System Design for Improved Extra-Terrestrial Communications

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Abstract

This paper describes a proposed system for improved exo-ionospheric communications. The dynamic magneto-ionic character of the channel is considered, in particular, the multipath situation arising therefrom. An ideal matched filter is found, matched to the multipath structure and the dynamics of the exo-ionospheric channel. The improvement in the signal-to-noise ratio through the matched filter is calculated. It is seen to depend on the quotient of the input signal to the bandwidths of the "measuring" filter and the "integrating" filter. Further advantages are shown to accrue from signal processing at the transmitter involving both increases in range, and, in particular, secure coding possibilities.

Key Words—**Communications, design, extra-terrestrial, signal processing, space, system.**

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Introduction

In space communications, whether they be satellite-to-satellite or earth-to-satellite, there are two factors that are of paramount importance and that determine the necessary signal conditioning of the communication system. The first is the tremendous distance that it may be necessary for the signal to cover. This makes it necessary to be able to concentrate sufficient power in the transmitted signal to assure its reaching the receiver at the required power level. The second involves the characteristics of the channel or medium through which the signal must propagate [1]–[3]. We can assume that at least a portion of the channel is through the ionosphere. This is a medium that has special peculiarities [4] which must be taken into account both from the point of view of avoidance of pitfalls and from the point of view of positive actions in the signal conditioning process. This, of course, involves also such relatively passive acts as choice of frequency, etc.

In the ionosphere, the index of refraction is a function of the operating frequency. Hence, the ionosphere is a dispersive medium and the various spectral components of a radiated waveform travel at different velocities. It is necessary to minimize or compensate for this effect in the signal conditioning.

Further, the earth's magnetic field makes the ionosphere an anisotropic medium. This means that the dielectric "constant" is not only frequency-dependent but a tensorial quantity as well. This leads naturally into consideration of the various anisotropic propagation phenomena caused by the presence of the magnetic field, whose relative importance will, of course, depend upon the operating frequency. This includes the Faraday effect, although this is not a matter of overriding concern.

The Multipath Situation in the Ionosphere

The reception of any amount of power, if it arrives in the form of noise, is useless. Hence, we consider first the multipath situation encountered in the ionosphere. Experience indicates that the Faraday effect will not be a problem. The rotation of the plane of polarization of a transmitted waveform gives no difficulty to receiving antennas of present-day systems.

Because of the frequency dispersion characteristic of the ionosphere, transmission using this medium will be afflicted with multipath. At a given receiving point, many rays of varying strength will arrive after having traveled through different paths. The communication system, then, must be antimultipath. It will be seen that this approach also contributes to increase of the range per unit power, since it is proposed to use the multipath structure essentially to augment the signal-to-noise ratio.

The heart of the proposed system is a number of appropriately designed correlation detectors at uniform delay increments. These delays can be made adjustable but must remain uniform. We are, in fact, able to achieve a reconstitution or collapsing of the multipath delays, thus effectively eliminating intersymbol interference. Signals arriving over each path are added coherently, but the noise associated with each path remains incoherent after addition. Hence, strong signals that have been delayed in time actually contribute to the useful signal output (rather than cause deterioration). The algebraic rather than the vectorial addition of the signals, properly delayed, phase-corrected, and weighted for maximum S/N ratio from the various paths, effectively eliminates the phenomenon of selective fading.

We seek, then, a matched receiver [5] with knowledge of the random elements in the transmitter and ionospheric channel. Such a matched receiver is by definition the optimum that can be achieved in terms of minimum average cost of wrong decision in a given coded sequence. Such a receiver essentially computes the a posteriori probabilities in the sequence and decides in favor of that which is most probable. Exact measurement of such randomly varying channel characteristics, as in the ionosphere, cannot be made as long as noise is present. Nevertheless, good measurement accuracy [6]–[8] of the varying multipath characteristics can be achieved by procedures involving cross correlation. It is sensible, then, to employ the results of statistical decision theory in constructing the ideal receiver.

If the transmitted signal has a bandlimited spectrum $S(w)$ and the multipath is varying slowly compared to the transmission bandwidth,

$$H'(w) = \begin{cases} \Phi_{xy}(w)/S(w), & \text{in transmission band} \\ 0, & \text{outside transmission band} \end{cases} \tag{1}$$

where Φ_{xy} is the Fourier transform of the cross-correlation function of the transmitted signal $x(t)$ and the received signal $y(t)$. Then, for a constant spectrum, $S(w)$, within the transmission bandwidth, W_0

$$h'(t) = A'\phi_{xy}(t) = A' \frac{1}{2T} \int_{-T}^{T} x(\tau)y(\tau + t)d\tau \tag{2}$$

where A' is a scaling factor, and $2T$ is limited to $1/W_0$.

