Microwave Bytes

Cutting Tapers

■ **Steve C. Cripps**

I suppose I have always had an eye for the unconventional. Indeed, I know that some have characterized me as a "rebel" in as much as I'm always wasting too much time trying to avoid doing things the same way that everyone else does. But for some reason I have always enjoyed exploring the back routes, the less well-trodden paths that may, or (as usually it turns out) may not, lead to a better solution to a given problem than the boring old conventional approaches. I think also, sometimes, this is coupled to another well-known conundrum, which has been characterized as the "solution without a problem" syndrome. One is aware of this gizmo, this gadget or technique, that in itself appears clever, or elegant, but never seems to get used. One somehow gets besotted and spends the rest of your life trying to find a use for it, and/or wasting energy trying to get others to see it through the same rose-tinted spectacles through which you have always viewed it.

Being a "solution without a problem" seems to be a rite of passage that just about every newfangled microwave technology has to go through. I think I first heard the phrase applied to GaAs MMICs sometime back in the 1990s, and it has more recently been applied to Gallium Nitride, and MEMs technologies. For once, I will not get embroiled in such mainstream debates, since I am here thinking on a more personal, maybe even slightly "wacky" basis. Something that I have always thought looks clever, useful, elegant, but hardly ever gets used. Several candidates come to mind, and it occurs to me that one or two of them have already featured in this column. The mercury wetted relay switch (see my December 2009 column, [1]) has always fascinated me, and I suppose I have always yearned for a project that had an unquestionable need for a microwave tube amplifier (I mean one of those fascinating disk-sealed triodes, not a TWT!). But for the present offering I have selected the tapered transmission line transformer.

I first encountered it during my postgraduate days. I was playing around with a device that could generate a very short, high power pulse, hundreds of volts and quite a few amps, with about a 100 picosecond pulse length. The trouble was it would only work into a resistive load of a few Ohms, so although I could monitor the performance using a 50 Ohm sampling system, most of the power was wasted in a resistor and could not be "extracted," So my supervisor suggested that I should make a transmission line whose characteristic impedance gradually and smoothly changed, from the appropriate low value of a few Ohms, to 50 Ohms at the other end. At this particular time the circuit I was using was entirely made up from machined air-spaced coaxial lines, so the long tapered section would have to be machined to slide into a 50 Ω airline, giving a gradual increase in the inner coaxial dimension. Although this was an era where numerically controlled machine tools were becoming more widely available, it certainly posed a nice little challenge for the machine shop. This particular shop (unlike the standard breed, I have

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to say) seemed rather intrigued by the challenge and actually used a "mimic" lathe to cut the taper, where the tool is made to follow a profile that is cut from a metal sheet. I still remember the conversation about how thick the end should be, that is, where the insert diameter tapers down to zero. I seem to remember, in my misguided youthful zeal, asking for a ridiculously thin end dimension that buckled out of shape the first time I inserted it into the 50 Ω sleeve. However, all was well after I had sheepishly asked for the mangled part to be machined off, and I managed to get some mileage out of my resulting giant pulses [2].

More conventionally however, tapered transformers are realized using planar microstrip structures, as shown in Figure 1. Intuitively, one expects that if a transmission line is tapered slowly enough, the fields will readjust themselves and a broadband match to the appropriate terminating impedances will result at each end, so long as the taper is "slow" enough. An obvious question immediately arises: what is the optimum profile that gives the shortest length for a given transformation ratio. This is a problem that is a bit more complex than it may at first appear. As always (and it is becoming a regular feature in this column!), I reach for my trusty old copy of Ramo, Whinnery, Van Duzer (RWV) [3], which analyzes an exponential taper. This particular species happens to lend itself to a straightforward closed-form analytical treatment, but this is no guarantee that it represents the optimum solution. Finding the "perfect" taper profile has attracted a few like-minded workers over the years. As long ago as 1955 the problem was essentially solved, albeit with some restrictions, in elegant analytical style by Klopfenstein [4]. More recently Mahon and Elliot [5] extended Klopfenstein's analysis to a more general result. I must admit, despite finding this work interesting, I have always felt that there is an element of hair-splitting when comparing the responses of various taper profiles, and I have always tended to stick to the simple exponential variety.

