APPENDIX

THEOREM *If the activity plane is parallel to the image lines, and if all the measuring points are on two horizontal image lines, the Fisher information matrix* (25) *is singular.*

PROOF Let $S \subseteq \{1,\ldots,N\}$ be the set of measurements such that the Y_k coordinate of measurement points takes constant values on S and on its complement. Let $V \subseteq R^N$ be the subspace of vectors with components constant on S and on its complement. V has dimension at most 2. From (9) we see that the last two columns of $M(\alpha)$ belong to V. Using $a_x = 0$ we see that $(M'(\alpha)a)_k = (Y_k \cos(\alpha) + B\sin(\alpha))(a_y Y_k/B + a_z)$ so $M'(\alpha)a$ also belongs to V. We conclude that the Fisher information matrix (25) has rank at most 3.

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REFERENCES

[1] Sage, K., and Young, S. (1999)
Security applications of computer vision.
IEEE AES Systems Magazine, (Apr. 1999), 19–24.

[2] Weng, J., and Cogen, P. (1992)
Camera calibration with distortion models and accuracy evaluation.
IEEE Transactions on Pattern Analysis and Machine Intelligence, **14**, 10 (1992), 965–980.

[3] Wan, X., and Xi, G. (1996)
Camera parameters estimation and evaluation in active vision system.
Pattern Recognition, **29**, 3 (1996), 439–447.

[4] Kendall, M., and Stuart, A. (1977)
The Advanced Theory of Statistics (4th ed.).
New York: Macmillan, 1977.

[5] Golub, G. H., and Van Loan, C. F. (1989)
Matrix Computations (2nd ed.).
Baltimore, MD: The Johns Hopkins University Press, 1989.

Active Fault Tolerant Control with Actuation Reconfiguration

This paper presents an integrated approach to fault detection, isolation, and fault tolerant control (FTC). In the proposed approach a constrained Kalman filter based fault detection and isolation (FDI) method [6] is modified to reduce its computation load and then applied to detect and isolate a faulty actuator or sensor. Explicit algorithms are derived to estimate the effectiveness factor of a detected faulty actuator or sensor. Furthermore, a novel design concept of actuation reconfiguration is proposed and implemented in the derived control scheme to restrain the workload of a faulty actuator while recovering the prefault system performance. Simulation results on an aircraft dynamic model have demonstrated the effectiveness of the proposed method.

I. INTRODUTION

A fundamental task of fault tolerant control (FTC) is to maximize system reliability by taking full advantage of system redundancy, either in hardware or analytical form. Depending upon how redundancy is being utilized, FTC systems can be categorized into passive [7, 9, 13] and active [4, 8, 10–12] types. In passive FTC, potential system component faults are known a priori and are all taken into consideration in the control system design stage. In contrast, active FTC systems rely on a fault detection and isolation (FDI) scheme to detect the occurrence of faults in the system and to identify the source and seriousness of the faults. Based on the output of FDI, a fault accommodation scheme can then be designed to maintain a certain degree of control performance of the postfault system. An effective FDI scheme is critical for designing high performance active FTC systems, and many model-based FDI techniques have been developed based on analytical redundancy, state estimation, and parameter identification. In [5], a full-order Kalman filter is used to isolate multiple sensor faults in discrete-time stochastic systems. In [6], a bank of constrained Kalman filters is introduced

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to detect and isolate actuator and/or sensor faults. In [2], an input observer is combined with fault detection filters and used for FDI.

With the assumption that system faults can be detected and isolated, several reconfigurable control methods have been developed in the literature, including eigenstructure assignment [4, 8, 10, 12], reference model following [11], and pseudoinverse method [1, 3].

In the proposed integrated approach to FDI and FTC, a constrained Kalman-filter-based FDI method [6] is modified to reduce computation load and then applied to detect and isolate a faulty actuator or sensor. Explicit algorithms are derived to estimate the effectiveness factor of a faulty actuator or sensor after it is detected and isolated. Furthermore, a novel concept of actuation reconfiguration is proposed and integrated into the developed control scheme, so that the control scheme not only recovers the prefault system performance similarly as in [12] but also restrains the workload that is required of the faulty actuator. The actuation reconfiguration guarantees an adequate load sharing among the actuators of the postfault system, which is essential in practical implementations. Simulation results on an aircraft dynamic model have demonstrated the effectiveness of the proposed method.

II. SYSTEM MODELING

Consider a class of discrete-time linear systems:

$$x(k+1) = Ax(k) + Bu(k) + w(k+1) \tag{1}$$

$$y(k) = Cx(k) + v(k) \tag{2a}$$

$$z(k) = C_r x(k) \tag{2b}$$

where $x(k) \in R^{n\times 1}$ denotes the state vector at time k, $u(k) \in R^{p\times 1}$ denotes the input, $y(k) \in R^{q\times 1}$ denotes the sensor measurement output, $z(k) \in R^{m\times 1}$ represents system output without measurement noise. The system disturbance is denoted by $w(k+1)$, and the measurement noise is denoted by $v(k)$. Both w and v are assumed to be white Gaussian noise with zero mean and covariance Σ_{ww} and Σ_{vv}, respectively. The matrices $A \in R^{n\times n}$, $B \in R^{n\times p}$, $C_r \in R^{m\times n}$ and $C \in R^{q\times n}$ are the state transition, input, output, and sensor measurement matrices, respectively.