Now, from the sampling theorem [9] it is clear that since $h'(t)$ is bandwidth-limited to the transmission bandwidth, W_0, it is theoretically only necessary to sample-measure at points $1/W_0$ apart in time, irrespective of the presence or absence of additive channel noise. What we have really done is to relax a theoretical but impossible requirement that there be an infinite number of discrete paths in random variation whose time delays are known a priori to the receiver. The time-delay taps corresponding to known path delays are replaced by the aforementioned uniform tap spacing. This is close enough to capture all paths as they occur according to the sampling theorem.

We have arrived essentially at a system where a multiplicity of delayed signals are detected [10]–[12] separately, weighed, phase-corrected, and then added. This requires multipath resolution, which, in turn, requires wideband transmission. This is another way of saying that we want to be able to separate, distinguish, and operate on those signal echoes arriving within multiples of $1/W_0$ in time. A test of whether we have chosen a wide enough transmission band will then evidently be whether there is no selective fading, i.e., W_0 is wide enough to "average out" the selective fading.

Consider that we have a transmission bandwidth W_0 Hz. If this band can ideally be expressed as

$$S(w) = \begin{cases} \text{rect}\ (w), & \text{inside transmission band} \\ 0, & \text{outside transmission band} \end{cases} \tag{3}$$

where rect (w) is used to represent the normalized transmission as described in the foregoing, then in the time domain, this corresponds to

$$s(t) = C' \frac{\sin k\pi W_0\tau}{k\pi\tau W_0} \tag{4}$$

where C' is a scaling factor and k an integer. This is the familiar $(\sin x)/x$ expression, and for a bandwidth W_0, equal, for example, to 10 kHz, such as the one we can arbitrarily choose, the value decreases to zero as τ is increased from zero to $1/W_0$ or 100 μs. That is, the value of the amplitude of the signal voltage after correlation depends on the time difference τ between the signal and the reference.

It should be particularly noted in Fig. 1 that the use of two heterodyne operations removes the random phase angles of the input signal. This is part of the phase correction that allows the multipath signal contributions to be added in phase. It can be shown that with signal and Gaussian noise, coherent signal addition in phase after squaring gives optimum weighting. The noise adds incoherently.

As a matter of fact, at point A in Fig. 1 (i.e., following the measuring filter) the voltage is

$$e_A(t) = C'' \frac{\sin W_0 k\pi\tau}{W_0 k\pi\tau} e^j\,(\omega_r t - \omega_0 t) \;- j\theta + j\theta_r \tag{5}$$

where

C'' is scaling factor
ω_r is the reference frequency
ω_0 is the filter center frequency.

At point B this becomes:

$$e_B(t) = C''' \left(\frac{\sin k\pi W_0\tau}{k\pi W_0\tau}\right)^2 e^{j}\omega_r t + j\theta_r. \tag{6}$$

This shows that the contribution of each path to the output signal appears at the same frequency and phase with an amplitude dependent on the original signal amplitude and the relative τ for that path. It should be noted that (7) involves $((\sin x)/x)^2$. A positive processing gain can be expected from this scheme.

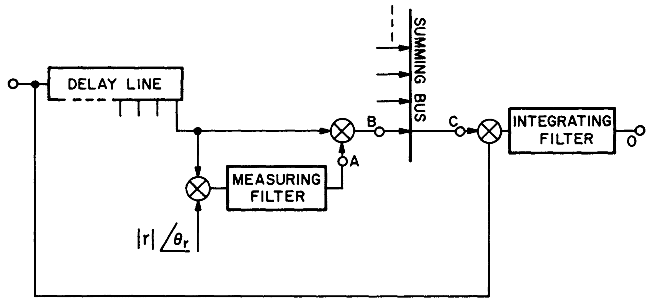

Fig. 1. Filter matched to multipath structure.

Signal-to-Noise Estimate

At the input of the multipath matched filter shown in Fig. 1

$$P_{\text{in}} = (s_k + N_k)\left(\frac{W_0}{W_0}\right). \tag{7}$$

Now, the fluctuation term before filtering is the second term below:

$$s_{kc} + (s_{kn} + N_{kn})\,\frac{W_0}{W_0} = P_c. \tag{8}$$

After filtering, the fluctuation power is again the second term below.

$$s_{kc} + (s_{kn} + N_{kn})\,\frac{I}{W_0} = P_{cp}. \tag{9}$$

However,

$$s_{kn} + N_{kn} = s_k + N_k = P_{\text{in}}. \tag{10}$$

Hence

$$(s_{kn} + N_{kn})\,\frac{I}{W_0} = P_{\text{in}}\,\frac{I}{W_0}$$
$$= (\text{noise power at output of filter}). \tag{11}$$

