Joint Integrated Probabilistic Data Association: JIPDA

A new recursive filter for multi-target tracking in clutter is presented. Multiple tracks may share the same measurement(s). Joint events are formed by creating all possible combinations of track-measurement assignments and the probabilities for these joint events are calculated. The expressions for the joint event probabilities incorporate the probabilities of target existence of individual tracks, an efficient approximation for the cluster volume and a priori probability of the number of clutter measurements in each cluster. From these probabilities the data association and target existence probabilities of individual tracks are obtained, which allows track state update and false track discrimination. A simulation study is presented to show the effectiveness of this approach.

I. INTRODUCTION

In many radar, sonar, and other target tracking applications, measurements (detections) may originate from targets, whose existence and trajectory are generally not known a priori, and also from other random sources, usually termed clutter. Target measurements are present in each scan with a certain probability of detection. In a multi-target situation, the measurements may have also originated from one of various targets. Automatic tracking in this environment initiates tracks using both target and clutter measurements. If a track follows a target, we call it a true track otherwise we call it a false track. To discriminate between true and false tracks, a track quality measure is necessary.

This paper presents the joint integrated probabilistic data association (JIPDAF) filter for multi-target tracking in a cluttered environment. JIPDAF contains recursive expressions for the track quality measure and data association coefficients. JIPDAF and all other algorithms mentioned here use the probabilistic data association (PDA) [3] approximation, which uses all validated measurements of the track being updated and approximates the track state estimate probability density function (pdf) with a single Gaussian pdf. JIPDAF, in a manner similar to integrated PDAF (IPDAF) [9, 12], recursively calculates the probability of target existence for the track quality measure. Target existence is modeled as a Markov process, with two propagation models, Markov Chain One and Markov Chain Two. When applying JIPDAF to an isolated track, JIPDAF becomes IPDAF and in this sense it is a multi-target generalization of IPDAF. JIPDAF and IPDAF integrate seamlessly in the sense that a track can be processed by either as the circumstances dictate, with no transition effects when switching from one to the other.

JIPDAF handles the possible presence of multiple targets in a joint PDAF (JPDAF) [6] manner. The JPDAF algorithm allows for the possibility that a measurement may have originated from one of a number of candidate tracks or from clutter. In each scan, JPDAF partitions tracks into clusters, where tracks in each cluster have common measurements. It generates all possible joint measurement to track assignments, which are called joint events, and calculates the a posteriori probability of each joint event. From these probabilities, the data association coefficients of each track are calculated and then used to update the track estimates. JIPDAF enumerates the same joint events as JPDAF and adds the target existence concept to the JPDAF. When no a priori information on clutter measurement density is available, nonparametric JPDAF uses the volume of the whole surveillance area and the number of all measurements present in its equations. Nonparametric JIPDAF uses an efficient approximation of the volume of the cluster area and estimates the number of clutter measurements within the cluster. Parametric JPDAF assumes a uniform clutter density with an a priori known clutter measurement density, whereas parametric JIPDAF, in the manner of IPDA-MAP [11], allows use of spatially nonuniform a priori known clutter measurement density [11, 15]. In the manner of IPDA-MAP, JIPDA-MAP is used to denote the parametric form of JIPDAF.

Integrated JPDAF (IJPDAF) [4] is also an algorithm for multi-target tracking in clutter. IJPDAF provides a measure of track quality and handles multiple target measurement origin possibility by creating all possible joint events. The measure of track quality is calculated in a manner similar to EB-PDAF [5, 8]. It is assumed that a target exists 'behind' each track, and the probability of perceivability of the target is recursively calculated as the track quality measure. The propagation model for the perceivability is equivalent to Markov Chain One model for target existence propagation, no Markov Chain Two model is identified. The number of joint events of IJPDA is much larger than in J(I)PDA due to nonperceivable

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target possibility. Apart from the fact that it also has JPDA parametric and nonparametric limitations described above, the main difference between JIPDAF and IJPDAF is that JIPDAF calculates the track state estimate pdf conditioned on target existence, whereas IJPDAF track state estimate depends on the probability of target perceivability, in the manner of generalized pseudo-Bayesian estimator of the first order (GPB1) [2].