Consider the following system model after actuator and/or sensor faults [12]:

$$x(k+1) = Ax(k) + B_f u(k) + w(k+1) \tag{3}$$

$$y(k) = C_f x(k) + v(k) \tag{4a}$$

$$z(k) = C_r x(k) \tag{4b}$$

where in (3)

$$B_f = B(\mathrm{I} - \Gamma_a), \qquad \Gamma_a = \mathrm{diag}(\gamma_{a1}, \gamma_{a2}, \ldots, \gamma_{ap}) \tag{5}$$

where $\gamma_{ai} \in [0,1]$, $i = 1,2,\ldots,p$, are the effectiveness factors that indicate the seriousness of actuator faults. If $\gamma_{ai} = 0$, the actuator is fault free. If $\gamma_{ai} = 1$, the actuator has completely failed. In (4a),

$$C_f = (\mathrm{I} - \Gamma_c)C, \qquad \Gamma_c = \mathrm{diag}(\gamma_{c1}, \gamma_{c2}, \ldots, \gamma_{cq}) \tag{6}$$

where $\gamma_{ci} \in [0,1]$, $i = 1,2,\ldots,q$, are the effectiveness factors that indicate the seriousness of sensor faults. Again, $\gamma_{ci} = 0$ means that the sensor is fault free, and $\gamma_{ci} = 1$ represents a complete failure.

III. FAULT DETECTION, ISOLATION AND EFFECTIVENESS FACTOR ESTIMATION

A. Fault Detection and Isolation

For FDI, a bank of constrained Kalman filters is used in [6], with one separate filter assigned for each actuator or sensor that may potentially malfunction. In order to minimize the computation load and speed up the FDI process, the approach in [6] is modified to generate a two-step FDI scheme. In the first step, actuators and sensors of the system are grouped according to a criterion to be derived. Each group is then monitored with a constrained Kalman filter. If a fault is detected from a group, a second step is invoked to isolate the faulty actuator or sensor.

The standard Kalman filter for system state estimation consists of the following two steps. The first step is a one-step-ahead prediction of the plant states

$$\hat{x}(k+1 \mid k) = A\hat{x}(k \mid k) + Bu(k) \tag{7}$$

where $\hat{x}(k+1 \mid k)$ denotes the estimate of $x(k+1)$ given data available through time k, and $\hat{x}(k \mid k)$ denotes the estimates of $x(k)$ given all actuator and sensor data available through time k.

The second step is a correction,

$$\hat{x}(k+1 \mid k+1) = \hat{x}(k+1 \mid k) + g[y(k+1) - C\hat{x}(k+1 \mid k)] \tag{8}$$

where g is an updating gain matrix.

Let $e(k) = x(k) - \hat{x}(k \mid k)$, from (7) and (8),

$$e(k+1) = (\mathrm{I} - gC)Ae(k) + (\mathrm{I} - gC)w(k+1) - gv(k+1). \tag{9}$$

Through optimizing the gain matrix g, $e(k)$ is minimized, and the state estimation is henceforth obtained.

In the first step, the actuation input u is partitioned into "included" and "excluded" inputs, denoted by u_{incl} and u_{excl}, respectively. The input matrix B is partitioned to input matrices B_{incl} and B_{excl}, correspondingly. Similarly, the sensor output y is also partitioned into included and excluded outputs, denoted by y_{incl} and y_{excl}, respectively. The two corresponding output matrices are C_{incl} and C_{excl}.

After the partitioning, the system equations (1) and (2a) can be rewritten as

$$x(k+1) = Ax(k) + B_{\text{incl}}u_{\text{incl}}(k) + B_{\text{excl}}u_{\text{excl}}(k) + w(k+1) \tag{10}$$

$$y = \begin{bmatrix} y_{\text{incl}} \\ y_{\text{excl}} \end{bmatrix} = \begin{bmatrix} C_{\text{incl}} \\ C_{\text{excl}} \end{bmatrix} x(k) + v(k). \tag{11}$$

For the partitioned system (10) and (11), the predictor-corrector equations for the state estimation become

$$\hat{x}(k+1 \mid k) = A\hat{x}(k \mid k) + B_{\text{incl}}u_{\text{incl}}(k) \tag{12a}$$

$$\hat{x}(k+1 \mid k+1) = \hat{x}(k+1 \mid k) + g[y_{\text{incl}}(k+1) - C_{\text{incl}}\hat{x}(k+1 \mid k)]. \tag{12b}$$