Figure 2 shows a simple example of the matching properties of an exponential transmission line taper. This has been computed for a case where the length of the taper at 2 GHz is a full electrical wavelength, and the taper goes from 10 Ω up to 50 Ω. The resulting input reflection at the 50 Ω end, with a 10 Ω termination at the remote end is shown. I should comment that I have computed this, and all of the other responses in this piece using a home-brew program which divides and cascades the tapered line using 100 short sections of regular transmission line. Clearly, as the structure gets longer in terms of wavelengths, the impedance trajectory spirals inwards towards the 50 Ω point. In fact, Figure 2 does suggest that the exponential taper may not be the optimum solution, since the almost-circular loops of the spiral are not actually centered but sit slightly above the "bulls eye," but it takes a good half wavelength for the taper to scream up to full revs. And if we are restricted by space, even a single quarter wave length can be huge. A critical test, which I have to admit I have often thought about but never actually performed, is shown in Figure 3. Here I compare directly a regular uniform quarter wave transformer with an exponential taper of equal length. The uniform transformer has a characteristic impedance that is the geometric mean of the terminating impedances, 22.4 Ω in this case. The taper has an exponential variation of Z_o ranging between the termination impedances. (I note in passing that Klopfenstein's more optimum impedance profile does not start and finish at the precise termination values.) Clearly, looking around the design frequency at which the conventional uniform transformer becomes a quarter wavelength (0.5 GHz in this case), the quarter-wavelength taper falls well short in matching performance.

But as we have already observed, the tapered transformer starts to excel as its length is increased. To keep a level playing field, when for example comparing the half-wave cases, we must design the conventional transformer

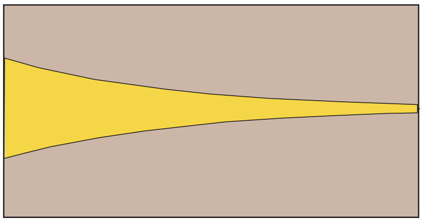

Figure 1. *A tapered microstrip transformer.*

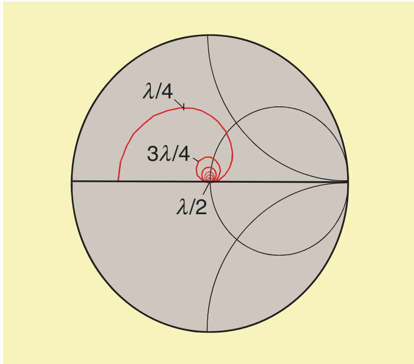

Figure 2. *Input impedance of a 10 Ω to 50 Ω tapered transformer; markers indicate electrical length at measurement frequency.*

as a two-section quarter wave cascade. My old rule of thumb says that for a two-section quarter wave transformer the intermediate impedance Z_M has to form a harmonic progression between the source and load terminations, that is to say

$$\frac{1}{R_s} - \frac{1}{Z_M} = \frac{1}{Z_M} - \frac{1}{R_L},$$

and the transformer Z_o values are then given by

$$Z_1 = \sqrt{R_S Z_M}, \; Z_2 = \sqrt{R_L Z_M},$$

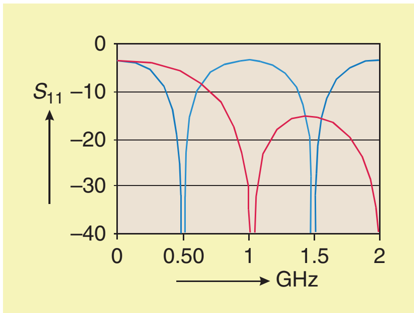

Figure 3. *Matching response of a conventional quarter-wave transformer (blue) and a tapered transformer having the same physical length (red); design frequency 0.5 GHz, 10 Ω to 50 Ω transformation.*

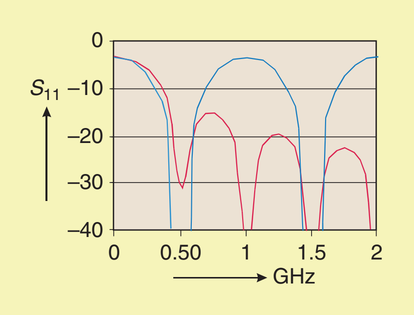

Figure 4. *Comparison between conventional two-section transformer (blue) and tapered transformer of equal physical length (red).*

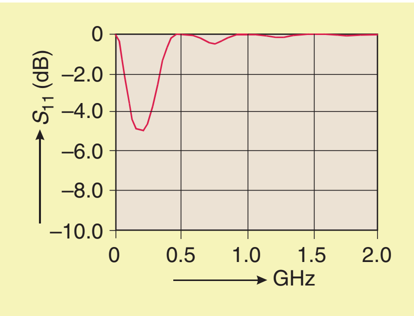

Figure 5. *"Back-to-back" transmission response of two 10 Ohm tapers.*

so for the case in question here,

$$Rs = 10, R_L = 50, Z_M = 16.7, \quad Z_1 = 12.9, Z_2 = 28.9.$$

Figure 4 shows the comparison between the two section and the tapered transformers, each now having the same overall electrical length of a half wavelength at 0.5 GHz. For my money the taper now wins, especially if we are allowed to stretch it just a little bit more.

