## A Fast Leak Locating Method Based on Wavelet Transform<sup>\*</sup>

GE Chuanhu (葛传虎), YANG Hongying (杨红英)<sup>†</sup>, YE Hao (叶 昊)<sup>\*\*</sup>, WANG Guizeng (王桂增)

**†** Department of Automation, Tsinghua University, Beijing 100084, China; Department of Computer Science and Technology, Tsinghua University, Beijing 100084, China

Abstract: The problem of leak location is actually a time delay estimation (TDE) problem. Since most existing TDE methods may encounter the problem of high computational complexity when used for online leak location. This paper presents a fast leak locating method based on wavelet transform (WT). The method first gets a rough estimate of the time delay from the WT coefficients of the pressure signals at the largest scale, then keeps refining the estimate using WT coefficients on smaller and smaller scales. Quantitative analyses and test results based on real data show that the method reduces the computational complexity while maintaining the time delay estimation accuracy.

Key words: leak location; time delay estimation; wavelet transform

## Introduction

Pipelines are of great importance in the chemical and petrochemical industries. In recent years, many kinds of leak detection and location systems have been developed and used in real applications. Among the various leak detection and location methods, the so-called negative pressure wave (NPW)-based method has become the most prevalent method due to its relatively low cost, easy implementation, and detection sensitivity<sup>[1]</sup>. In the NPW-based method, searching online the difference between the time of NPW's arrival at the upstream and downstream ends is the key to accurately locate the leak. Thus, this is actually a time delay estimation (TDE) problem.

There are many TDE algorithms<sup>[2]</sup>, such as correlation methods, adaptive filtering methods<sup>[3,4]</sup>, and high order statistical methods<sup>[5]</sup>, whose basic idea is to view one signal as a shifted version of the other to estimate the time delay by optimizing a cost function with the time delay as the independent variable. However, most of the existing TDE methods may encounter the problem of high computational complexity<sup>[2,6,7]</sup>, and so they cannot be used online for locating leaks, as explained in Section 2.

This paper presents a fast leak locating algorithm based on multi-scale wavelet transform (WT). The method uses the multi-resolution characteristics of the WT to effectively reduce the computational effort while maintaining the time delay estimation accuracy.

It is worth to notice that WT has been widely applied in signal analyses including TDE. But different from the method proposed in this paper, most WT methods for TDE are usually used as a tool for noise suppression<sup>[8,9]</sup>, which makes the TDE algorithms more computationally exhausting.

## 1 Problem Formulation and Preliminaries

#### 1.1 Problem formulation

When a leak occurs in a pipeline, a NPW will travel from the leak point towards the two ends of the

Received: 2008-06-18; revised: 2009-04-27

<sup>\*</sup> Supported by the National Natural Science Foundation of China (No. 60574085), the National High-Tech Research and Development (863) Program of China (No. 2006AA04Z428), and the China Postdoctoral Science Foundation (No. 20090450402)

<sup>\*\*</sup> To whom correspondence should be addressed. E-mail: haoye@tsinghua.edu.cn; Tel: 86-10-62790497

pipeline, and the location of the leak point can be determined by

$$d = \frac{L + \Delta t' \times v}{2} \tag{1}$$

where *d* denotes the distance from the leak point to the upstream end, *L* denotes the pipeline length, *v* denotes the speed of the NPW, and  $\Delta t'$  denotes the difference between the time of the NPW's arrival at the upstream and downstream ends.

Taking the sampling rate,  $F_s$ , into account, Eq. (1) becomes

$$d = \frac{L + (\Delta t / F_{\rm s}) \times v}{2} \tag{2}$$

where  $\Delta t$  is the time difference between sampling points corresponding to  $\Delta t'$ , which will be called the time delay here. In Eq. (2), *L* and *v* are constants which can be easily determined based on prior knowledge, so the leak location problem is actually converted to that of determining the time delay,  $\Delta t$ , i.e., the TDE problem.