The error dynamic equation (9) becomes

$$e(k+1) = (\mathrm{I} - gC_{\text{incl}})Ae(k) + (\mathrm{I} - gC_{\text{incl}})B_{\text{excl}}u_{\text{excl}}(k) + (\mathrm{I} - gC_{\text{incl}})w(k+1) - gv(k+1). \tag{13}$$

From (13), it can be seen that the following constraint equation makes the error dynamics impervious to the excluded inputs

$$(\mathrm{I} - gC_{\text{incl}})B_{\text{excl}} = 0. \tag{14}$$

From the knowledge of matrix algebra, a solution g to (14) exists if and only if $\operatorname{rank}(C_{\text{incl}}) \geq \operatorname{rank}(B_{\text{excl}})$, which represents the constraint of the constrained Kalman filters dedicated to FDI.

According to matrix algebra theory, the solution to (14) can be expressed as

$$g = g_{\perp} + sg_{\Pi} \tag{15}$$

where

$$g_{\perp} = B_{\text{excl}}(C_{\text{incl}}B_{\text{excl}})^{+} \tag{16}$$

where the superscript "+" denotes the pseudoinverse of the matrix. The matrix g_{Π} is the left kernel space of $C_{\text{incl}}B_{\text{excl}}$. The objective of the constrained Kalman filter derivation is to optimize the matrix s, such that the resulting error covariance is minimized. Squaring both sides of (13) and taking the stochastic mean yields

$$\Sigma_{ee} = (\mathrm{I} - gC_{\text{incl}})A\Sigma_{ee}A^{T}(\mathrm{I} - gC_{\text{incl}})^{T} + (\mathrm{I} - gC_{\text{incl}})\Sigma_{ww}(\mathrm{I} - gC_{\text{incl}})^{T} + g\Sigma_{vv}g^{T} \tag{17}$$

where Σ_{ee} is the covariance of the state estimation error, Σ_{ww} is the covariance of the plant disturbance, and Σ_{vv} is the covariance of the measurement noise. Taking the first-order variation of (17) and combining it with (15), the optimal s for the minimum error covariance is obtained if the following equation is satisfied:

$$[(\mathrm{I} - g_{\perp}C_{\text{incl}})(A\Sigma_{ee}A^{T} + \Sigma_{ww})C_{\text{incl}}^{T} - g_{\perp}\Sigma_{vv}]g_{\Pi}^{T} = sg_{\Pi}[C_{\text{incl}}(A\Sigma_{ee}A^{T} + \Sigma_{ww})C_{\text{incl}}^{T} + \Sigma_{vv}]g_{\Pi}^{T}. \tag{18}$$

From (15), (17), and (18), the optimal constrained updating gain g can be determined.

In prefault operations, $e(k)$ is small, but it will become relatively large in the presence of actuator and/or sensor faults, and Σ_{ee} could be a residual for fault detection. A fault in an actuator or a sensor can then be localized to one group of actuators or sensors as the first step. Once a faulty group is detected, the second step of FDI is taken to isolate the fault within the group by repeating the process described by (12)–(18). However, in the second step, a single actuator or sensor is excluded for each constrained Kalman filter as in [6], so that the specific faulty actuator or sensor can be isolated.

B. Estimation of Effectiveness Factor

In case of a fault in the ith actuator, the input matrix $B = [b_1, \ldots, b_i, \ldots, b_p]$ changes to $B_f = [b_1, \ldots, (1-\gamma_{ai})b_i, \ldots, b_p]$, where $b_j \in R^{n\times 1}$, $j = 1, 2, \ldots, p$, and γ_{ai} is the effectiveness factor of the faulty actuator. The Kalman filter equation with consideration of the fault in the ith actuator is

$$\hat{x}(k+1 \mid k+1) = A\hat{x}(k \mid k) + Bu(k) + gCAe(k) + gC(B_f - B)u(k) + gCw(k+1) + gv(k+1). \tag{19}$$

And the corresponding state estimation error is given by

$$e(k+1) = (\mathrm{I} - gC)Ae(k) + (\mathrm{I} - gC)(B_f - B)u(k) + (\mathrm{I} - gC)w(k+1) - gv(k+1). \tag{20}$$

Denoting $\Delta B = B - B_f = \gamma_{ai}[0, \ldots, b_i, \ldots, 0] \stackrel{\Delta}{=} \gamma_{ai}\Phi_a$, (20) can be rewritten as

$$e(k+1) = (\mathrm{I} - gC)Ae(k) + (\mathrm{I} - gC)w(k+1) - gv(k+1) + (\mathrm{I} - gC)\Delta Bu(k). \tag{21}$$

