□ THE SEARCH FOR EFFECTIVE ALGORITHMS FOR RECOVERY FROM LOSS OF SEPARATION

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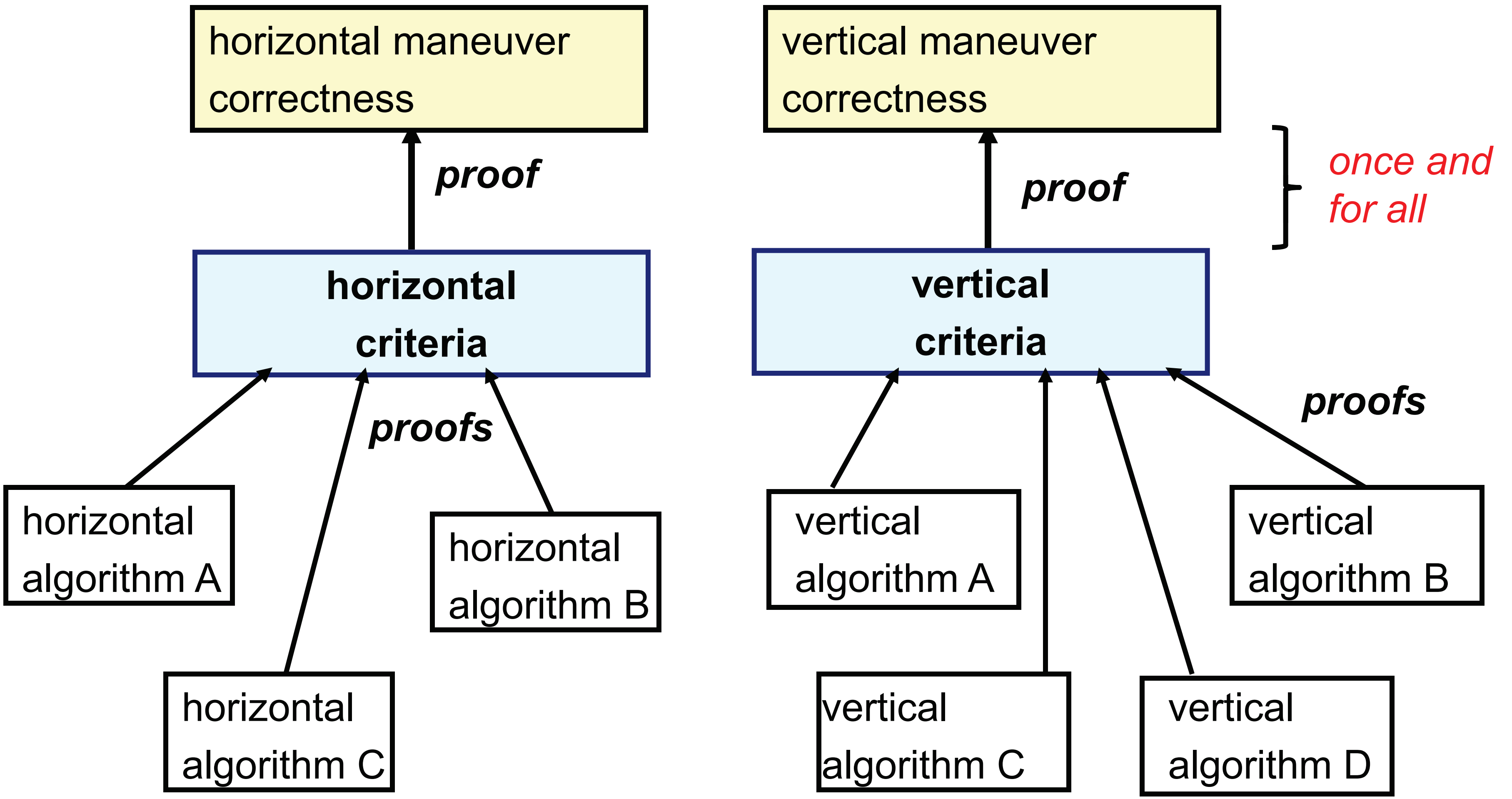
Outline

- The criteria approach to self separation
- Original loss of separation (LoS) criteria
- Problems with the original criteria
- New LoS criteria
- Proposed algorithms that satisfy the new criteria
- The new algorithms solve many of the problems

Criteria Concept



Our Approach

Rigorous mathematical definition of correctness for vertical and horizontal maneuvers

- State-based
- Independent (when only one aircraft maneuvers)
- Implicitly coordinated (when both aircraft maneuver)

Definition: IMPLICIT COORDINATION

both aircraft maneuver and

-- the *combined* effect resolves the conflict

-- there is no communication between the CD&R algorithms on the two aircraft

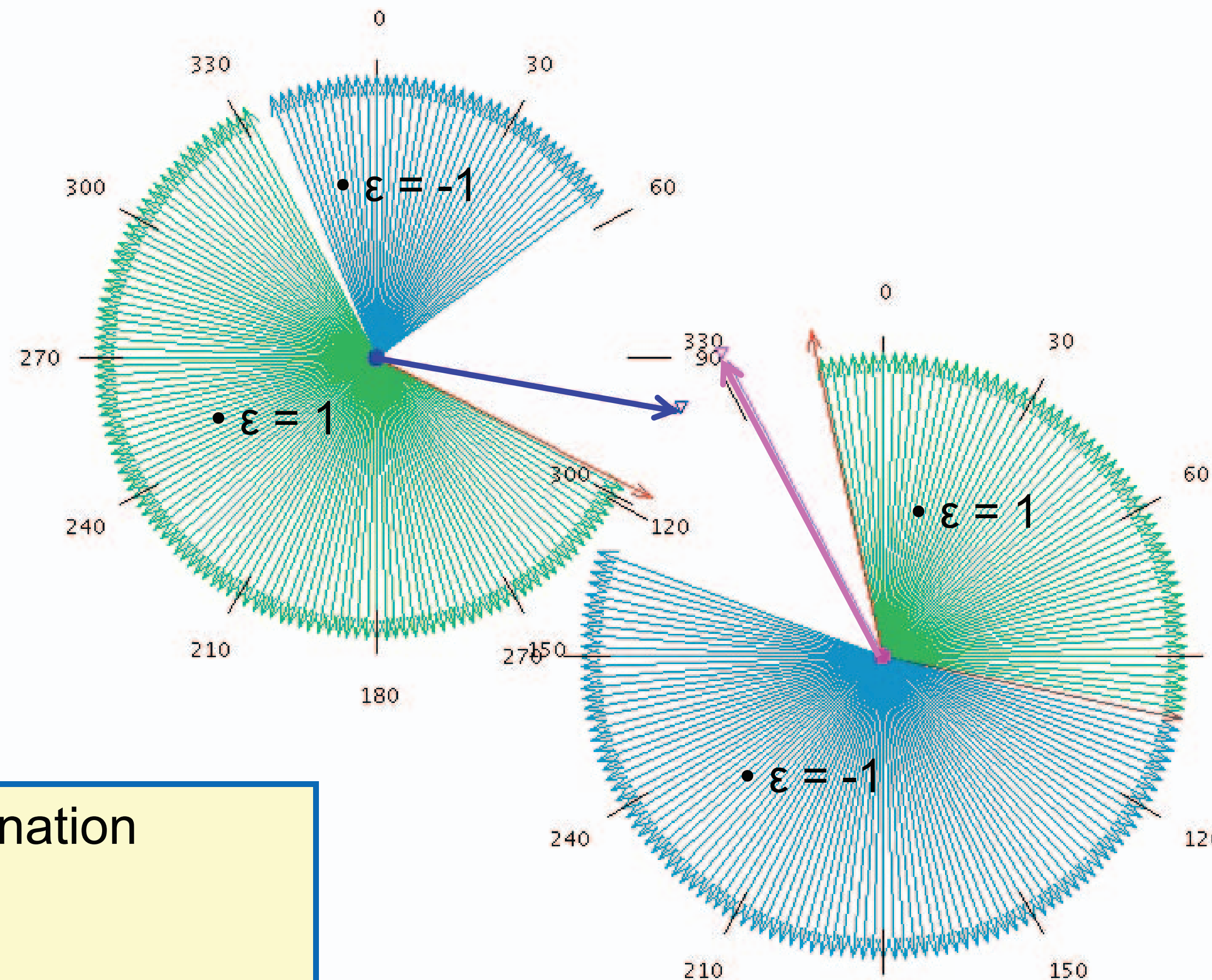
-- the only information used by the algorithms is ADS-B surveillance data

Horizontal Criteria for Conflict

$$(s \bullet v) \geq \varepsilon \; R\,(s^{\perp} \bullet v) \quad \text{where } \varepsilon = \{-1,1\}$$

and

$$R = \frac{\sqrt{s^2 - D^2}}{D}$$



- ε is the key to coordination
- We recommend:

$$\varepsilon = \text{sign}(s \bullet v^{\perp})$$

Original Criteria for Loss of Separation

- The original concept of correctness for loss of separation recovery was centered on divergence and timeliness.
- The correctness theorems depend upon the following assumptions:
  1. that aircraft state data is perfectly known,
  2. the solution vectors can be achieved instantaneously, and
  3. all aircraft compute their solutions using exactly the same data.