Various TDE approaches can be used to estimate  $\Delta t$  by optimizing a cost function with  $\Delta t$  as the independent variable,

$$\min_{\Delta t \in H} J(\Delta t) \tag{3}$$

where *H* denotes the searching interval of  $\Delta t$  and  $J(\Delta t)$  denotes the cost function, with the time delay,  $\Delta t$ , estimated by trying every possible value in  $H^{[10]}$ .

Since a leak may occur at any point in a pipeline,  $\Delta t$  may take any value within  $[-LF_s/v, LF_s/v]$ . Therefore, the searching interval *H* should also be set as  $[-LF_s/v, LF_s/v]$ , which means at least  $2LF_s/v$ searches are needed with existing optimization-based TDE methods. However, with a long pipe, high NPW speeds and high sampling rates, the number of searching operations, i.e.,  $2LF_s / v$ , can be very large, which will lead to very high computational complexity when solving Eq. (3); therefore, existing TDE approaches are not practical for online leak locating. For example, for a 100-km pipeline with a NPW's speed of 1 km/s and a sampling rate of 100 Hz,  $2LF_s / v$  is equal to  $2 \times 100 \times 100 / 1 = 2 \times 10^4$ . With the TDE based on the cross correlation function method<sup>[2,11]</sup>, the computational complexity will be at least  $O(10^8)^{[6]}$ .

To solve the problem, a fast leak locating method based on WT is proposed in this paper.

# **1.2** Brief introduction of the wavelet transform method

The wavelet transform method is a time-frequency analysis signal method with many desirable properties that is widely used in many fields. The wavelet transform of a signal  $s_0(k)$  can be calculated based on the Mallat algorithm shown in Fig. 1<sup>[12]</sup>, where  $d_j(k)$ and  $s_j(k)$ , j = 1,...,N are called the detail and approximation WT coefficients of  $s_0(k)$  at scale j, N is the number of scales,  $h'_0(k)$  and  $h'_1(k)$  are low-pass and high-pass filters, and (2) denotes the downsampling operation. With the downsampling, the lengths of  $d_j(k)$  and  $s_j(k)$  are half that of  $d_{j-1}(k)$  and  $s_{j-1}(k)$ (j = 1,...,N), consequently, the sampling rate of  $d_j(k)$  and  $s_j(k)$  is twice that of  $d_{j-1}(k)$  and  $s_{j-1}(k)$ (j = 1,...,N).

$$s_{0}(k) \xrightarrow{\qquad h_{1}'(k) \qquad (2) \qquad d_{1}(k) \qquad (k) \qquad (k)$$

Fig. 1 Mallat algorithm for the wavelet transform method

## 2 Leak Location Based on the Wavelet Transform Method

#### 2.1 Basic idea

As mentioned in Section 1.1, the time delay  $\Delta t$  may take any value within  $[-LF_s / v, LF_s / v]$ , whose length is  $2LF_s / v$ . Since most existing TDE methods search

 $\Delta t$  by trying every possible value, the search must be executed  $2LF_s/v$  times, which makes existing TDE methods computationally complex when used for leak locating for long pipes, high NPW speeds and high sampling rates.

Instead of directly estimating the time delay based on the original pressure data, the proposed method first gets a rough estimate of the time delay based on the WT approximation coefficients for the original pressure data at the largest scale, i.e., N. Since the sampling rate at scale N is reduced to  $F_s/2^N$  due to the downsampling for N times, the searching interval of the time delay becomes  $[-LF_s/(v2^N), LF_s/(v2^N)]$  at scale N. Therefore, the number of searching operations at scale N,  $2LF_s/(v2^N)$ , is much less than for the original data,  $2LF_s/v$ , which greatly reduces the computational effort at scale N. For convenience, let  $\Delta t_N^*$  denote the estimation result at scale N.