Squaring both sides of (9) and taking stochastic mean yields

$$\Sigma_{ee} = (\mathrm{I} - gC)A\Sigma_{ee}A^{T}(\mathrm{I} - gC)^{T} + (\mathrm{I} - gC)\Sigma_{ww}(\mathrm{I} - gC)^{T} + g\Sigma_{vv}g^{T} \tag{22}$$

where Σ_{ee} is the covariance of the fault free state estimation error. Squaring both sides of (21) and taking stochastic mean leads to

$$\Sigma'_{ee} = (\mathrm{I} - gC)A\Sigma'_{ee}A^T(\mathrm{I} - gC)^T + (\mathrm{I} - gC)\Sigma_{ww}(\mathrm{I} - gC)^T + g\Sigma_{vv}g^T + (\mathrm{I} - gC)\Delta B\{E[u(k)u^T(k)]\}\Delta B^T(\mathrm{I} - gC)^T \tag{23}$$

where Σ'_{ee} is the covariance of the state estimation error in the presence of an actuator fault.

Denote

$$\Delta\Sigma = \Sigma'_{ee} - \Sigma_{ee}. \tag{24}$$

From (22) and (23), we have

$$\Delta\Sigma = (\mathrm{I} - gC)A\Delta\Sigma A^T(\mathrm{I} - gC)^T + (\mathrm{I} - gC)\Delta B\Sigma_{uu}\Delta B^T(\mathrm{I} - gC)^T \tag{25}$$

where

$$\Sigma_{uu} = E[u(k)u^T(k)]. \tag{26}$$

Let

$$T_a = \Delta\Sigma - (\mathrm{I} - gC)A\Delta\Sigma A^T(\mathrm{I} - gC)^T \tag{27}$$

and

$$\Psi_a = (\mathrm{I} - gC)\Phi_a\Sigma_{uu}\Phi_a^T(\mathrm{I} - gC)^T. \tag{28}$$

Then

$$T_a = \gamma_{ai}^2\Psi_a. \tag{29}$$

Let the total number of non-zero elements in T_a (or Ψ_a) be N_a. Rearranging the non-zero elements of T_a into an array T_{ak}, $k = 1,2,\ldots,N_a$, and the corresponding non-zero elements of Ψ_a into an array Ψ_{ak}, $k = 1,2,\ldots,N_a$. From (29), the effectiveness factor γ_{ai} is then determined as a mean value by

$$\gamma_{ai} = \left(\sum_{k=1}^{N_a}\sqrt{\frac{T_{ak}}{\Psi_{ak}}}\right)\cdot\frac{1}{N_a}. \tag{30}$$

In case of a fault in the ith sensor with an effectiveness factor γ_{ci}, the sensor measurement matrix $C = [c_1^T,\ldots,c_i^T,\ldots,c_q^T]^T$ changes to $C_f = [c_1^T,\ldots,(1-\gamma_{ci})c_i^T,\ldots,c_q^T]^T$, and $\Delta C = C - C_f = \gamma_{ci}[0,\ldots,c_i^T,\ldots,0]^T \stackrel{\Delta}{=} \gamma_{ci}\Phi_s$, where $c_j \in R^{1\times n}$, $j = 1,2,\ldots,q$. The following equations are derived similarly as for the effectiveness factor estimation for a faulty actuator and can be used to estimate a sensor effectiveness factor γ_{ci}:

$$\Sigma_{xx} = E\{x(k+1)x^T(k+1)\} \tag{31}$$

$$T_s = \Delta\Sigma - (\mathrm{I} - gC)A\Delta\Sigma A^T(\mathrm{I} - gC)^T \tag{32}$$

$$\Psi_s = g\Phi_s\Sigma_{xx}\Phi_s^T g^T \tag{33}$$

$$T_s = \gamma_{ci}^2\Psi_s. \tag{34}$$

The effectiveness factor of a faulty sensor can then be calculated as

$$\gamma_{ci} = \left(\sum_{k=1}^{N_s}\sqrt{\frac{T_{sk}}{\Psi_{sk}}}\right)\cdot\frac{1}{N_s} \tag{35}$$

where N_s is the total number of non-zero elements in T_s (or Ψ_s). T_{sk}, $k = 1,2,\ldots,N_s$, is an array made up of the non-zero elements of T_s, and Ψ_{sk}, $k = 1,2,\ldots,N_s$, are the corresponding non-zero elements of Ψ_s.

IV. CONTROL DESIGN WITH ACTUATION RECONFIGURATION

For the prefault system defined by (1) and (2), consider the following control law [12]:

$$u(k) = -Kx(k) + K_r r(k) \tag{36}$$

where K and K_r are gain matrices, and $r(t) \in R^{p\times 1}$ denotes the reference input.

The closed-loop system under the control law (36) becomes

$$x(k+1) = (A - BK)x(k) + BK_r r(k). \tag{37}$$

The closed-loop system eigenvalues are

$$\{\lambda_i(A - BK),\ i = 1,2,\ldots,n\} \in \Omega \tag{38}$$

where Ω is a stable eigenvalue set that is determined based on desired closed-loop system performance.