Section II outlines categories of JIPDAF, and Section III defines the cluster. The joint events and associated a posteriori probabilities are presented in Section IV. Simulation is used in Section V to compare JIPDAF with IPDAF, EB-PDAF and IJPDAF algorithms in crossing targets situations in a dense and nonuniform clutter, followed by concluding remarks in Section VI. The nonparametric JIPDAF with Markov Chain One target existence model derivation is presented in the Appendix.[1]

II. VARIOUS CATEGORIES OF JIPDAF

Let z_k denote the set of measurements at scan k, and let $z_{k,i}$ denote the ith measurement of z_k, with $Z^k = z_k \cup Z^{k-1}$ denoting the set of sets of measurements up to and including scan k. At any scan k, and based on data Z^l, $l \leq k$, each track t is described by a state estimate $\hat{x}^t_{k|l}$, an error covariance matrix $\hat{P}^t_{k|l}$ and a probability of target existence. We model the target existence propagation as a Markov chain, and examine two possible cases. The first, Markov Chain One, has two states, the target may exist and is detectable with a known probability of detection P^t_D (event χ^t_k), or the target may not exist. This model is first used in [1] and is given by

$$P\{\chi^t_k\} = p^{(1)}_{11}P\{\chi^t_{k-1}\} + p^{(1)}_{21}(1 - P\{\chi^t_{k-1}\}). \quad (1)$$

The Markov Chain Two model [9, 12] distinguishes three possibilities. The first, denoted by $\chi^{t,d}_k$, is that the target exists at scan k and is detectable (term also used in IMM-PDA (Interacting Multiple Model PDA) [1] in a similar context); i.e. the target measurement is present in z_k with a known probability of detection P^t_D. The second, denoted by $\chi^{t,n}_k$, is that the target exists in scan k but is not detectable, i.e. the target measurement is present in z_k with probability zero. The third possibility is that the target does not exist at scan k. Possible uses of this model are discussed in [9, 12, 14]. This model is given by

$$P\{\chi^{t,d}_k\} = p^{(2)}_{11}P\{\chi^{t,d}_{k-1}\} + p^{(2)}_{21}P\{\chi^{t,n}_{k-1}\} + p^{(2)}_{31}(1 - P\{\chi^t_{k-1}\}) \quad (2)$$

$$P\{\chi^{t,n}_k\} = p^{(2)}_{12}P\{\chi^{t,d}_{k-1}\} + p^{(2)}_{22}P\{\chi^{t,n}_{k-1}\} + p^{(2)}_{32}(1 - P\{\chi^t_{k-1}\}) \quad (3)$$

$$P\{\chi^t_k\} = P\{\chi^{t,d}_k\} + P\{\chi^{t,n}_k\}. \quad (4)$$

For both models above, the Markov chain coefficients must satisfy

$$0 \leq p^{(1)}_{11}, p^{(1)}_{21}, p^{(2)}_{11} + p^{(2)}_{12}, p^{(2)}_{21} + p^{(2)}_{22}, p^{(2)}_{31} + p^{(2)}_{32} \leq 1. \quad (5)$$

The track t update on scan k starts with the track predicted state $\hat{x}^t_{k|k-1}$, and state covariance matrix $\hat{P}^t_{k|k-1}$, and the a priori probability $P\{\chi^t_k \mid Z^{k-1}\}$ for Markov Chain One or $P\{\chi^{t,d}_k \mid Z^{k-1}\}$ and $P\{\chi^{t,n}_k \mid Z^{k-1}\}$ for Markov Chain Two. The estimation algorithm provides the a priori pdf of the predicted measurement position $f^t(z \mid Z^{k-1})$. The expected measurement position $\hat{z}^t_{k|k-1}$ is calculated and a selection window (validation gate) is defined around $\hat{z}^t_{k|k-1}$ such that the probability of detected target measurement being selected is P^t_W. Let V^t_k denote the volume of the window at scan k. If $p^t(z \mid Z^{k-1})$ denotes the a priori pdf for the predicted measurement position, conditioned on the measurements being selected, then

$$P^t_W = \int_{V^t_k} f^t(z \mid Z^{k-1}) dz \quad (6)$$

$$p^t(z \mid Z^{k-1}) = \begin{cases} \frac{1}{P^t_W} f^t(z \mid Z^{k-1}), & z \in V^t_k \\ 0, & z \notin V^t_k \end{cases}. \quad (7)$$

No a priori knowledge of clutter measurement density results in the nonparametric version of JIPDAF. If we assume known clutter measurement density in the surveillance region, we obtain the parametric version of JIPDAF (JIPDA-MAP).