One obvious place where one might consider using such a tapered transformer is in a traditional, "Wilkinson" power combiner. One still quite often sees these items designed using multiple cascades of quarter-wave transformers, in an effort to increase their bandwidth. This was one of many occasions in my own career that I thought I might explore an alternative path to the conventional well-trodden (and slightly boring) highway. I had been tasked with designing a set of power combiners that were compatible with an existing amplifier product line. It was not a particularly exciting assignment, and I could not resist the temptation to try something a bit different. On discovering my covert plan to use a taper instead of a double quarter-wave transformer, my boss at the time made the comment "but how would you cut it?" This was not, as one may at first assume, a use of the "cutting it" phrase in the usual colloquial sense, but referred to the fact that in that era all microwave circuits started their existence in the form of a "rubylith," something that my younger readers may have never heard of. The term refers to a large transparent plastic sheet that had a dark red (hence "ruby") film of thinner plastic material attached. Your circuit, for which you had to supply a detailed set of coordinates for every vertex, was "cut" using a large machine called a "coordinatograph." Essentially, this consisted of a cutting tool that scored through the ruby film, thus producing a much magnified version of your circuit layout; 50 or 100 times magnification was quite commonly used. The hapless circuit designer then had the wonderful task of "peeling" the ruby, so that the metal traces became windows in the sheet. I must say, having made my usual brief excursion into nostalgia, I always remember the "ruby cutting room" as quite a social place where designers spent time putting the world to rights; perhaps this was not a universal phenomenon, but seemed quite an institution at the company in question! Anyway, you get the point. The coordinatograph was only able to move in orthogonal directions and couldn't cut curves. Although I daresay around that time computer-controlled rubylith cutters became available, the CAD revolution was well under way and computer layout software such as AutoCad largely wiped the last rubyliths off the face of the planet. I can't say I miss them, although the rubylith room chit-chat certainly made some tougher days at the office more bearable.

Despite the admirable transforming properties that these structures offer to the GHz frequency designer (gosh, we really have to struggle not to call ourselves "microwave engineers" any more do we not!), they seem generally to be little used. The most obvious snag is that they really are quite long. As shown in Figures 3 and 4, the structure has to have an electrical length of maybe about a half wavelength at the low end of the required frequency range in order to get a match equivalent to a uniform quarter-wave transformer. But there is something else. It seems that these structures hide a dark secret; deadly nightshade lurks in this quiet, dark corner of electromagnetic theory. I remember this being brought up on most of the occasions in my career that I have dared to suggest the possible use of a tapered transformer. "Ah but," prefaces my friendly colleague as he prepares to blast me and my tapered transformer out of the water, "those things have a cut-off, just like a waveguide, you have to be careful about that."

I have always suspected that this marginal truth has played a major part in relegating tapered transformers to the realms of only very special, or quirky, applications. But is it even true? My ageing copy of RWV actually falls open on the pages (53–56 in my edition, art. 1.26) where tapered transmission lines are given a brief formal treatment. Surprisingly, I would even say astonishingly, these structures *do* appear to show cut-off behavior. In brief mathematical terms, the spatial transmission coefficient γ has the form

$$\gamma = -\frac{q}{2} \pm \sqrt{\left(\frac{q}{2}\right)^2 - \omega^2 L_o C_o},$$

where q, L_o, and C_o characterize the length and impedance profile of the taper. RWV comment, almost in passing, that this expression shows *"the interesting property of cutoff again,"* that is to say at a frequency given by

$$\omega^2 L_o C_o = \left(\frac{q}{2}\right)^2$$

the transmission coefficient crosses from imaginary to real values.

My first reaction, inevitably, is to start putting in some typical values in order to find this mysterious cut-off frequency, although I must admit I have always found it a bit tricky to unravel the taper profile information contained in the q, L_o, and C_o parameters in relation to a physical piece of hardware sitting on the bench in front of me. I will thus not subject my readers to this piece of algebraic administration, and give the answer in the form of a generalization that for a 10:1 taper impedance ratio, the cut-off frequency would appear to correspond to where the structure has an electrical length of about a fifth of a wavelength.

The cut-off would thus certainly appear to be below the frequency at which the matching properties of the taper cease in any case to be useful, but there is more to it than that. The transmission coefficients of the forward and reverse waves take on "real" values, thus making the wave evanescent. Oh dear, that wretched "e" word. I remember another early career experience, in a lab where there was a group working on evanescent mode filters. Here were these filters, assembled from lengths of waveguide, which from their dimensions were clearly being used well below their cut-off frequency. So here was a structure of some considerable length, certainly many free space wavelengths, that could still transmit a signal from one end to the other, despite being assembled mainly using a cut-off structure. So I suppose I have always treated the "e word" with some circumspection. The most obvious way of analyzing a tapered transmission line is to break it up into short sections of line, with appropriate successive steps in the characteristic impedance value. Given that each individual section behaves in a classical TEM fashion, with transmission extending all the way down to dc how can such an approach yield any kind of cut-off behavior? How can an electrically short piece of metal, albeit of nonuniform shape, "block" the propagation of an electromagnetic wave?