The postfault system model is given by

$$x(k+1) = Ax(k) + B_f u(k) \tag{39}$$

where B_f is the postfault input matrix.

The objective of reconfigurable control system design is to synthesize a new feedback gain matrix so that the closed-loop reconfigured system recovers the prefault system performance. Naturally it is desirable to make the postfault system eigenvalues the same as those of the prefault system [12], i.e.

$$\lambda_i(A - BK) = \lambda_i(A - B_f K_f) \in \Omega, \qquad i = 1,2,\ldots,n \tag{40}$$

where K_f is the postfault state feedback control gain matrix. To maintain the same influence of the reference input, let

$$BK_r = B_f K_{rf} \tag{41}$$

where K_{rf} is the postfault system control gain matrix.

If the control gain matrices K_f and K_{rf} are determined to satisfy (40) and (41), respectively, the postfault control input

$$u_f(k) = -K_f x(k) + K_{rf} r(k) \tag{42}$$

will recover the closed-loop system eigenvlaues, and henceforth the performance of prefault system.

Consider that a fault occurs in the ith actuator with an effectiveness factor γ_{ai}. From (40),

$$|\lambda\mathrm{I} - (A - BK)| = |\lambda\mathrm{I} - (A - B_f K_f)|. \tag{43}$$

A sufficient condition for (43) is

$$BK = B_f K_f. \tag{44}$$

Rewrite

$$K = [k_1^T, k_2^T, \ldots, k_p^T]^T, \qquad k_i \in R^{1\times n}, \quad i = 1,2,\ldots,p \tag{45}$$

$$K_f = [k_{f1}^T, k_{f2}^T, \ldots, k_{fp}^T], \qquad k_{fi} \in R^{1\times n}, \quad i = 1,2,\ldots,p \tag{46}$$

$$K_r = [k_{r1}^T, k_{r2}^T, \ldots, k_{rp}^T]^T, \qquad k_{ri} \in R^{1\times p}, \quad i = 1,2,\ldots,p \tag{47}$$

$$K_{rf} = [k_{rf1}^T, k_{rf2}^T, \ldots, k_{rfp}^T], \qquad k_{rfi} \in R^{1\times p}, \quad i = 1,2,\ldots,p. \tag{48}$$

Since $B = [b_1,\ldots,b_i,\ldots,b_p]$ and $B_f = [b_1,\ldots,(1-\gamma_{ai})b_i,\ldots,b_p]$, the modified control gain parameters

$$k_{fi} = (1-\gamma_{ai})^{-1}k_i \tag{49}$$

and

$$k_{rfi} = (1-\gamma_{ai})^{-1}k_{ri} \tag{50}$$

will satisfy (44) and (41). The reconfigured control input $u_{fi}(k)$ to the ith actuator is

$$u_{fi}(k) = -k_{fi}x(k) + k_{rfi}r(k). \tag{51}$$

Combining (49), (50), and (51) yields

$$\begin{aligned} u_{fi}(k) &= -k_i(1-\gamma_{ai})^{-1}x(k) + k_{ri}(1-\gamma_{ai})^{-1}r(k)) \\ &= (1-\gamma_{ai})^{-1}u_i(k). \end{aligned} \tag{52}$$

Equation (52) implies that, when an actuator is detected faulty with an effectiveness factor γ_{ai}, the commanded input to the faulty actuator is increased by $(1-\gamma_{ai})^{-1}$ times (through the increase of gains) in order to maintain the originally desired actuation. This agrees with common sense and may recover the prefault system performance under certain circumstances. However, there are several potential problems associated with this FTC scheme. First, the faulty actuator is virtually not distinguished from the healthy actuators and is still loaded the same as a healthy actuator. For a faulty actuator such as a motor with welded windings or a damaged control surface of an aircraft, the maximum available actuation is reduced as the result of the fault, and it is essential to reduce correspondingly the commanded loading to the faulty actuator. Secondly, when the actuator fault is severe, i.e. $(1-\gamma_{ai})^{-1}$ is large, the value of u_{fi} may become too large to implement and lead to control saturation. For example, it may be well beyond the range of a digital-to-analog interface port that is often used to provide commanded input to actuators.