III. CLUSTER OVERVIEW

In each scan, tracks are partitioned into clusters [6]. A cluster is a set of tracks which share no measurements with any track not belonging to the cluster. A trivial cluster is the set of all the tracks, for computational reasons each cluster should contain the minimal set conforming to the definition. Let T denote the number of the cluster tracks, let m_k and m^t_k denote the total number of cluster measurements and the number of measurements in the window of track t, respectively. For Markov Chain One model, the a priori estimated number of clutter measurements $\hat{m}_k$ in the cluster is

$$\hat{m}_k = \sum_{i=1}^{m_k} \left(\prod_{t=1}^{T} \left(1 - \frac{P^t_D P^t_W P\{\chi^t_k \mid Z^{k-1}\}}{m^t_k} \right)^{\mu(k,t,i)} \right) \quad (8)$$

[1]Parts of this paper have appeared as [10], however this paper contains considerable extensions.

where $\mu(k,t,i)$ is one if measurement i is in the window of track t at scan k and zero otherwise. For Markov Chain Two, $P\{\chi_k^{t,d} \mid Z^{k-1}\}$ replaces $P\{\chi_k^t \mid Z^{k-1}\}$. The cluster volume V_k, is the union of individual track windows. The approximate expression for V_k used in JIPDAF is

$$V_k = \max\left(\frac{m_k}{\sum_{t=1}^{T} m_k^t}\sum_{t=1}^{T} V_k^t, \max_t(V_k^t)\right) \tag{9}$$

where $\max_t(V_k^t)$ is the biggest window volume of individual tracks.

IV. JIPDAF DATA ASSOCIATION

A joint event is one possible mapping of the cluster measurements to the cluster tracks. JIPDAF generates the same joint events as the JPDAF algorithm [6]. In each joint event

1) each cluster track can be assigned zero measurements or one of the measurements which falls in the selection window of the track,

2) each measurement can be allocated to zero or one of the cluster tracks.

Two joint events are different if assignment of at least one measurement is different. The joint events are mutually exclusive, and they should form a complete set. Let χ_i and X denote the joint event i and the number of joint events in the cluster, respectively. Let T_0^i and T_1^i denote the set of tracks allocated no measurements and one measurement, respectively, in the joint event χ_i. For Markov Chain Two, nonparametric version, the a posteriori probability of joint event χ_i becomes

$$P\{\chi_i \mid Z^k\} = C^{-1}\prod_{t\in T_0^i}(1 - P_D^t P_W^t P\{\chi_k^{t,d} \mid Z^{k-1}\}) \times \prod_{t\in T_1^i}\left(P_D^t P_W^t P\{\chi_k^{t,d} \mid Z^{k-1}\}\frac{p_i^t V_k}{\hat{m}_k}\right) \tag{10}$$

where $p_i^t = p^t(z_{m(i,t)} \mid Z^{k-1})$ and $m(i,t)$ denotes the measurement allocated to track t under joint event i. The joint events form a complete set and the normalization constant C is calculated using

$$\sum_{j=1}^{X} P\{\chi_j \mid Z^k\} = 1. \tag{11}$$

The a posteriori probability of an individual track event is obtained by summing the a posteriori probabilities of all joint events containing the track event. Denote by $\Xi(t,i)$ the possibly empty set of joint events in which track t has been allocated measurement i (0 denoting no measurement). The a posteriori probabilities of no measurement originating from the track t, of the event of target t existence and nondetectability, of the event of target t existence and detectability with no measurement originating from the target, and of the event of target t existence and detectability with measurement i originating from the target are, respectively,