Original Criteria for Loss of Separation (cont.)

The original criteria for both horizontal and vertical recovery required divergence, where horizontal divergence is defined as follows:

$$\text{xy\_divergent?}(\mathbf{s},\mathbf{v}) \equiv \forall t : t > 0 \implies \|\mathbf{s}\| < \|\mathbf{s} + t\mathbf{v}\|.$$

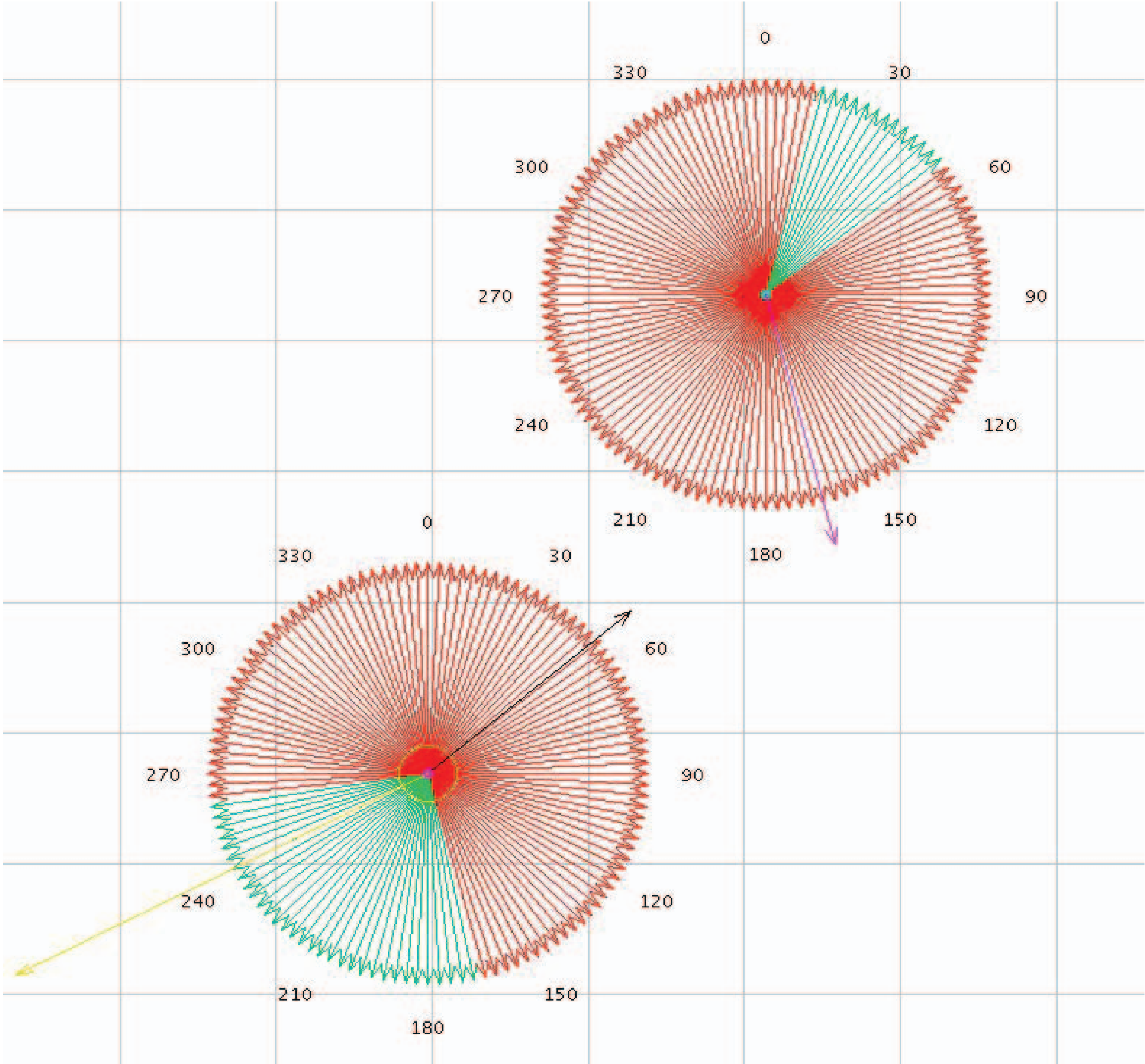
This property means that for all time in the future, the relative horizontal distance between the aircraft is greater than it was initially. In a similar manner, the vertical divergence is defined as follows:

$$\text{z\_divergent?}(\mathbf{s},\mathbf{v}) \equiv \forall t : t > 0 \implies |s_z| < |s_z + v_z t|.$$

Original Criteria → *Overly Restricted Maneuvers*

As we explored the criteria concept with hundreds of test cases, we began to notice that there were situations where the maneuvers were extreme when using our original criteria for recovery from loss of separation.