Actuation reconfiguration is a proposed design solution to address these problems when there are redundant actuators in the system. It means a load sharing redistribution among redundant actuators after a faulty actuator is detected and isolated. The load shared by redundant actuators is redistributed in such a way that the share of the faulty actuator is restrained. An adequate way of restraining the faulty actuator load is proposed as follows:

$$k_{fi} = k_i \tag{53}$$

$$k_{rfi} = k_{ri} \tag{54}$$

$$k_{fj} = k_j + \Delta_j, \qquad j \neq i \quad \text{and} \quad j = 1,2,\ldots,p \tag{55}$$

$$k_{rfj} = k_{rj} + \Delta_{rj}, \qquad j \neq i \quad \text{and} \quad j = 1,2,\ldots,p \tag{56}$$

where Δ_j are the vector elements of the gain increment matrix

$$\Delta = [\Delta_1^T \cdots \Delta_{i-1}^T \quad \Delta_{i+1}^T \cdots \Delta_p^T]^T = K_f - K \tag{57}$$

and Δ_{rj} are the vector elements of the gain increment matrix

$$\Delta_r = [\Delta_{r,1}^T \cdots \Delta_{r,r-1}^T \quad \Delta_{r,r+1}^T \cdots \Delta_{r,p}^T]^T = K_{rf} - K_r. \tag{58}$$

From (44), (45), (46), (53), and (55), we have

$$[b_1 \cdots b_i \cdots b_p]\begin{bmatrix} k_1 \\ \cdots \\ k_i \\ \cdots \\ k_p \end{bmatrix} = [b_1 \cdots (1-\gamma_{ai})b_i \cdots b_p]\begin{bmatrix} k_1 + \Delta_1 \\ \cdots \\ k_i \\ \cdots \\ k_p + \Delta_p \end{bmatrix}. \tag{59}$$

Hence

$$\sum_{j=1}^{p} b_j k_j = \sum_{\substack{j=1 \\ j\neq i}}^{p} b_j k_j + \sum_{\substack{j=1 \\ j\neq i}}^{p} b_j \Delta_j + (1-\gamma_{ai})b_i k_i. \tag{60}$$

And Δ satisfies the following relation

$$\Delta = \gamma_{ai}[b_1,\ldots,b_{i-1},b_{i+1},\ldots,b_p]^+ b_i k_i. \tag{61}$$

Similarly, aligning (41), (47), (48), (54), and (56), we have

$$\Delta_r = \gamma_{ai}[b_1,\ldots,b_{i-1},b_{i+1},\ldots,b_p]^+ b_i k_{ri}. \tag{62}$$

It should be noticed that the matrices Δ and Δ_r that satisfy (61) and (62) are not unique, since there are various ways to redistribute loading among redundant actuators. As a result, there are also various K_f and K_{rf} that can satisfy (43) and (44). Additional requirements can be specified in determining the components of the gain increment matrices Δ and Δ_r, such as sharing the load equally among the healthy actuators or using weight factors, etc.

Consider the case of equal actuation redistribution among the healthy actuators in postfault system, i.e., $\Delta_1 = \cdots = \Delta_{i-1} = \Delta_{i+1} = \cdots = \Delta_p = \hat{\Delta}$, and $\Delta_{r,1} =$

$\cdots = \Delta_{r,i-1} = \Delta_{r,i+1} = \cdots = \Delta_{r,p} = \hat{\Delta}_r$. From (61) and (62),

$$\hat{\Delta} = \gamma_{ai}\left(\sum_{j=1 j\neq i}^{p} b_j\right)^{+} b_i k_i \tag{63a}$$

$$\hat{\Delta}_r = \gamma_{ai}\left(\sum_{j=1 j\neq i}^{p} b_j\right)^{+} b_i k_{ri}. \tag{63b}$$

In the above analysis, B_f is assumed to have the same dimension as that of B. In case of a complete actuator failure, the corresponding column of B_f will be zero, and the load of the failed actuator will be completely taken over by the other actuators.

V. SIMULATIONS

Computer simulations are conducted using a linearized vertical takeoff and landing (VTOL) aircraft model in the vertical plane as given in [12]:

$$\dot{x}(t) = A_t x(t) + B_t u(t) + w(t)$$
$$z(t) = Cx(t) + v(t)$$
$$y(t) = C_r x(t)$$

where the state vector $x = [u \;\; v \;\; q \;\; \theta]^T$ consists of the horizontal velocity u, vertical velocity v, pitch rate q, and pitch angle θ; the control input $u = [\delta_c \;\; \delta_l]^T$ includes the blade angle of collective pitch control δ_c, and the blade angle of longitudinal cyclic pitch control δ_l. The model parameters are given as [12]

$$A_t = \begin{bmatrix} -0.0336 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.707 & 1.420 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}$$

$$B_t = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.52 & 4.49 \\ 0.0 & 0.0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}, \qquad C_r = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The zero hold equivalent system can be represented by

$$x(k+1) = Ax(k) + Bu(k) + w(k)$$
$$z(k) = Cx(k) + v(k)$$
$$y(k) = C_r x(k)$$

where $A = e^{A_t T}$, $B = (\int_0^T e^{A_t \tau} d\tau) B_t$. The sampling period is $T = 0.1$ s. The initial state vector is $x(0) = [20 \;\; 10 \;\; 8 \;\; 1]$. The four desired eigenvalues are given as $\{0.449, 0.662, 0.7617, 0.8308\}$. A Gaussian white noise is used to simulate the sensor measurement noise v and actuation disturbance w. Assume $\Sigma_{ww} = \mathrm{diag}\{0.01^2, 0.01^2, 0.01^2, 0.01^2\}$, $\Sigma_{vv} = \mathrm{diag}\{0.2^2, 0.2^2, 0.2^2, 0.2^2\}$, and

$$BK_r = B_f K_{rf} = \begin{bmatrix} 0.0240 & -0.0154 \\ -0.0472 & 0.1344 \\ -0.1167 & -0.0007 \\ -0.0059 & -0.0001 \end{bmatrix}.$$

