

convergence, random number generators, and Markov processes (pp. 505–551). The text is quite readable, despite difficult topics covered. The majority of important theorems, ideas of algorithms, methods, and issues are well illustrated by over 130 examples. The author puts a lot of well-balanced comments in the text and this type of lecturing definitively helps with understanding presented stochastic methods. The level of mathematical complication is reasonably low—perhaps the most formal chapter contains less than 20 equations. The focus is on theoretical foundations of the algorithms rather than on formal mathematical proofs. That type of presentation will be welcome by less theoretically oriented readers. Despite of that, the presentations of algorithms and theorems are rigorous, and, in many instances, the references to original papers or relevant monographs are given. The list of 415 references covers all major literature related to stochastic search, optimization, control, and analysis.

In chapter 1, a background for understanding stochastic search and optimization is given. Basic notions of loss function and global optimization are presented and the meaning of "stochastic" is explained. Direct search methods are covered in chapter 2. Especially interesting here is a part devoted to random search with noisy loss measurements and the nonlinear simplex algorithm. In chapter 3, recursive optimization algorithms for linear models are described. It would be nice to have linear models working perfectly in all areas of optimization and modeling; unfortunately, quite often they are not adequate. Anyhow, the least-mean-square solutions to recursive estimation problem are very popular. I believe that all readers will enjoy a beautiful example of application of recursive-least-squares algorithms for estimation of quality of oboe reed—a part of musical instrument.

In chapters 4–7, core issue of stochastic approximation in nonlinear search and optimization is discussed. First, the famous root-finding Robbins–Monro (1951) algorithm is presented. Convergence problems are extensively discussed. Then, implementations of stochastic gradient algorithms are described such as neural network training or image restoration. A separate chapter 6 is devoted to problems when the stochastic gradient is not available and finite-difference approximation has to be used. If a large number of variables is to be optimized, good efficiency may be obtained using simultaneous perturbation stochastic approach described in chapter 7. In order to avoid trapping in a local minimum, sometimes random perturbations are "injected" into algorithms giving a better chance for global optimization. Such annealing-type algorithms are described in chapter 8. This chapter should be most useful for biophysical community using simulated annealing molecular dynamics for finding optimum structures of proteins or nucleic acids. Chapters 9 and 10, devoted to evolutionary computations, are particularly valuable: in a concise way, they present both the traditional schema theory and a Markov-chain-based approach. Fans of genetic algorithms will be, however, disappointed: using no-free-lunch theorem, the author shows that often claimed superiority of such an approach towards optimization is not true. Learning problems, very important, for example, in artificial intelligence, are only slightly touched upon in chapter 11, which presents reinforcement learning via temporal differences. In this part, I have found quite interesting discussion on why temporal difference methods can work better than supervised learning.

Chapter 12 sits a little bit aside. It is devoted to a specific class of optimization where parameters being optimized take only discrete values. The chapter examines several approaches to multiple comparison tests and gives some background for decision supporting software. Numerous readers will find chapter 13 to be the most important. It discusses such fundamental issues as mathematical model selection, tradeoff between bias and variance in choosing model form, cross validation as a tool for selecting a model, Fischer information matrix,

etc. Chapters 14 and 15 focus on computer-simulation-based optimizations. They offer substantial cost savings but should be used wisely. The large part on common random numbers, though dealing with important problem of reduction of variance, is, in my opinion, too specialized. In chapter 15, it is shown how to exploit the simulation process information on the gradient of the loss function. Most examples are built here on multiple gradient evaluations within the sample path optimization method. Markov chain Monte Carlo (MCMC) methods for computing statistical estimates, integrals, etc., are very well described in chapter 16. Gibbs sampling is discussed here in great detail. Chapter 17 is devoted to a very popular problem of optimal design for experimental inputs, a very important subject in statistics, quality control, medical clinical trials, and agriculture. The author presents main algorithms for finding optimal designs for linear and nonlinear cases and gives some useful examples.

It is worth noting that over 250 exercises form an integral part of the book. Only a small part of them have solutions included in the volume, but the author published a separate solution manual (J. C. Spall, *Solutions Manual for Introduction to Stochastic Search and Optimization: Estimation, Simulation, and Control*, pp. 145, 2003). He also maintains a very informative webpage [www.jhuapl.edu/ISSO](http://www.jhuapl.edu/ISSO) which contains MATLAB files for many algorithms, instructional material, data files, and errata.

The book may serve as either a reference for researchers and practitioners in many fields or as a textbook. It requires previous knowledge of basic probability and statistics, multivariate calculus, and some matrix algebra. Thus, students of virtually all graduate courses in science and engineering departments may profit from this useful book. Exploiting knowledge on stochastic search and optimization presented in this book should contribute to solid advancements of many areas of experimental and practical activity.

**Towards a Unified Modeling and Knowledge-Representation Based on Lattice Theory: Computational Intelligence and Soft Computing Applications (Studies in Computational Intelligence)**—Vassilis G. Kaburlasos (Berlin, Germany: Springer-Verlag, 2006, pp. 245, ISBN: 10-3-540-34169-2). *Reviewed by Gerhard X. Ritter*

The author presents a novel proposal in computational intelligence including neural computing based on lattice theory. With the proliferation of computers, a variety of paradigms emerged under computational intelligence and soft computing. An advancing technology is fragmented in different domains due, as the author also claims, to the need to cope with disparate types of data including vectors, Boolean data, symbols, images, text, graphs/structures, (fuzzy) sets, etc. The author proposes lattice theory as a sound theoretical basis for unifying disparate (lattice-ordered) data including both numeric and nonnumeric data.