Original Criteria → *Overly Restricted Maneuvers*

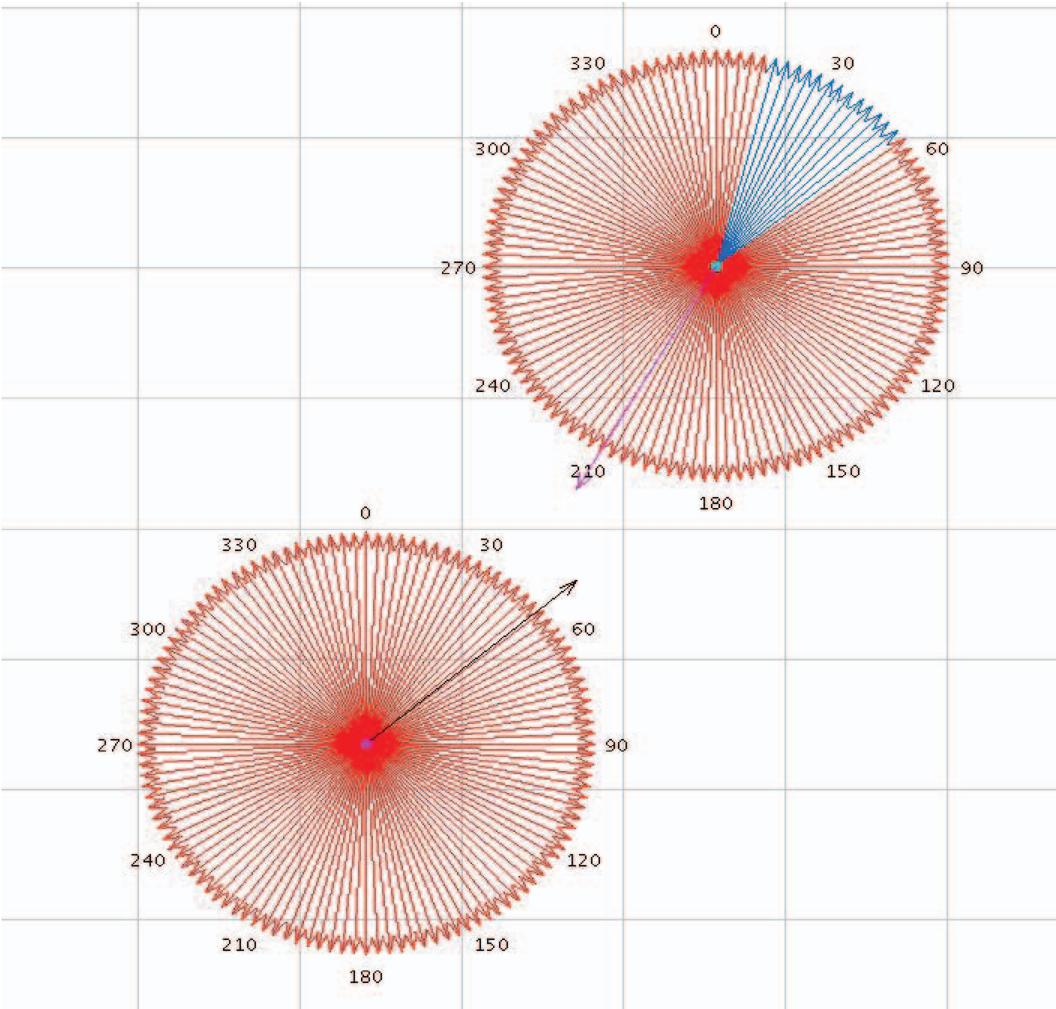


Green Vectors = Allowed Resolution Vectors

Original Criteria → *Overly Restricted Maneuvers*

If original track of the traffic aircraft is changed from 167 to 210 degrees

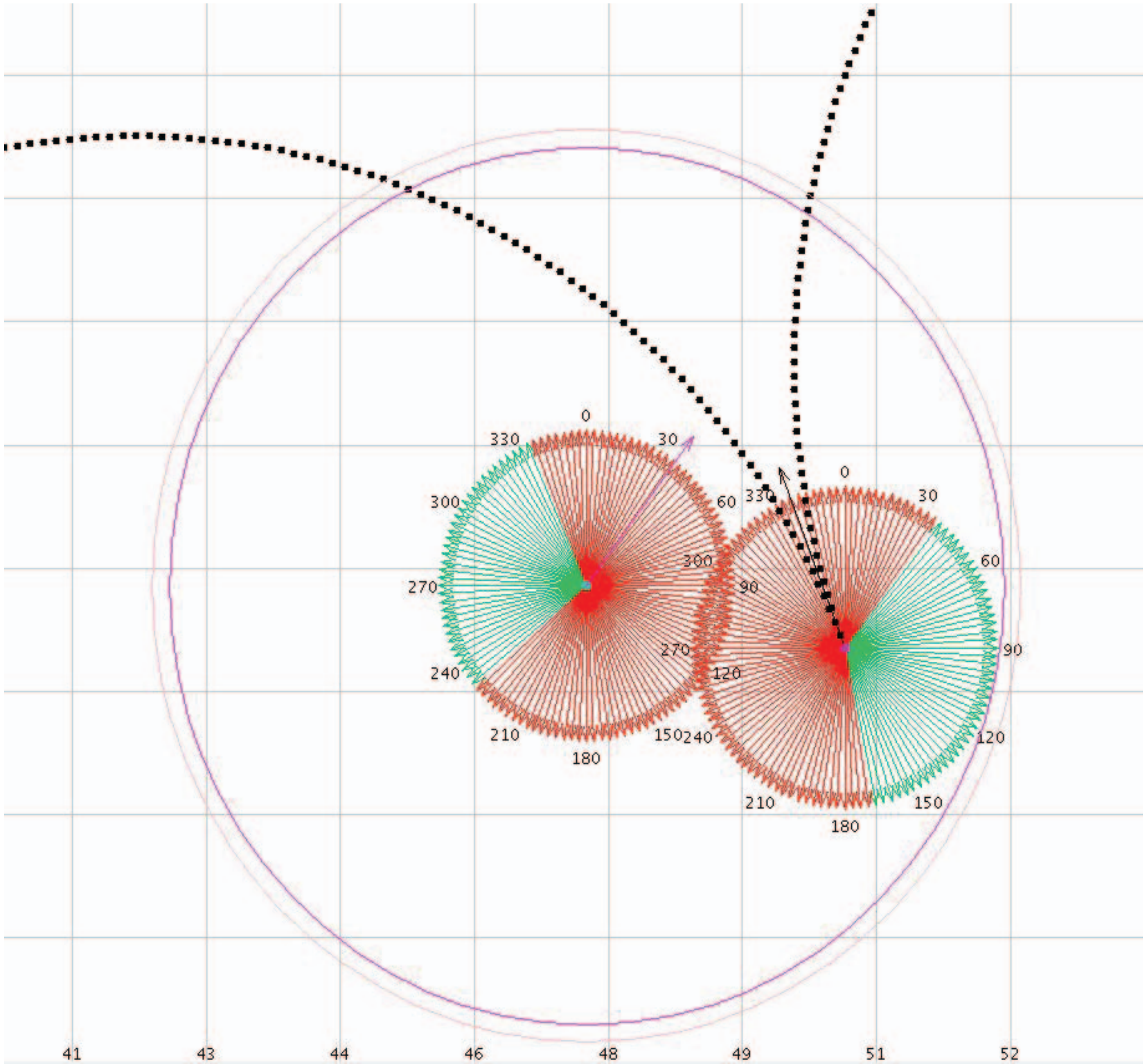
BIG CHANGE!



Blue Vectors = Allowed Resolution Vectors

Impact of Ignoring Maneuver Direction

The dotted line in Figure 5 shows the path of the aircraft for both left and right turns to reach the allowed green region.

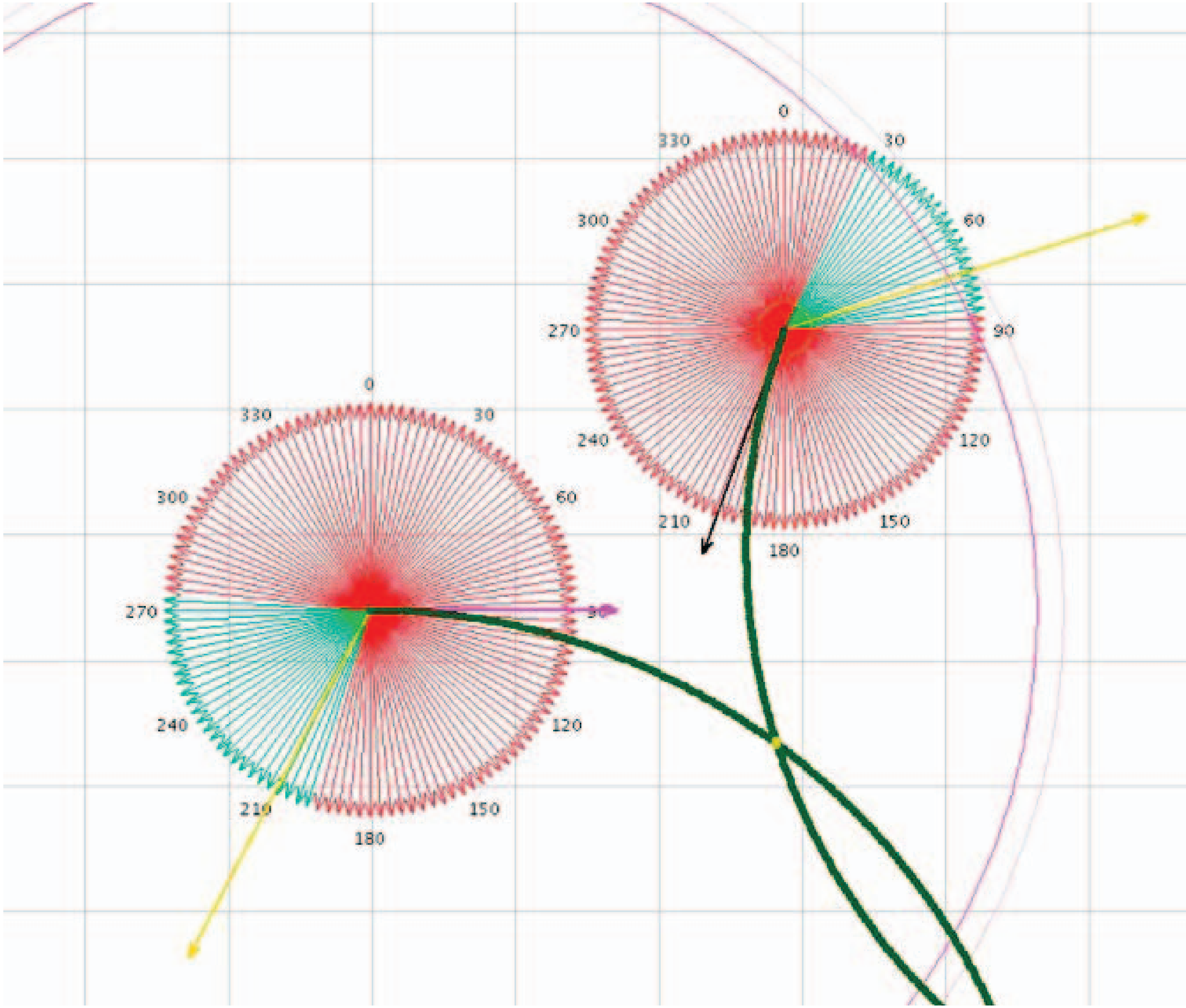


Distance at the closest point of approach:

left turn: 0.44 nm

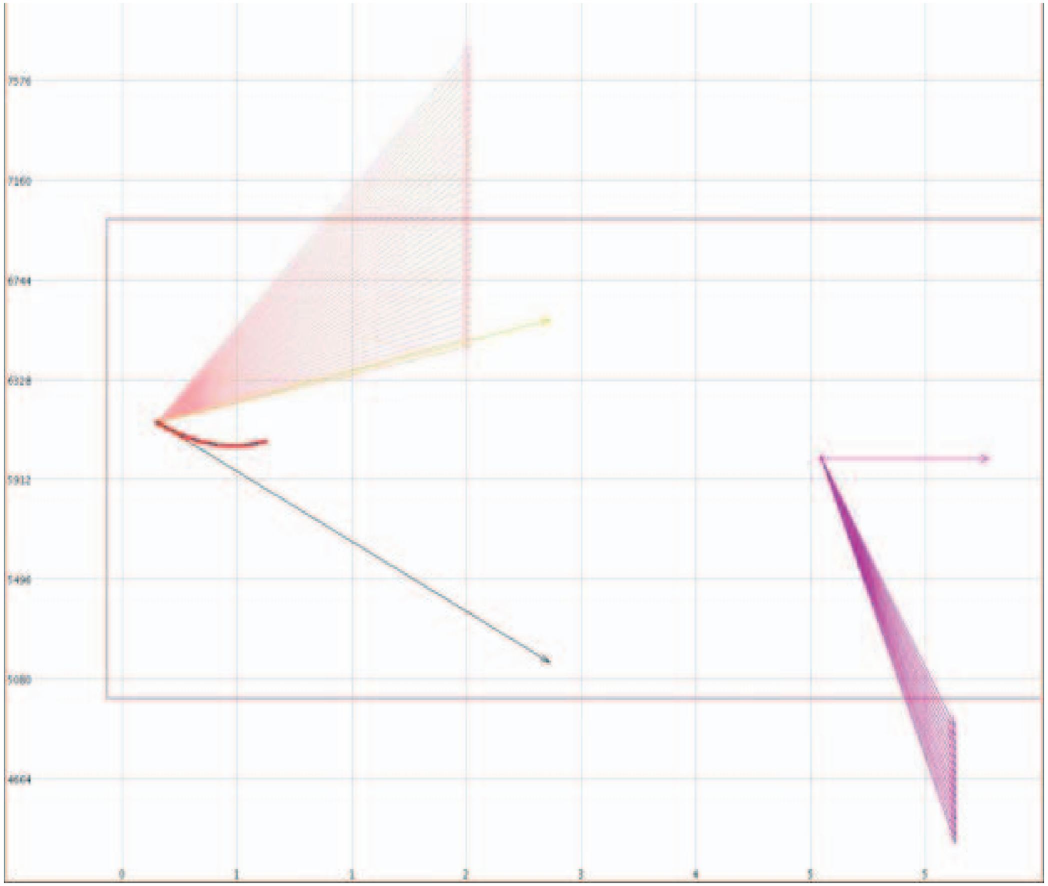
right turn: 0.19 nm

Another Example of Close Encounter

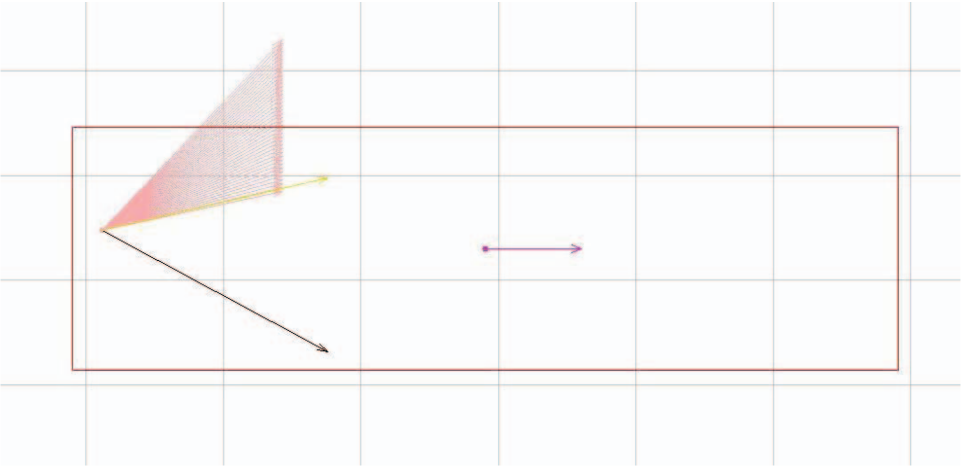


Vertical Path Of The Aircraft Under Constant Acceleration

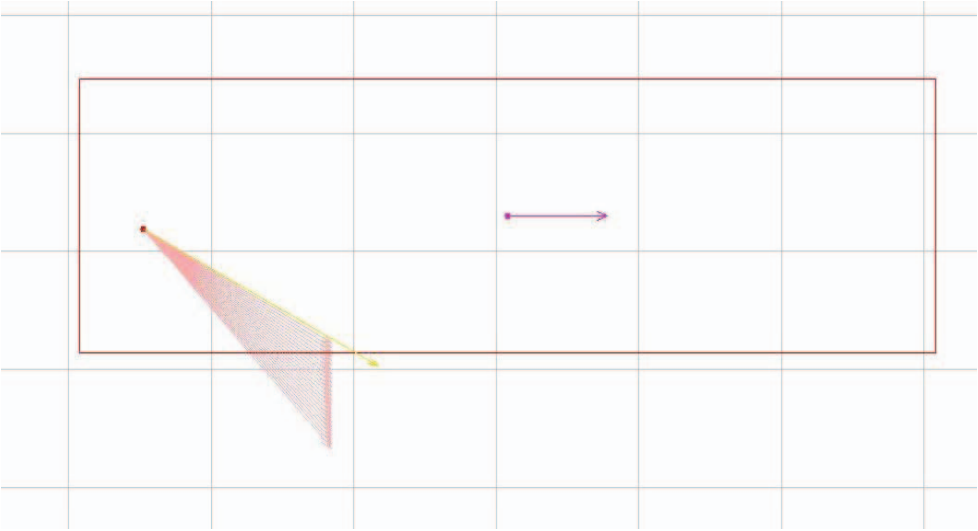
The instantaneous vertical maneuver is to ascend. But long before the target vertical speed is reached, the aircraft will be below the other one and diverging.