In this simulation, the two actuators and four sensors are divided into three groups for FDI. The two actuators are taken as one group, and the four sensors are split into two groups. For this system, the original algorithm in [6] requires the computation of six Kalman filters. Using the proposed scheme, only three Kalman filters are computed to detect and isolate an actuator fault, and five Kalman filters for detecting and isolating a sensor fault. The computation load reduction will be even more significant if there is a large number of actuators and sensors in the system.

In the simulation we consider a single fault in each test run. The reference input is selected as $r(k) = [20 \;\; 20]^T$. A fault is introduced at $t = 10$ s in each scenario. Fig. 1 shows the simulation results with an actuator fault with an effectiveness factor of 0.8. The last estimate of the effectiveness factor is 0.7904. The simulation results for a sensor fault with an effectiveness factor of 0.35 are shown in Fig. 2. The last estimate of the effectiveness factor is 0.3579.

The results have demonstrated that the proposed method can efficiently estimate the effectiveness factor of a faulty actuator or sensor in the presence of the simulated system disturbance and measurement noise.

Fig. 3(a) shows the control inputs when a conventional eigenvalue assignment FTC as given in (52) is used. Using the proposed control scheme with actuation reconfiguration, the control inputs are as shown in Fig. 3(b). A clear reduction in the loading to the faulty actuator $u_2(t)$ can be seen by comparing the results.

VI. CONCLUSIONS

A systematic approach to integrated fault detection, isolation and reconfigurable control system design with consideration of both sensor and actuator faults is presented. Using grouped constrained Kalman filters, an actuator or sensor fault is detected and isolated. The effectiveness factor of a detected faulty actuator or sensor is calculated with a derived explicit algorithm, and the result is integrated in the FTC. A proposed concept of actuation reconfiguration is

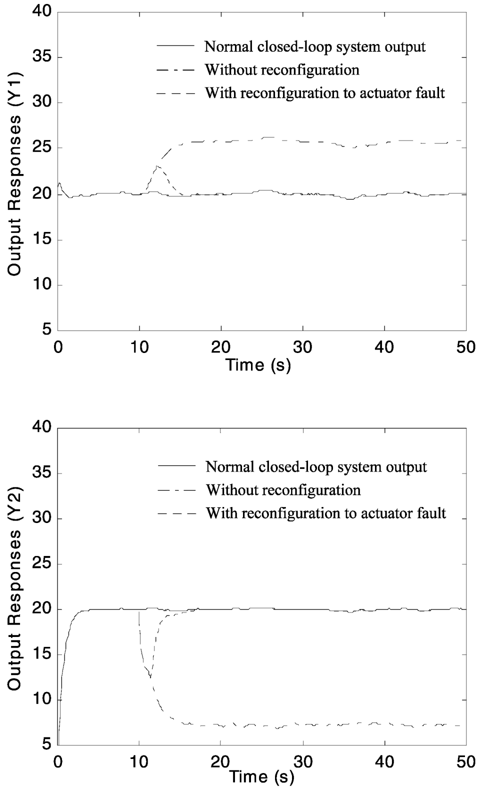

Fig. 1. Output responses for actuator fault at $t = 10$ s.

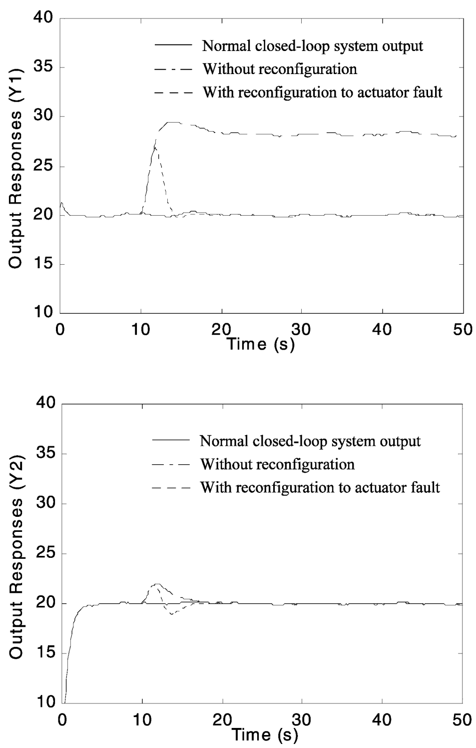

Fig. 2. Output responses for sensor fault at $t = 10$ s.

implemented in the control design for the postfault system to moderate the workload of a faulty actuator. Simulation results for an aircraft example have confirmed the effectiveness of the proposed approach.