$$P\{\chi_{k,0}^t \mid Z^k\} = \sum_{e\in\Xi(t,0)} P\{\chi_e \mid Z^k\} \tag{12}$$

$$P\{\chi_k^{t,n} \mid Z^k\} = \frac{P\{\chi_k^{t,n} \mid Z^{k-1}\}}{1 - P_D^t P_W^t P\{\chi_k^{t,d} \mid Z^{k-1}\}} P\{\chi_{k,0}^t \mid Z^k\} \tag{13}$$

$$P\{\chi_k^t \chi_{k,0}^{t,d} \mid Z^k\} = \frac{(1 - P_D^t P_W^t) P\{\chi_k^{t,d} \mid Z^{k-1}\}}{1 - P_D^t P_W^t P\{\chi_k^{t,d} \mid Z^{k-1}\}} P\{\chi_{k,0}^t \mid Z^k\} \tag{14}$$

$$P\{\chi_k^t \chi_{k,i}^t \mid Z^k\} = \sum_{e\in\Xi(t,i))} P\{\chi_e \mid Z^k\}. \tag{15}$$

The a posteriori probability that target t exists and is detectable is

$$P\{\chi_k^{t,d} \mid Z^k\} = P\{\chi_k^{t,d}\chi_{k,0}^t \mid Z^k\} + \sum_{i\in\{\mu(k,t,i)>0\}} P\{\chi_k^t \chi_i^t \mid Z^k\} \tag{16}$$

where $\{\mu(k,t,i) > 0\}$ denotes the set of measurements falling in the window of track t at scan k. The a posteriori probability of target existence of track t is

$$P\{\chi_k^t \mid Z^k\} = P\{\chi_k^{t,d} \mid Z^k\} + P\{\chi_k^{t,n} \mid Z^k\}. \tag{17}$$

The β data association probabilities [3] for track t are

$$\beta_0^t = \frac{P\{\chi_k^t \chi_{k,0}^t \mid Z^k\}}{P\{\chi_k^t \mid Z^k\}} = \frac{P\{\chi_k^{t,d}\chi_{k,0}^t \mid Z^k\} + P\{\chi_k^{t,n} \mid Z^k\}}{P\{\chi_k^t \mid Z^k\}} \tag{18}$$

$$\beta_i^t = \frac{P\{\chi_k^t \chi_{k,i}^t \mid Z^k\}}{P\{\chi_k^t \mid Z^k\}}, \qquad i \in \{\mu(k,t,i) > 0\}. \tag{19}$$

The βs are used to update track estimates in a standard PDA manner [3, 8, 12] which is not repeated here for the reasons of brevity. The last step in the JIPDAF recursion is calculation of the predicted state estimate and predicted probability of target existence for scan $k+1$. Predicted target existence probabilities are calculated using (2)–(4).

Formulae for JIPDAF Markov Chain One with a nonparametric clutter model are obtained from JIPDAF Markov Chain Two formulae (10)–(19) by substituting

$$P\{\chi_k^{t,d}\} = P\{\chi_k^t\} \qquad P\{\chi_k^{t,n}\} = 0. \tag{20}$$

For JIPDAF Markov Chain Two, with a parametric clutter model, (10) becomes

$$P\{\chi_i \mid Z^k\} = C^{-1}\prod_{t\in T_0^i}(1 - P_D^t P_W^t P\{\chi_k^{t,d} \mid Z^{k-1}\}) \times \prod_{t\in T_1^i}\left(P_D^t P_W^t P\{\chi_k^{t,d} \mid Z^{k-1}\}\frac{p_i^t}{\rho_i^c}\right) \tag{21}$$

where $\rho_i^c = \rho^c(z_{m(i,t)})$ denotes the a priori clutter measurement density at the $z_{m(i,t)}$ location. Equations (11)–(19) remain unchanged. JIPDA Markov Chain One parametric formulae are again obtained by applying (20) to JIPDA Markov Chain Two parametric formulae.