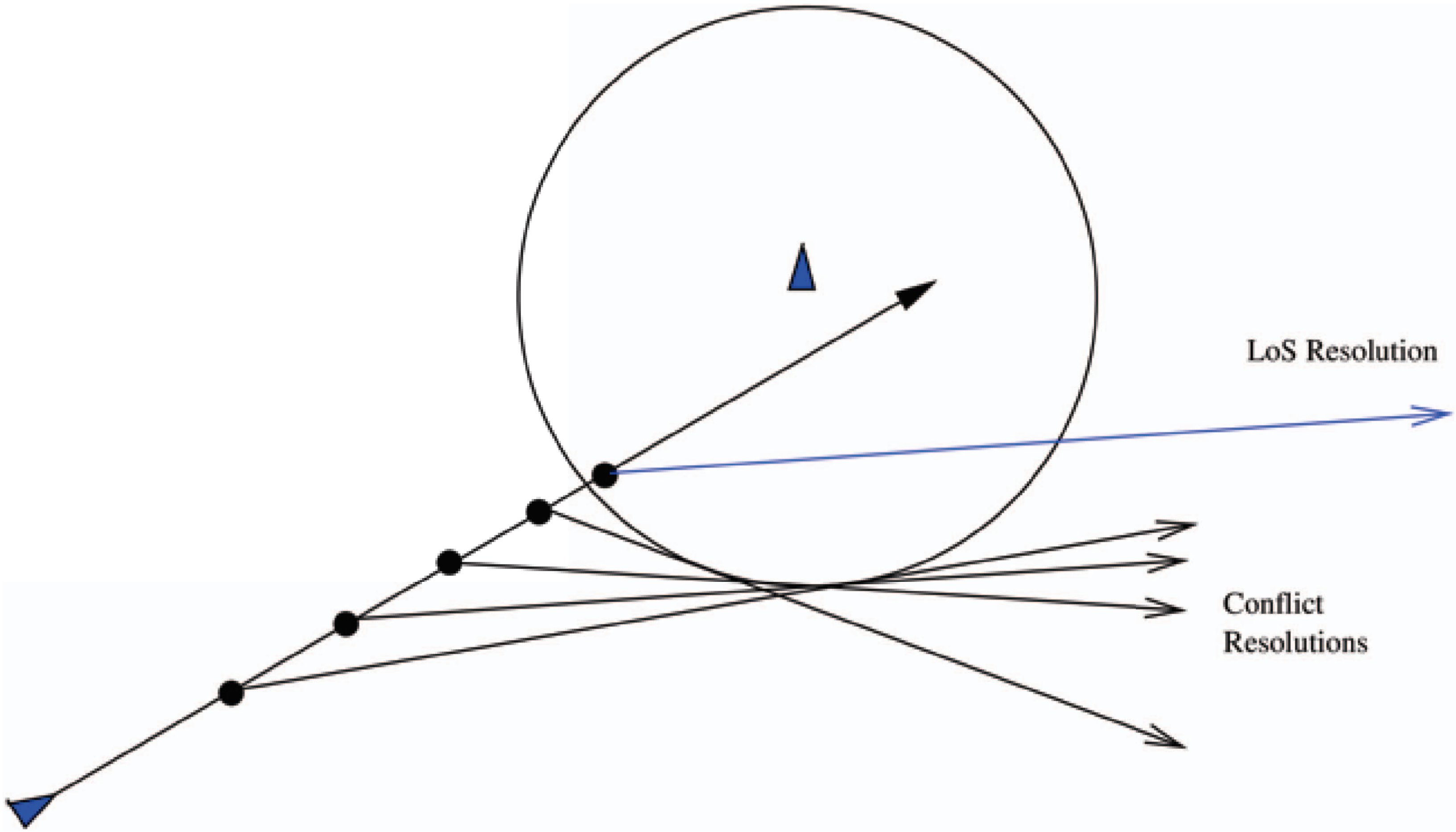
Old Criteria: Vertical (relative frame)



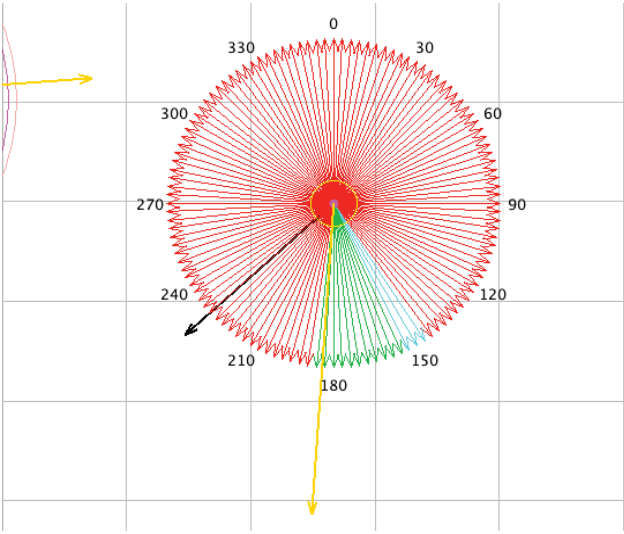
+ 10 sec



Rapid Resolution Change During Transition from Conflict to LoS

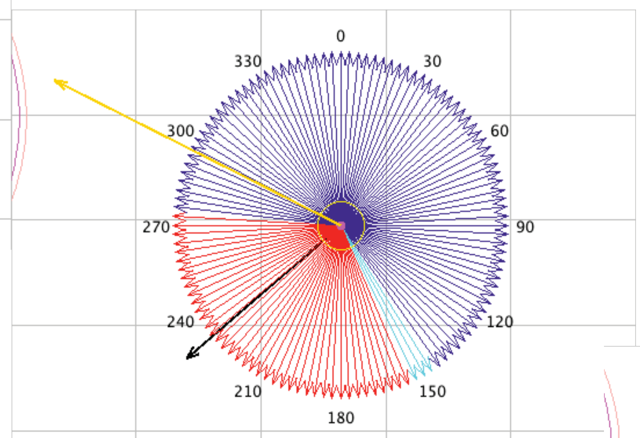


Critical point at 229.5



Turn Right

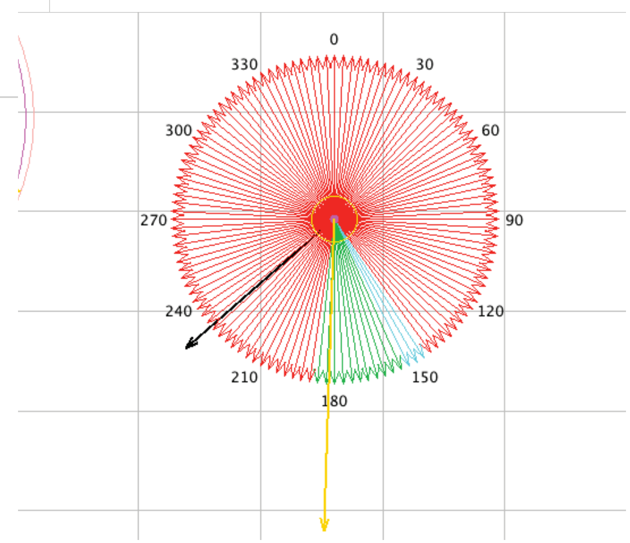
(Track 229)



+1 sec: Turn Left

(Track 230)

(Track 229)



+2 sec: Turn Right

A New Approach: Criteria Based on Repulsion

$$\text{horizontal\_los\_criterion}(\mathbf{s}, \mathbf{v}, \epsilon_h)(\mathbf{v}') : \text{bool} =$$

$$(\epsilon_h \, \mathbf{s} \cdot \mathbf{v}^{\perp} \leq 0 \wedge \mathbf{s} \cdot \mathbf{v} < 0 \wedge$$

$$(\mathbf{s} \cdot \mathbf{v}' \geq 0 \vee \text{horizontal\_entry?}(\mathbf{s}, \mathbf{v}')) \wedge \epsilon_h \mathbf{v}' \cdot \mathbf{v}^{\perp} < 0)$$

$$\vee$$

$$(\mathbf{s} \cdot \mathbf{v} \geq 0 \wedge \mathbf{s} \cdot \mathbf{v}' > \mathbf{s} \cdot \mathbf{v})$$

Repulsion insures that the distance at the time of closest approach for the resolution is greater than the initial state.