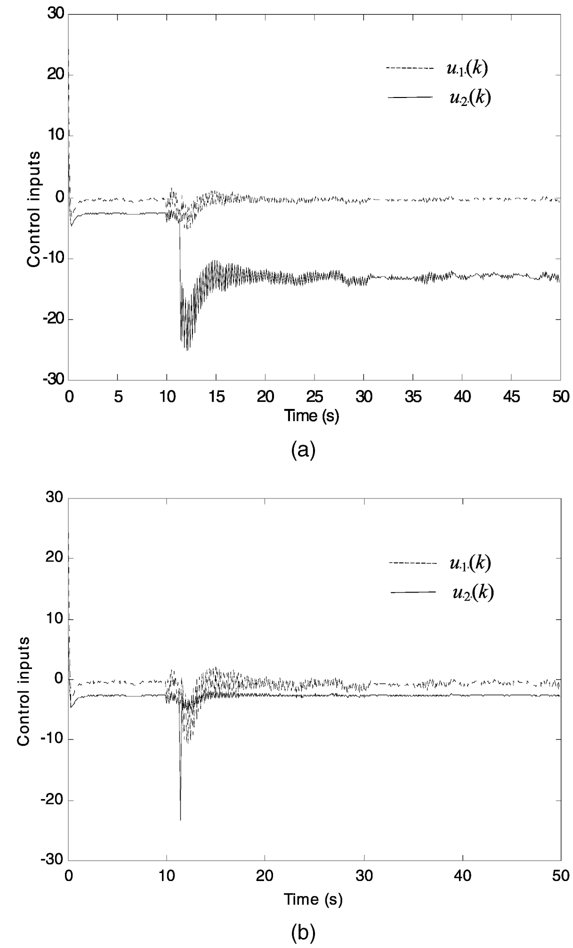

Fig. 3. Control inputs. (a) Under conventional control law without actuation reconfiguration. (b) Under proposed control law with actuation reconfiguration.

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REFERENCES

[1] Bao, G., Liu, G., Jiang, J., and Lam, C. (2002)
Active fault-tolerant control system design for a two-engine bleed air system of aircraft.
In *23rd Congress of The International Council of the Aeronautical Sciences*, Canada, 2002, 6102.1–6.

[2] Chen, J., Patton, R. J., and Zhang, H. Y. (1996)
Design of unknown input observers and robust fault detection filters.
International Journal of Control, **63**, 1 (1996), 85–105.

[3] Gao, Z., and Antsaklis, P. J. (1991)
Stability of the pseudo-inverse method for reconfigurable control systems.
International Journal of Control, **53** (1991), 717–729.

[4] Jiang, J., and Zhao, Q. (1998)
Fault tolerant control system synthesis using imprecise fault identification and reconfigurable control.
In *Proceedings of the 1998 IEEE ISIC/CIRA/ISAS Joint Conference*, Gaithersburg, MD, 1998, 14–17.

[5] Keller, J-Y. (1999)
Fault isolation filter design for linear stochastic systems.
Automatica, **35** (1999), 1701–1706.

[6] Larson, E. C., Parker, B. E., Jr., and Clark, B. R. (2002)
Model-based sensor and actuator fault detection and isolation.
In *Proceedings of the American Control Conference*, Anchorage, AK, 2002, 4215–4219.

[7] Liu, G. (2001)
Control of robot manipulators with consideration of actuator performance degradation and failures.
In *Proceedings of the IEEE International Conference on Robotics and Automation*, Korea, 2001, 2566–2571.

[8] Noura, H., Sauter, D., Hamelin, F., and Theilliol, D. (2000)
Fault-tolerant control in dynamic system: application to a winding machine.
IEEE Control System Magazine, **20** (2000), 34–49.

[9] Tao, G., Joshi, S. M., and Ma, X. L. (2001)
Adaptive state feedback and tracking control of systems with actuator failures.
IEEE Transactions on Automatic Control, **46**, 1 (2001), 78–95.

[10] Wu, N. E., Zhang, Y. M., and Zhou, K. M. (2000)
Detection, estimation, and accommodation of loss of control effectiveness.
International Journal of Adaptive Control, **14** (2000), 775–795.

[11] Zhang, Y. M., and Jiang, J. (2001)
Integrated active fault-tolerant control using IMM approach.
IEEE Transactions on Aerospace and Electronic Systems, **37**, 4 (2001), 1221–1235.

[12] Zhang, Y. M., and Jiang, J. (2002)
Active fault tolerant control system against partial actuator failures.
IEE Proceedings Control Theory Application, **149**, 1 (2002), 95–104.

[13] Zhao, Q., and Jiang, J. (1998)
Reliable state feedback control system design against actuator failures.
Automatica, **34**, 10 (1998), 1267–1272.