane, versus load resistance: $L = 0.487\lambda$, $c = 0.001\lambda$, $a = 0.5\lambda$, $d = 0.25\lambda$.

$$\eta_{receive} = \frac{P_L}{P_L + P_S}. \tag{9}$$

This definition may appear reasonable, but in fact is not useful unless the loads are conjugately matched to the antennas. Since Equation (9) is not normalized to the incident power, it is possible for this efficiency to approach unity, even though the array is receiving very little power from the incident wave. This can happen, for example, for an array in free space with very high load impedances.

A better definition uses the incident power to normalize the absorbed power:

$$\eta_{aperture} = \frac{P_L}{P_{inc}} = \frac{P_L}{A_{eff} S_{inc}}, \tag{10}$$

where A_{eff} is the effective area of the array, and $S_{inc} = |E_0|^2 / \eta_0$ is the power density incident upon the array in W/m^2. We refer to this quantity as the *receiving aperture efficiency*. This definition incorporates the physically appealing idea that maximizing the absorbed power for a given incident field should maximize receiving aperture efficiency. Note that the definition is consistent with the standard definition of effective area for a conjugately matched receiving antenna.

The effective area of the array is subject to interpretation. The straightforward approach of using the physical area $A_{eff} = N^2 a^2$ is reasonable for large arrays, but loses meaning for small arrays (what is the effective area of an array with one element?), and can result in Equation (10) being slightly greater than unity. We have found that basing the effective area on the directivity, D, of the array when transmitting works better. Thus, we use $A_{eff} = \lambda^2 D / 4\pi$.

Figure 1 shows the resulting aperture efficiency of Equation (10) for dipole arrays of one and nine elements, with and without a ground plane, versus load resistance. Observe that the maximum efficiency in both cases occurs near conjugate matching (in the range of 70 – 90 Ω), but that the free-space arrays achieve a maximum receive aperture efficiency of 50%, while the arrays with a ground plane achieve 100% efficiency. It is also interesting to observe that this situation applies to single elements as well as to arrays of larger size. Note that the scattered fields and power of Equations (5)-(8) are not used directly in Equation (10), but numerical results show that the total scattered power does indeed drop to zero for the case of a conjugately matched array over a ground plane, and it is the cancellation of the re-radiated dipole fields with the fields reflected from the ground plane that causes this effect.

In closing, I think that Allan Love's intuition that it is possible for a receiving antenna to "capture" all available incident power, without re-radiating or scattering any of that incident power, is correct, and is demonstrated by these results. In fact, of course, practical antenna performance would be seriously hampered if this were not the case.

[Editor's note: In connection with the above contribution, readers may also be interested in the following contributions that have recently appeared in the *Magazine*:

Jørgen Bach Andersen and Rodney G. Vaughan, "Transmitting, Receiving, and Scattering Properties of Antennas," *IEEE Antennas and Propagation Magazine*, **45**, 4, August 2003, pp. 93-98.

Allan W. Love, "Comment on 'Limitations of the Thevenin and Norton Equivalent Circuits for a Receiving Antenna'," *IEEE Antennas and Propagation Magazine*, **45**, 4, August 2003, pp. 98-99.

R. E. Collin, "Remarks on 'Comment on "Limitations of the Thevenin and Norton Equivalent Circuits for a Receiving Antenna"'," *IEEE Antennas and Propagation Magazine*, **45**, 4, August 2003, pp. 99-100.

R. E. Collin, "Limitations on the Thevenin and Norton Equivalent Circuits for a Receiving Antenna," *IEEE Antennas and Propagation Magazine*, **45**, April, 2003, pp. 119-124.

Allan W. Love, "Comment: On the Equivalent Circuit of a Receiving Antenna," *IEEE Antennas and Propagation Magazine*, **44**, October, 2002, pp. 124-125; also see the "Correction," *IEEE Antennas and Propagation Magazine*, **44**, December, 2002, p. 146.

J. Van Bladel, "On the Equivalent Circuit of a Receiving Antenna," *IEEE Antennas and Propagation Magazine*, **44**, January, 2002, pp. 164-165.

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