A New Approach: Criteria Based on Repulsion

$$
\begin{aligned}
&\texttt{vertical\_los\_criterion?}(\mathbf{s}, \mathbf{v}, \epsilon_v, \texttt{minRelVspd})(\mathbf{v}') = \\
&\quad (|s_z| < H) \;\; \wedge \\
&\quad (\texttt{IF} \;\; \epsilon_v v_z > 0 \;\; \texttt{THEN} \\
&\qquad\qquad \epsilon_v v'_z \geq \epsilon_v v_z \wedge -\epsilon_v v_z(\mathbf{v}' \cdot \mathbf{v}) + \epsilon_v v'_z(\mathbf{v} \cdot \mathbf{v}) \geq 0 \\
&\qquad \texttt{ELSE} \\
&\qquad\qquad \epsilon_v v'_z \geq 0) \;\; \wedge \\
&\quad (\texttt{IF} \;\; \epsilon_v v_z \leq 0 \;\; \texttt{THEN} \\
&\qquad\qquad \epsilon_v v_z \geq \texttt{minRelVspd} \\
&\qquad \texttt{ELSE} \\
&\qquad\qquad \epsilon_v v_z \geq max(\texttt{minRelVspd}, |v_z|))
\end{aligned}
$$

The New Repulsive Algorithms

Our new LoS algorithms are based on two basic ideas

- Search (e.g. varying track) for a solution that is in the repulsive direction
- Stop search when no longer repulsive or the relative separation speed is greater than a specified amount

$$\texttt{right\_turn\_repulsive?}(\mathbf{s}, \mathbf{v_o}, \mathbf{v_i}) =$$
$$\qquad \texttt{repulsive\_along?}(\mathbf{s}, \mathbf{v_o} - \mathbf{v_i})({\mathbf{v_o}}^{\perp}))$$

$$\texttt{left\_turn\_repulsive?}(\mathbf{s}, \mathbf{v_o}, \mathbf{v_i}) =$$
$$\qquad \texttt{repulsive\_along?}(\mathbf{s}, \mathbf{v_o} - \mathbf{v_i})(-{\mathbf{v_o}}^{\perp}))$$

$$\texttt{repulsive\_along?}(\mathbf{s}, \mathbf{v})(\mathbf{v}') =$$
$$\qquad (\mathbf{s} \cdot \mathbf{v} < 0 \wedge -\texttt{sign}(\mathbf{s} \cdot \mathbf{v}^{\perp})(\mathbf{v}' \cdot \mathbf{v}^{\perp}) < 0)$$
$$\qquad \vee (\mathbf{s} \cdot \mathbf{v} \geq 0 \wedge \mathbf{s} \cdot \mathbf{v}' > 0);$$

The New Repulsive Algorithms

- After a solution is found for the most urgent conflict, the search will continue over the remaining unexplored repulsive vectors until all secondary conflicts are avoided as well.
- If this is not possible then the solution for the most urgent aircraft is returned.
- A flag is set if the kinematic search enters a LoS with another aircraft before it is able to find a resolution for the primary.

Resolution Continuity Over TIme

Under the stress of a near term loss of separation, pilots may become confused if the resolutions change too fast.

Therefore, it is important that CD&R algorithms do not switch from *turn right* to *turn left* (and vice versa) or from go up to go down (or vice versa) solutions too quickly as time progresses.

We call this property *continuity*.

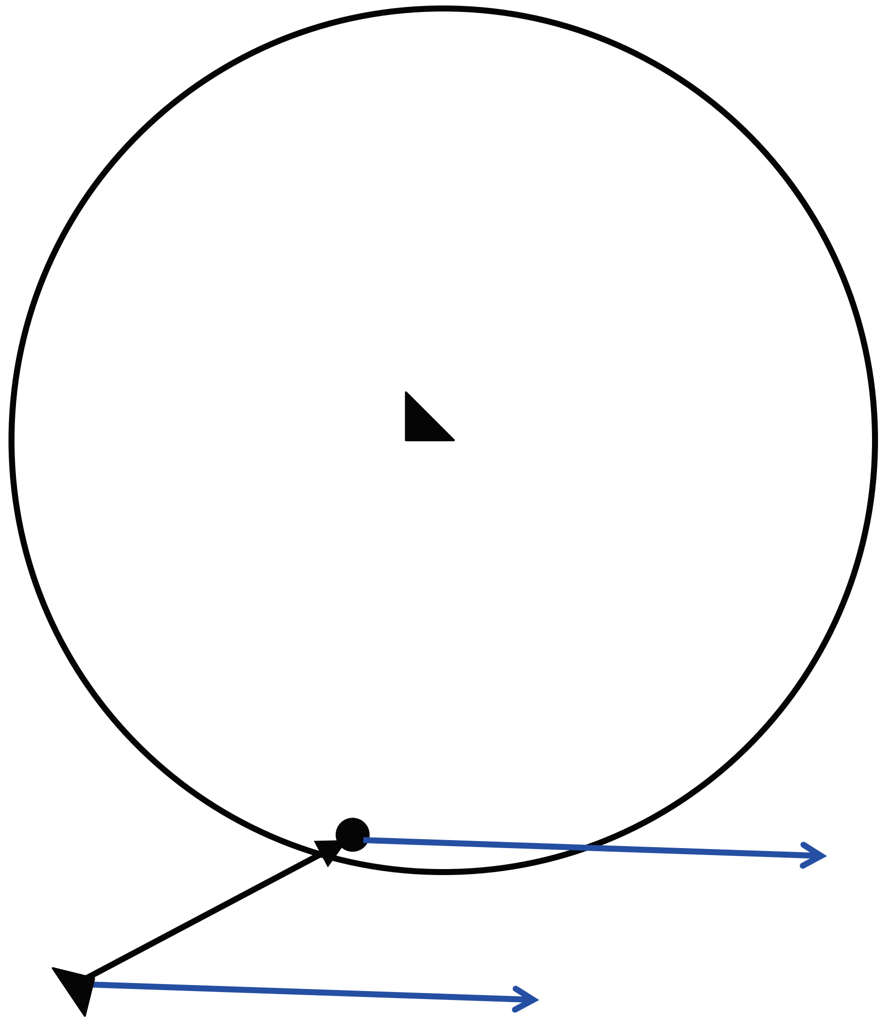
(1) Continuity Into LoS

(2) Continuity Near A Critical Point

Achieving *Continuity Into LoS*

- At some point during the entry into LoS, the track maneuver becomes infeasible, i.e. the aircraft is not able to complete the proposed maneuver without entering LoS.
- At this point it is probably more prudent to produce a resolution that is consistent with the resolution that will be issued once LoS has occurred.
- The goal is no longer to avoid loss of separation, but rather to minimize the depth of the entry.
- In these cases, we project the state vectors linearly to a time exactly 1/2 second after entry into LoS.
- We then use the LoS resolution algorithms on this time-projected data.

Achieving Continuity into LoS



Project aircraft positions to ½ second after LoS

Compute resolution for this projected position

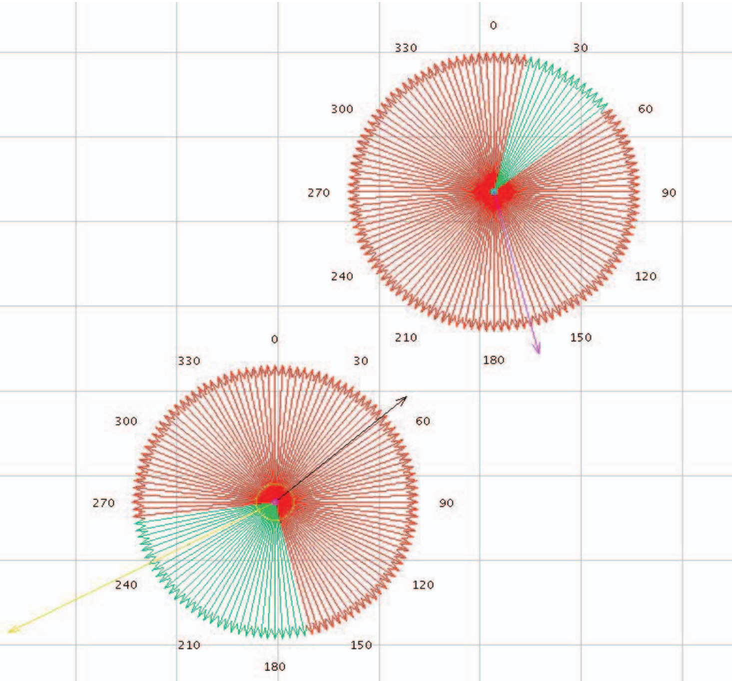
Three Possible Approaches to critical point problem:

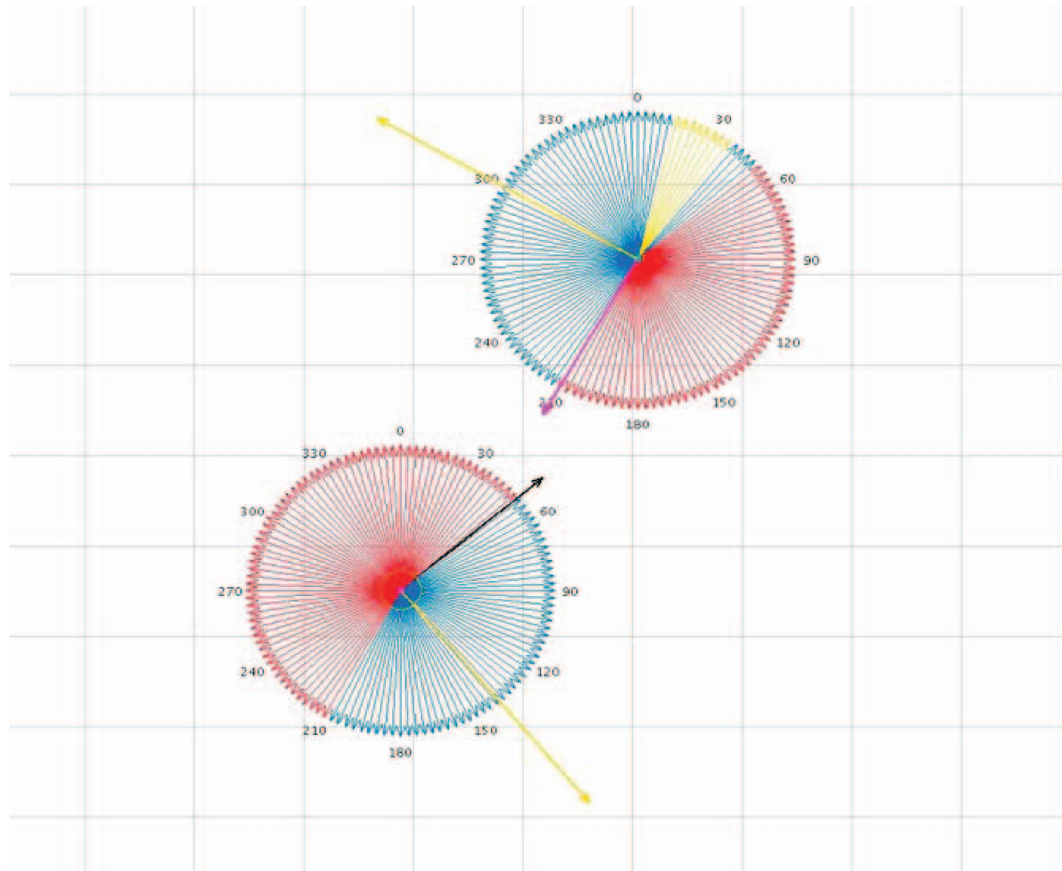
1. Use dead bands. In this case, the algorithm freezes the resolutions while the aircraft stays on a critical point.
2. Filter the direction. In this case, the algorithm freezes the direction of the resolutions.
3. Use future resolutions. In this case, the algorithm computes a resolution in the future and holds it as long as it is still valid.

We are using 2.

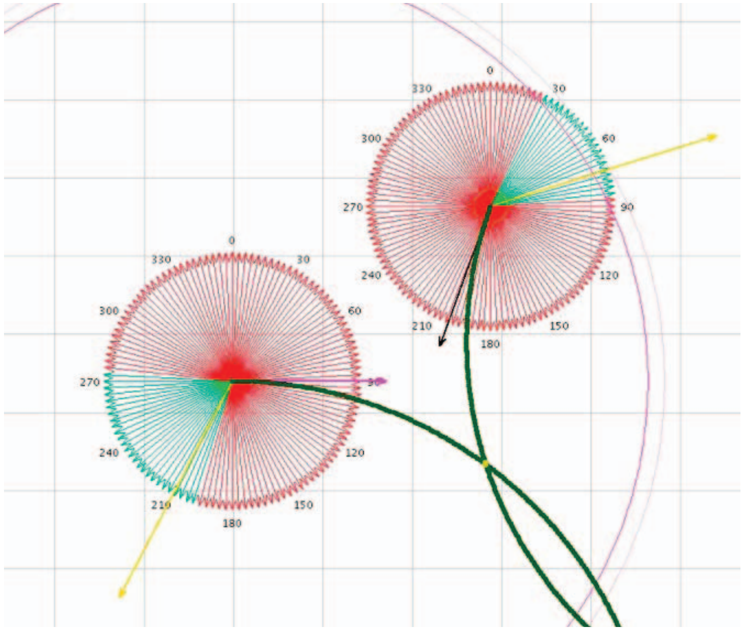
Revisiting Earlier Examples Using the New Algorithms