V. SIMULATION

Simulation is used to compare the JIPDAF algorithm with IPDAF, EB-PDAF, and IJPDAF with respect to track discrimination and target crossing performance, in a heavy and nonhomogenous clutter environment. All algorithms compared are nonparametric. IPDA and EB-PDA both use the expected value of the number of selected clutter measurements, referred to as the "heuristic clutter density estimator" in [7], for the estimator of the number of clutter measurements. The tracks are initiated automatically, using two-point differencing and initial track probability assignment as described in [1] and [11]. Two different clutter scenarios were implemented. In the "lighter clutter environment" JIPDAF and IJPDAF algorithms are implemented on all tracks, from initiation onwards. In the "heavier clutter environment," JIPDAF and IJPDAF are applied to confirmed tracks only, and IPDAF and EB-PDAF, respectively, are applied to nonconfirmed tracks. In the IPDAF and EB-PDAF experiments, IPDAF and EB-PDAF are applied to all tracks after initialization in all scenarios. A two-dimensional surveillance situation was considered. The area under surveillance was 1000 m long and 400 m wide. The clutter measurements satisfied a uniform Poisson distribution with a base density with two patches with higher clutter density. The high clutter density patches are rectangular with corner coordinates $(x_{\min}, x_{\max}, y_{\min}, y_{\max})$ of (330, 490, 203, 303) m and (718, 840, 100, 200) m. The heavier clutter environment has a base density of 1.0×10^{-4}/scan/m^2 with a sevenfold increase in clutter density in the clutter "patches"; the lighter clutter environment has a base density of 0.2×10^{-4}/scan/m^2 with a fivefold increase in clutter density in the clutter patches.

The experiments consisted of 1000 and 800 runs each in the heavier and lighter clutter situations, respectively. Each run consists of 24 scans. In each run target one reappears in scan 1 with an initial state of $x'(1) = [130\text{ m}\ \ 35\text{ m/s}\ \ 200\text{ m}\ \ 0\text{ m/s}]$, and maintains constant speed thereafter. Target two follows a uniform speed trajectory which intersects the first target trajectory in scan 19, with a crossing angle of 10°. The true track situation is observed on scan 14 and then again on scan 24 to determine the effects of target crossing. The target motion model in Cartesian coordinates is

$$x(k+1) = Fx(k) + \nu(k) \tag{22}$$

where $x(k)$ consists of the position and the velocity in each of the 2 coordinates

$$x' = [x \quad \dot{x} \quad y \quad \dot{y}] \tag{23}$$

with the transition matrix

$$F = \begin{bmatrix} F_T & 0 \\ 0 & F_T \end{bmatrix}, \qquad F_T = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \tag{24}$$

where T is the sampling period. The plant noise $\nu(k)$ is zero-mean white Gaussian noise with

$$E[\nu(k)\nu(j)'] = Q\delta(k,j) \tag{25}$$

$$Q = q\begin{bmatrix} Q_T & 0 \\ 0 & Q_T \end{bmatrix}, \qquad Q_T = \begin{bmatrix} T^4/4 & T^3/2 \\ T^3/2 & T^2 \end{bmatrix} \tag{26}$$

where $\delta(k,j)$ is the Kronecker delta function and $q = 0.75$. The detection probability was 0.9 and the sensor introduced independent errors in the x and y coordinates with an rms error of 5 m. The tracking estimation filter was a simple Kalman filter based on the described trajectory and sensor models. The gate selection probability was $P_W = 0.9999$. All algorithms used a Markov Chain One target existence propagation model with

$$[p_{11}^1 \quad p_{21}^1] = [0.98 \quad 0]. \tag{27}$$

The JIPDAF appears to be relatively robust to the choice of the Markov Chain parameters. Parameter $p_{11}^{(1)}$ defines a priori expected length of true track life τ

$$\tau = \frac{T}{1 - p_{11}^{(1)}}. \tag{28}$$

After initialization, tracks evolve according to the algorithm simulated. The tracks are confirmed if the probability of target existence/perceivability exceeds the confirmation threshold and are terminated if the probability falls below the termination threshold. Thresholds and initial probability of target existence/perceivability were automatically optimized separately for each algorithm. For each algorithm the sum of confirmed false track scans was kept approximately the same; for the heavier clutter simulations it was 600 over 24000 scans, and for the lighter clutter simulations it was 60 over 19200 scans. The false track discrimination comparison is illustrated in Figs. 1 (heavier clutter) and 2 (lighter clutter). Each curve shows the number of scans in which a confirmed track followed target one. The horizontal axis depicts the time in scans from the start of the simulation run. The target crossing comparison is shown in Tables I and II for the heavier and lighter clutter situation, respectively. Only the cases where two confirmed tracks were following each of the two targets at scan 14 were considered. Five possible outcomes were recognized on scan 24:

a) both tracks continue to follow their original targets,

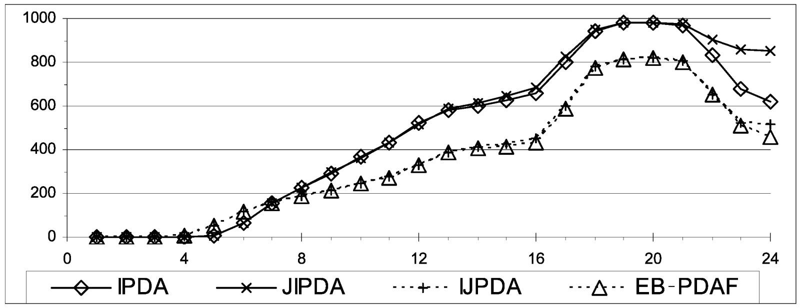

Fig. 1. Target one discrimination comparison—heavier clutter situation (1000 runs).

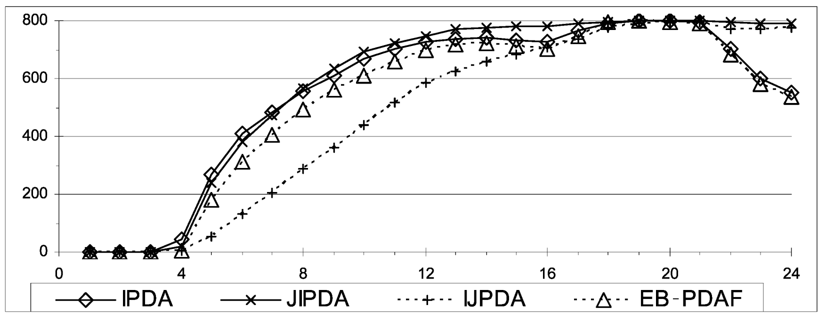

Fig. 2. Target one discrimination comparison—lighter clutter situation (800 runs).

TABLE I
Trajectory Intersection Results—Heavier Clutter Case

	IPDAF	JIPDAF	EB-PDAF	IJPDAF
Total	478	487	271	280
(a)	230	470	186	271
(b)	225	17	84	6
(c)	0	0	0	2
(d)	17	0	1	1
(e)	6	0	0	0
Time(s)	8412	8712	8491	8776

TABLE II
Trajectory Intersection Results—Lighter Clutter Case

	IPDAF	JIPDAF	EB-PDAF	IJPDAF
Total	685	747	655	540
(a)	316	738	301	533
(b)	327	9	322	6
(c)	0	0	0	1
(d)	27	0	21	0
(e)	15	0	11	0
Time(s)	400	844	392	6340

b) only one track continues to follow the original target,

c) both tracks switch targets,

d) one track switches the target, the other track becomes false or terminated,

e) both tracks become false or terminated.

The JIPDAF clearly improves the IPDAF track discrimination and track crossing performance. In this environment they appear to compare favorably with IJPDAF and EB-PDAF, respectively. Simulation durations in seconds for the algorithms are also presented in Tables I and II.

VI. CONCLUSION

This paper presents the joint IPDA algorithm for tracking multiple targets in clutter. JIPDA is suitable for automatic target tracking (sub)systems, as it provides the probability of target existence on each scan which can then be used as a measure of track quality useful for false track discrimination. JIPDAF integrates seamlessly with the IPDAF; a track can be followed by either of the algorithms as the situation dictates, and changing from one to the other requires no change to the track state and incurs no transient effects. When used in tandem with IPDAF on confirmed tracks only, JIPDAF can be used to track a small number of targets in very dense clutter situations without excessive increase in computational requirements.