Lattice theory has, traditionally, been popular in logic as well as in image processing due to the applications of mathematical morphology in image analysis. It was in the late 1980s that Davidson and Hejman showed that, for all practical purposes, mathematical morphology forms a subalgebra of general lattice theory. In this book, the author

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pursues unified modeling, i.e., function approximation, as well as unified knowledge-representation in clustering, classification, and regression applications based on lattice theory. Useful mathematical tools are introduced. Comparative experimental results are presented extensively. Interdisciplinary work by other authors is also presented comparatively. All required mathematics is well covered in this volume with additional mathematical background provided in the appendices.

This book is organized in four parts. Part I, including chapters 1 and 2, describes the origins as well as a relevant literature review. In particular, chapter 1 highlights the chronicle of this work starting from the author's doctoral work on fuzzy adaptive resonance theory (ART) neural network; the latter computes lattice-ordered hyperboxes in the unit hypercube. Chapter 1 also outlines the development of lattice theory since the introduction of Boolean algebra in the mid-19th century. Chapter 2 stakes out the domain of interest, that is model induction as well as knowledge discovery. An extensive literature review is presented from classic system modeling to human-oriented modeling. Computational intelligence, or equivalently, soft computing, emerges as a pluralistic coalition of methodologies to deal with real-world problems. In this context, lattice theory is proposed as a sound foundation for unified system analysis and design. A review of lattice theory applications is presented.

Part II, including chapters 3–7, details novel mathematical tools as well as a number of algorithms for clustering, classification, and regression. More specifically, chapter 3 summarizes basic lattice theory. An instrumental novelty here is the employment of positive valuation functions in order to introduce both a metric and an inclusion measure function in a lattice. Chapter 4 shows the relevance of lattice theory by presenting popular lattices including the Euclidean space  $R^N$ , a Boolean lattice of propositions, a probability space, a linear metric space, a space of graphs, etc. Special attention is paid to the lattice of *fuzzy interval numbers (FINs)*. The author introduces rigorously an analysis of FINs based on novel *generalized intervals*. Chapter 5 focuses on knowledge representation. The author makes a distinction between “data” and “information”; the latter is the meaning a human attaches to data. Then, he defines “knowledge” as a set of rules, i.e., samples of a “lattice function” having both a lattice domain and a lattice range. In conclusion, the author argues how a (mathematical) lattice can represent tunable semantics. Chapter 6 presents the terms “function” and “model” as synonyms. Then, it formulates the modeling problem as an optimal function approximation problem based on (input, output) data pairs of a lattice function. Optimization can be pursued by genetic algorithms. Chapter 7 presents a number of algorithms for model induction. A family of algorithms, namely, *fuzzy lattice neurocomputing (FLN)*, is presented for clustering inspired from *fuzzy adaptive reso-*

*nance theory (ART)*. Another family of algorithms, namely, *granular self-organizing map (grSOM)*, is presented for clustering inspired from SOM. A number of FLN extensions are presented for classification including also a “majority voting” ensemble as well as a FIN-based nearest neighbor classifier. Regression can be pursued by classifica-

tion. Part III, including chapters 8–10, presents both experimental results and connections with established paradigms. In particular, chapters 8 and 9 demonstrate comparatively applications of the previous algorithms in a number of problems involving numeric- and non-numeric-data, respectively. Chapter 10 details inspiring connections with established paradigms as explained next. First, the FLN enhances the fuzzy-ART neural network. Second, the FLN improves on other hyperbox-based models including the min–max neural networks. Third, the grSOM improves on SOM neural networks. Fourth, the FLN improves over multilayer perceptrons as well as other neural networks. Fifth, FIN-based design implies various advantages regarding both fuzzy inference systems (FISs) and radial basis function (RBF) neural networks. Additional advantages include a novel treatment of ambiguity/uncertainty using FINs in algebraic modeling applications. Furthermore, connections are shown with various machine-learning techniques, database-processing techniques, and “applied mathematics” techniques.

Part IV, including chapters 11 and 12, discusses practical implementation issues. More specifically, chapter 11 explains the author's perspective regarding how lattice-theory modeling may result in hypercomputing towards human intelligence modeling. Finally, chapter 12 summarizes the contribution of this book. It also proposes a super-Turing machine modeled as a closed-loop feedback system.

Appendix A includes useful mathematical definitions. Appendix B details mathematical proofs of theorems in this book. Appendix C shows useful geometric interpretations on the plane.

I found the presentation in this research monograph clear, concise, and systematic with much of it based on the author's high-quality research published in major scientific journals and conferences. The material is presented in a straightforward and understandable style, with a nice variety of explanations and examples included. Additionally, the author provides pointers to many valuable references which will aid other investigators in the field of computational intelligence based on lattice theory.

The book includes 47 figures, 31 tables, a list with 72 acronyms and 151 symbols, 686 references in 34 pages, and an index with 249 terms.

This timely book is an excellent choice for readers who wish to familiarize themselves with a promising, emerging technology.