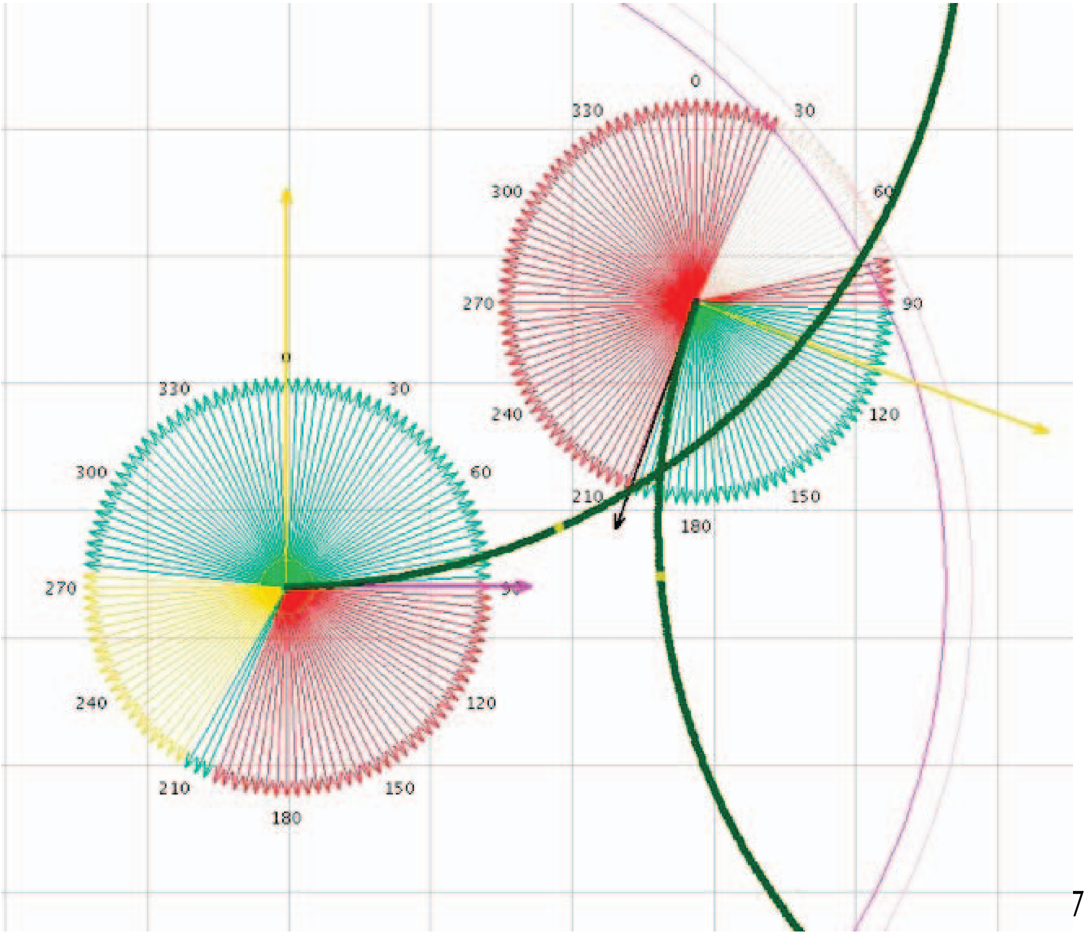
SOLVED: Overly Restricted Maneuvers



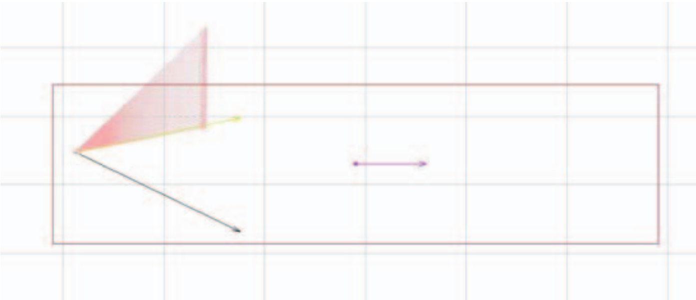


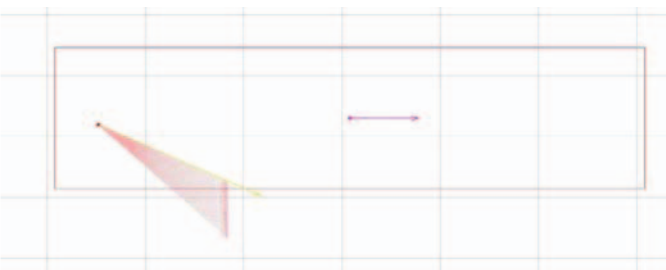
SOLVED: Close Encounter

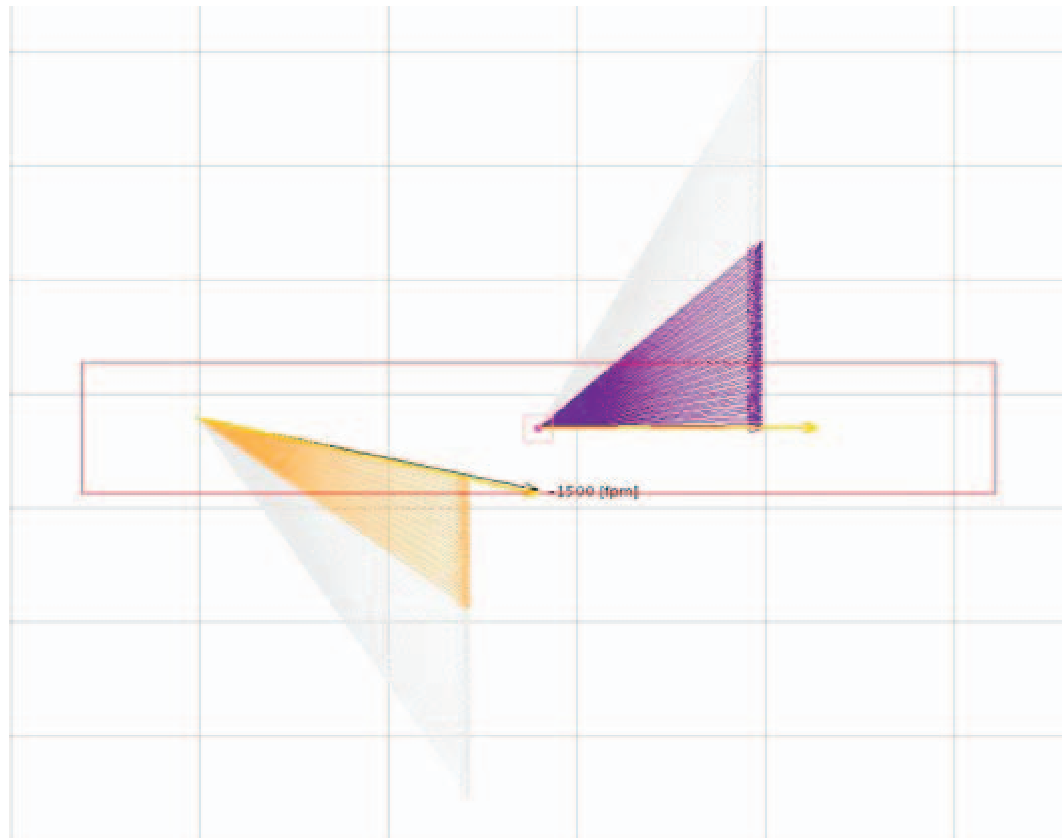




SOLVED: Momentum Switch Problem

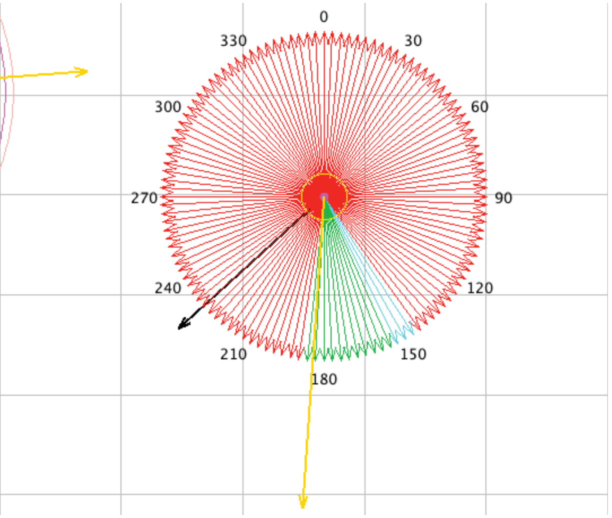






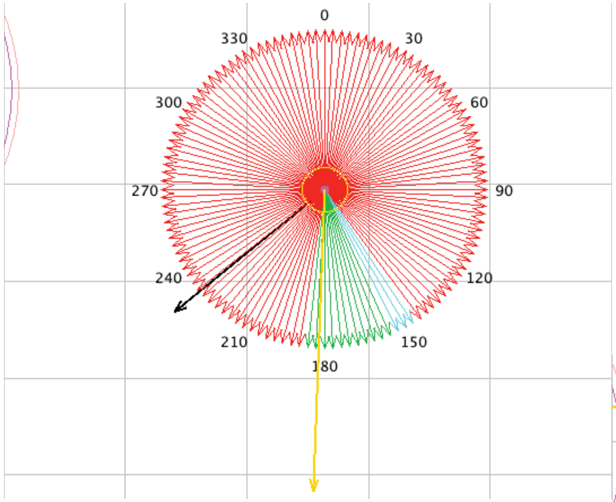
SOLVED: Continuity of Solutions near Critical Points

Critical point at 229.5



Turn Left to 183

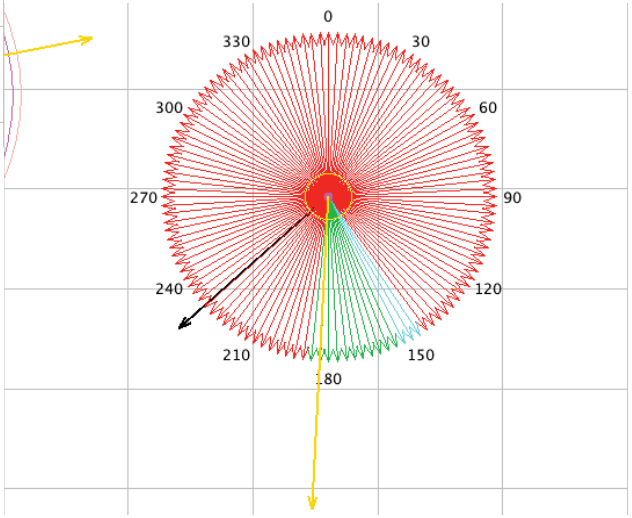
(Track 229)



+1 sec: Turn Left to 183

(Track 230)

(Track 229)



+2 sec: Turn Left to 183

Conclusion

- Our previous LoS criteria previously was based on the concepts of divergence and timeliness of the recovery maneuvers.
- Extensive experimentation has shown that even though that approach was satisfactory in many cases, it relied on overly optimistic assumptions, and thus resulted in poor performance in other cases.
- In this paper, new horizontal and vertical loss of separation recovery criteria are proposed.

  -- The new criteria are based on the concept of repulsiveness.

  -- Experimentation has shown that the new criteria successfully solves most of the problems encountered with the original approach.