APPENDIX. JIPDA NONPARAMETRIC MARKOV CHAIN ONE DERIVATION

The derivation of JIPDA with Markov Chain One target existence and propagation model, and with the nonparametric clutter model, is presented here. JIPDA with Markov Chain Two target existence and propagation model and/or parametric clutter model derivation follows in a similar manner and is not presented here for reasons of brevity. The derivations here assume that tracks have validated measurements and that clusters of tracks have been formed. Only one cluster is observed.

Let T denote the number of the tracks in the cluster, let m_k, m_k^t, and V_k^t denote the total number of measurements in the cluster, the number of measurements in the validation gate of track t and the validation gate volume of track t, respectively. The cluster area is formed as a union of validation gates of individual tracks and from then on, the cluster area becomes the common validation gate for each truck in the cluster. It should be noted here that the pdf of measurement position for each track is zero outside its individual validation gate; thus joint events should not be formed by allocating to tracks the measurements from outside its individual validation gate. An approximate expression for the cluster area

$$V_{ap} = \frac{m_k}{\sum_{t=1}^{T} m_k^t} \sum_{t=1}^{T} V_k^t \tag{29}$$

is exact in the case of infinitely dense and uniform clutter measurements within the cluster area. The approximation appears to function well in the simulated environment. For $m_k = 0$, cluster volume is not used and (29) is not calculated. The correction

$$V_k \approx \max(V_{ap}, \max_t(V_k^t)) \tag{30}$$

ensures V_k to be at least as big as the biggest validation gate in the cluster. Each measurement in the cluster may have originated from one or more tracks. With respect to one track t, the measurement i is a clutter measurement if it has not originated from this track. The a priori probability of measurement i not originating from track t is

$$P_{c,i}^t = \left(1 - \frac{P_D^t P_W^t P\{\chi_k^t \mid Z^{k-1}\}}{m_k^t}\right)^{\mu(k,t,i)} \tag{31}$$

where P_D^t is the probability of target detection, P_W^t is the selection probability, and $P\{\chi_k^t \mid Z^{k-1}\}$ is the a priori probability of target existence for track t. Expression $\mu(k,t,i)$ is one if measurement i is selected by track t at scan k and zero otherwise. The event of measurement not originating from track is independent across tracks and the a priori probability that measurement i is clutter becomes

$$P_{c,i} = \prod_{t=1}^{T} P_{c,i}^t. \tag{32}$$

The a priori expected number of clutter measurements in the cluster is obtained by

$$\hat{m}_k = \sum_{i=1}^{m_k} P_{c,i} = \sum_{i=1}^{m_k} \left(\prod_{t=1}^{T} \left(1 - \frac{P_D^t P_W^t P\{\chi_k^t \mid Z^{k-1}\}}{m_k^t}\right)^{\mu(k,t,i)} \right). \tag{33}$$

To find the a posteriori probability of a joint event χ_e, first separate cluster tracks in two sets:

1) set T_0^e of tracks with no allocated measurements,

2) Set T_1^e of tracks with one allocated measurements under the joint event χ_e. The number of tracks in this set is denoted by N_1^e. Denote with $m(e,t)$ the measurement allocated to track t under the joint event χ_e.

The a priori probability that no selected measurement originated from target t is,

$$P\{\chi_{k,0}^t \mid Z^{k-1}\} = 1 - P_D^t P_W^t P\{\chi_k^t \mid Z^{k-1}\}. \tag{34}$$

The a priori probability that no selected measurement is the target detection, and the target exists

$$P\{\chi_k^t, \chi_{k,0}^t \mid Z^{k-1}\} = (1 - P_D^t P_W^t) P\{\chi_k^t \mid Z^{k-1}\}. \tag{35}$$

The a priori conditional probability that allocated measurements are the correct target detections, given that these targets exist is

$$\begin{aligned} P&\left\{ \bigcap_{t\in T_1^e} \chi_{k,m(e,t)}^t \,\middle|\, \bigcap_{t\in T_1^e} \chi_k^t, m_k, Z^{k-1} \right\} \\ &= \prod_{i=1}^{N_1^e} \frac{1}{(m_k + 1 - i)} \prod_{t\in T_1^e} (P_D^t P_W^t) \\ &= \frac{(m_k - N_1^e)!}{m_k!} \prod_{t\in T_1^e} (P_D^t P_W^t). \end{aligned} \tag{36}$$