Hence, the signal-to-noise ratio at point A is

$$\frac{S_A}{N_A} = \frac{f_k}{P_{\text{in}}\,\frac{I}{W_0}} = \frac{s_k}{P_{\text{in}}}\,\frac{W_0}{I} = \frac{s_k}{s_k + N_k}\,\frac{W_0}{I}\,. \tag{12}$$

At point B we have

$$s_{kc} + (s_{kn} + N_{kn})\,\frac{W_0}{W_0} + s_{kc} + (s_{kn} + N_{kn})\,\frac{I}{W_0}$$
$$= s_{kc} + P_{\text{in}}\,\frac{W_0}{W_0} + s_{kc} + P_{\text{in}}\,\frac{I}{W_0}\,. \tag{13}$$

$$\text{Noise power} = P_{\text{in}}\,\frac{W_0}{W_0} + P_{\text{in}}\,\frac{I}{W_0}\,. \tag{14}$$

Hence, signal-to-noise ratio is

$$\frac{S_B}{N_B} = \frac{2s_k}{P_{\text{in}}\left(\frac{W_0}{W_0} + \frac{I}{W_0}\right)} = \frac{2s_k}{P_{\text{in}}\,\frac{W_0 + I}{W_0}}$$
$$= \frac{2s_k}{P_{\text{in}}}\,\frac{W_0}{W_0 + I}\,. \tag{15}$$

Obviously, the signal-to-noise ratio is degraded at point B with respect to that at point A, because of the necessity of phase correction, to approximately twice the input signal to the total signal plus noise power at the input. This can be written

$$\frac{S_B}{N_B} = \frac{2s_k}{P_{\text{in}}}\,\frac{W_0}{W_0 + I} = \frac{2s_k}{(s_k + N_k)}\,\frac{W_0}{W_0 + I}\,. \tag{16}$$

At point C there is an enhancement due to the coherent summing of signal strengths while the noise remains incoherent. This is a function of the variable multipath structure of the communication channel according to the ratio of the sum of the varying signal strengths to the root mean square of the sum of the respective noises. It is easily included as an indicated function, so that

$$\frac{S_C}{N_C} = \frac{2s_k W_0}{(s_k + N_k)I}\,F_M(s_k, N_k) \tag{17}$$

where $F_M\,(s_k, N_k)$ is the indicated function of the varying multipath structure.

Finally, at the output there is another large processing gain. That is, before filtering, the components of signal plus noise are

$$2s_k + (s_k + N_k)\,\frac{W_0 + I}{W_0} + (s_k + N_k)\,\frac{W_0}{W_0} = P_{c'}. \tag{18}$$

After filtering, the same components become

$$2s_k + (N_k + s_k)\,\frac{I'}{W_0} + (s_k + N_k)\,\frac{I'}{W_0} = P_{c'p'}. \tag{19}$$

Hence, the signal-to-noise ratio at the output of the multipath matched filter in Fig. 1 is

$$\frac{S_0}{N_0} = \frac{s_k}{P_{\text{in}}}\,\frac{W_0}{I'}\,F_M(s_k, N_k)$$
$$= \frac{s_k W_0}{(s_k + N_k)I'}\,F_M(s_k, N_k), \tag{20}$$

where, obviously, I represents the bandwidth of the measuring filter, and I' represents the bandwidth of the integrating filter in Fig. 1, with $I, I' \ll W_0$. The improvement in signal-to-noise ratio of output over input is therefore,

$$\left\{\begin{matrix}\text{improvement in}\\ \text{signal-to-noise}\\ \text{ratio}\end{matrix}\right\} = \frac{1}{N_k(s_k + N_k)} \frac{W_0}{I'} F_M(s_k, N_k)$$

$$= \frac{W_0}{P_{\text{in}} I'} F_M(s_k, N_k). \tag{21}$$

The Transmitter Signal

In the preceding section we have made use of the principles of statistical communication theory to find an ideal receiver matched to the random elements in the multipath propagation environment in the presence of noise and Doppler.

This amounts to signal processing at the receiver designed to take into account the channel medium characteristics. It is a happy by-product that it also conditions the signal for longer range reception by giving a processing gain at the output of the receiver over the signal-to-noise ratio at the receiver input.

Now, let us look at the transmitter. This gives the other possibility of signal processing. The major contribution of signal conditioning at the transmitter has to do with extending the range of the communication system. Hence, we are led to consideration of ways of increasing transmitter signal energy and, hence, detectability and precision. In general, such transmitter signal processing requires, as a corollary, compensatory and complementary signal processing at the receiver.

One method of such signal processing in a peak-power limited system consists of lengthening the signal pulse. Unfortunately, the information capacity of the system suffers when this is done since the bandwidth has been reduced. The solution, then, evidently is to effectively increase the length of the transmitted pulse, at the same time keeping the system bandwidth constant. There is an obvious analogy here to the radar range resolution problem and the question can be approached in a similar manner [13], [14]. That is, a stretched pulse can be received by the filter matched to the transmitted waveform, which acts to recompress it.