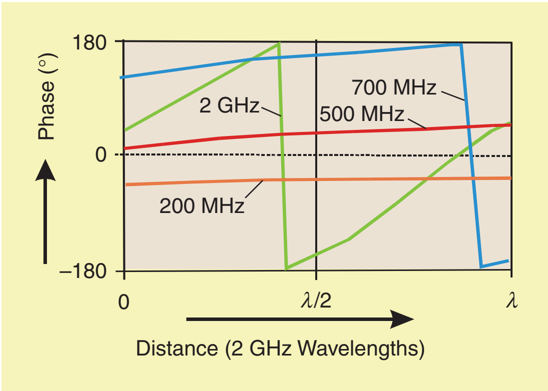

Figure 6. *Measured phase variation along the length of a tapered transformer.*

For a 10:1 taper impedance ratio, the cut-off frequency would appear to correspond to where the structure has an electrical length of about a fifth of a wavelength.

Well, even in the more familiar case of a waveguide cut-off it is worth noting that the process is not sudden. Indeed I rather think that the textbooks tend to promote this in a somewhat misleading manner. A cut-off waveguide that is less than a quarter wavelength long (and I'm talking free space wavelengths) certainly reflects some of the incident signal, but there is still some substantial transmission when the electrical length is short. In the case of the taper cut-off, we are dealing with a structure that is by definition always electrically quite short (i.e. less than a quarter wavelength), so that the cut-off process will never be fully revealed.

One way of exploring the cut-off behavior of a taper is to look at the transmission response of two tapered sections connected back-to-back, as shown in Figure 5. This plot, for me, does indeed have a "suggestion" of cut-off behavior, in this case around the one-fifth wavelength frequency. But even then, as the frequency gets even lower the cut-off subsides as dc is approached. I believe that the acid test of an evanescent wave is the disappearance of phase variation along the length of the structure; this is a mathematical consequence of having propagation constants that are real, rather than imaginary. It occurred to me that I could easily measure the variation of voltage along the length of a tapered section using the electric field probe that I described some time ago in this column [6]. Indeed, in the best tradition of academia I gave the project to a student who did a splendid job [7]. Figure 6 shows some of the results obtained, using a taper having similar properties to that described above for the half-wavelength case.

Clearly, as the frequency is lowered, the phase change along the length of the taper reduces, but the key is whether there is a greater reduction than that which would be

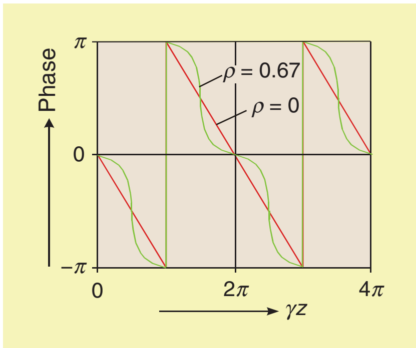

Figure 7. *Phase variation of voltage along a mismatched transmission line.*

TABLE 1. Phase variation due to various mechanisms.

Frequency (GHz)	Phase Variation (along taper)	Expected Phase Variation Due to Frequency Alone
2	360	360
0.7	100	126
0.5	36	90
0.2	10	36

expected due to frequency alone. This can be determined using Table 1.

Well, clearly there is a significant "foreshortening" effect in the apparent electrical length, although I am not quite sure whether these measurements truly bear out the theory, which would suggest an absolute cut-off in the spatial phase variation around 400 MHz for this particular structure. I also had to ask myself whether this behavior is not simply the interaction of forward and reverse waves as can be observed on a regular transmission line. We are all familiar with standing wave patterns on mismatched lines, but the textbooks tend to focus on the amplitude variations. I must admit I had not often (if ever at all) seen the corresponding phase plots, as shown in Figure 7. A terminated line shows the familiar linear phase variation, a familiar sight for network analyzer users. But as the reflection coefficient increases, the phase plot starts to distort and display a much flatter response over an increasing portion of the spatial cycle. So I have to say the measurement does not really convince me that we should really use terms such as "cut-off" and/or "evanescent waves," but it would appear to be a moot point inasmuch as the theoretical cut-off happens well below the frequency at which the device is useful for matching purposes.

I suppose in the end, it's size that matters. Tapers are wonderful impedance transformers, but the mainstream design establishment prefer to use inferior techniques that take up less space. My search for an application continues, although I note with minor satisfaction that the test and measurement community do use such structures in order to transform the ubiquitous 50 Ω to lower values, especially when making measurements on devices such as power transistors that have very low impedances. Small comfort, but probably more so than I will get from my quest to use a tube one day. The sand in that particular timer is running out faster than my own.

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