Thus, the a priori probability of joint event χ_e is

$$\begin{aligned} &P\{\chi_e \mid Z^{k-1}\} \\ &= P\left\{ \bigcap_{t\in T_0^e} \chi_{k,0}^t \bigcap_{t\in T_1^e} \chi_k^t, \chi_{k,m(e,t)}^t \,\middle|\, Z^{k-1} \right\} \\ &= \prod_{t\in T_0^e} (1 - P_D^t P_W^t P\{\chi_k^t \mid Z^{k-1}\}) \\ &\quad \times P\left\{ \bigcap_{t\in T_1^e} \chi_{k,m(e,t)}^t \,\middle|\, \bigcap_{t\in T_1^e} \chi_k^t, Z^{k-1} \right\} P\left\{ \bigcap_{t\in T_1^e} \chi_k^t \,\middle|\, Z^{k-1} \right\} \\ &= \prod_{t\in T_0^e} (1 - P_D^t P_W^t P\{\chi_k^t \mid Z^{k-1}\}) \frac{(m_k - N_1^e)!}{m_k!} \\ &\quad \times \prod_{t\in T_1^e} (P_D^t P_W^t P\{\chi_k^t \mid Z^{k-1}\}). \end{aligned} \tag{37}$$

If the number of false measurements follows a Poisson distribution [13], the a priori probability of m_c clutter measurements occurring in the window on scan k is

$$\begin{aligned} P_c\{m_c \mid Z^{k-1}\} &= e^{-\hat{m}_k} \frac{(\hat{m}_k)^{m_c}}{m_c!} \\ &= \frac{1}{m_c!(\hat{m}_k)^{m_k - m_c}} m_k! P_c\{m_k \mid Z^{k-1}\} \end{aligned} \tag{38}$$

and the a priori probability of m_k measurements in the cluster, given N_1^e target measurements is

$$P_c\{m_k - N_1^e \mid Z^{k-1}\} = e^{-\hat{m}_k} \frac{(\hat{m}_k)^{m_k - N_1^e}}{(m_k - N_1^e)!} = \frac{1}{(m_k - N_1^e)!(\hat{m}_k)^{N_1^e}} m_k! P_c\{m_k \mid Z^{k-1}\}. \quad (39)$$

The a priori pdf of each clutter measurement is

$$p_c(z_{k,i} \mid Z^{k-1}) = V_k^{-1}. \quad (40)$$

The a priori pdf of the measurements given that N_1^e measurements originated from the T_1^e targets is

$$p(z_k \mid \chi_e, m_k, Z^{k-1}) = V_k^{-m_k} \prod_{t \in T_1^e} (V_k p^t(z_{m(e,t)} \mid Z^{k-1})) \quad (41)$$

where $p^t(z \mid Z^{k-1})$ denotes the conditional pdf of the target measurement at the point of measurement z given that the measurement is selected by track t at scan k. The a posteriori probability of joint event χ_e is

$$\begin{aligned} P\{\chi_e \mid Z^k\} &= c_k^{-1} p(z_k \mid \chi_e, Z^{k-1}, m_k) \\ &\quad \times P\{m_k \mid \chi_e, Z^{k-1}\} P\{\chi_e \mid Z^{k-1}\} \\ &= C_k^{-1} \prod_{t \in T_0^e} (1 - P_D^t P_W^t P\{\chi_k^t \mid Z^{k-1}\}) \\ &\quad \times \prod_{t \in T_1^e} \left(P_D^t P_W^t P\{\chi_k^t \mid Z^{k-1}\} p^t(z_{m(e,t)} \mid Z^{k-1}) \frac{V_k}{\hat{m}_k} \right). \end{aligned} \quad (42)$$

Joint events are mutually exclusive and form a complete set, therefore

$$\sum_{j=1}^{X} P\{\chi_j \mid Z^k\} = 1. \quad (43)$$

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