An extended transmitted pulse can consist of a carrier frequency continuously and linearly increasing in frequency for the length of the pulse. In order that the information capacity not be degraded, it is desired that there be a peaking at the output of the reciever. Hence, passing such an extended pulse through its matched filter would cause a cancellation at all frequencies except in the center of the band. This would be ideal, giving a theoretically unimprovable information capacity with no increase in ambiguity. There are, however, inherent difficulties that preclude the building of a matched filter for such a waveform. It is, nevertheless, possible to build a stepped-frequency pulse compression system with consequent degradation from the performance of such an ideal linear-frequency system. The degradation would consist in the presence of undesired sidelobes to the main pulse.

A stepped-frequency waveform passed through its matched filter is compressed in real time with resultant processing gain at the receiver. The matched filter for such a system is shown in Fig. 2. It is seen that allowance has been made for some "mismatching" in order to reduce the unwanted sidelobes. This degrades somewhat the desired peaked signal but such degradation is slight and is a valid trade-off for the decrease in the sidelobes.

The basic stepped-frequency waveform is given by

$$e_1(t) = H^*(t) = \sum_{n=-M}^{M} \text{rect}\,(t - nT_p)e^{j2\pi(f_0+nf_p)t}, \tag{22}$$

where

asterisk indicates complex conjugate
subpulse length $= T_p$
stepping frequency $= f_p = \dfrac{1}{T_p}$
$N = 2M+1 =$ total number of steps
$f_0 =$ carrier frequency

The matched filter output is

$$\begin{aligned} e_2(t) &= (T_p - |t|)e^{-j2\pi(f_0-f/2)t}\left[\frac{\sin \pi f(T_p - t)}{\pi f(T_p - t)}\right] \\ &\quad \cdot \left[\frac{\sin N\pi t\left(\dfrac{1}{T_p} - f\right)}{\sin \pi t\left(\dfrac{1}{T_p} - f\right)}\right], \qquad 0 < t < T_p \\ &= 0, \qquad \text{for } |t| > T_p \end{aligned} \tag{23}$$

where f is the Doppler frequency.

In the case that the Doppler frequency can be neglected:

$$e_2(t) = (T_p - |t|)e^{-j2\pi f_0 t}\left[\frac{\sin N\pi \dfrac{t}{T_p}}{\sin \pi \dfrac{t}{T_p}}\right],$$

$$\text{for } 0 < t < T_p = 0, \qquad \text{for } |t| > T_p. \tag{24}$$

It is to be noted with reference to (24), that

$$\lim_{x\to 0} \frac{\sin Nx}{\sin x} = N.$$

This means that the compressed pulse peak amplitude is N times that of each subpulse. As a result, the *peak* power created is N^2 times the average power of the subpulse train.

A possible objection to such a system used in conjunction with the previously described "ideal receiver matched

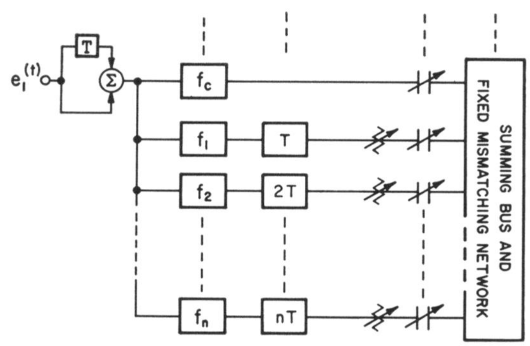

Fig. 2. Matched filter to stepped-frequency waveform with sidelobe suppression and amplitude and phase adjustments.

to the peculiarities of the exo-ionospheric channel" is the overall increase in complexity of the total system with respect to the subsystem involving only the channel characteristics. For example, the straightforward application of the theory described would call for a matched receiver for each step of the stepped pulse. These outputs would then be passed through shaping networks before finally being applied to the matched filter for the transmitted stepped-frequency pulse. Ingenuity, however, can reduce the number of component subsystems in practice. Further, the increasing distance involved in future space probes, etc., and the attendent difficulties in control and communication should make such a system described here both practical and welcome.

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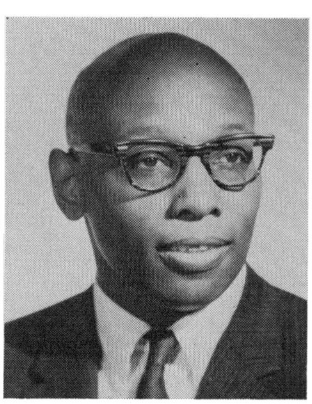

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