The low sampling rate at scale N greatly reduces the estimation accuracy of  $\Delta t_N^*$ , therefore, the approximation WT coefficients for the original pressure data at scale N-1 are used to refine the estimate, because the sampling rate at scale N-1,  $F_{e}/2^{N-1}$ , is twice that at scale  $N(F_s/2^N)$ . However, since a rough estimate of the time delay  $(\Delta t_N^*)$  has already been found, it is not necessary to still use  $[-LF_s/(v2^{N-1}), LF_s/(v2^{N-1})]$  as the searching interval now, which will lead to a higher computational effort compared with that at scale N. The searching interval with  $2\Delta t_N^*$  as the center,  $[2\Delta t_N^* - R, 2\Delta t_N^* + R]$ , can be adopted, which can still ensure a low computation amount when the radius R is properly selected. The reason why  $2\Delta t_N^*$  is used as the center of the searching interval at scale N-1 lies in that when taking the sampling rate of scale  $N(F_s/2^N)$ and scale  $(N-1)(F_s/2^{N-1})$  into account,  $\Delta t_N^* \times 2^N / F_s$ and  $2\Delta t_{N-1}^* \times 2^{N-1} / F_s$  correspond to the same time difference.

The procedure can be repeated at smaller and smaller scales, in which the estimation accuracy will become higher and higher. When *j*=0, the sampling rate will be the same as for the original data, so the estimated result,  $\Delta t_0^*$ , can be used to calculate the leak location according to Eq. (2).

#### 2.2 Optimization at each scale *j*

When two signals having the same shape match each other, their average square difference function (ASDF) will be minimized<sup>[13]</sup>. Therefore, the optimization objective at scale j, j = N,...,0 is defined as

$$\min_{\Delta t_j \in H_j} \sum_k [s_j^{\mathsf{u}}(k) - s_j^{\mathsf{d}}(k - \Delta t_j)]^2 \tag{4}$$

where  $H_i$  denotes the searching interval at scale j,

 $s_j^{u}(k)$  and  $s_j^{d}(k)$  denote the normalized approximation WT coefficients for the upstream and downstream pressure signals at scale *j*, and  $s_j^{d}(k - \Delta t_j)$  denotes the shifted version of  $s_j^{d}(k)$ . When *j*=0,  $s_0^{u}(k)$  and  $s_0^{d}(k)$  become the original normalized upstream and downstream pressure signals.

According to the idea in Section 2.1, the searching interval for each scale is then set as

$$\begin{cases} H_{N} = [-LF_{s} / (v2^{N}), LF_{s} / (v2^{N})], \\ H_{j} = [2\Delta t_{s,j+1}^{*} - R, 2\Delta t_{s,j+1}^{*} + R], \ j = N - 1, N - 2, \cdots, 0 \end{cases}$$
(5)

where  $\Delta t_{j+1}^*$  is the optimal solution of Eq. (4) at scale j+1 and the radius of the searching interval, *R*, is

$$R = \frac{LF_{\rm s}}{2^N v} \tag{6}$$

Therefore, the searching interval length at each scale is the same as for scale N.

In most existing TDE approaches,  $\Delta t_j^*$  at each scale *j* is determined by solving Eq. (4) for every possible value of  $\Delta t_j$  in  $H_j$  and choosing the one corresponding to the minimum cost function as  $\Delta t_j^*$ <sup>[2,10]</sup>.

#### 2.3 Estimation procedure

The complete time delay estimation procedure can be summarized as follows.

(1) Calculate the approximation WT coefficients for the upstream and downstream pressure series at scales j=1, ..., N.

(2) Normalize the original pressure signals and their approximation WT coefficients.  $s_0^u(k)$  and  $s_0^d(k)$  are used to denote the normalized results for the original pressure signals, while  $s_j^u(k)$  and  $s_j^d(k)$  are used to denote the normalized approximation WT coefficients of the original pressure signals at scale *j*, *j*=1, ..., *N*.

(3) Let j=N and set the searching interval,  $H_N$ , according to the first equation in Eq. (5), then solve the optimization problem in Eq. (4) with the existing ASDF-based TDE method<sup>[2,13]</sup>.

(4) Let j = j - 1 and set the searching interval  $H_j$  according to the second equation in Eq. (5), where  $\Delta t_{j+1}^*$  is the optimal solution in Eq. (4) at scale j+1. The radius, *R*, can be set according to Eq. (6), then

solve the optimization problem in Eq. (4) with existing ASDF-based TDE method.

(5) Repeat step (4) until  $j \le 0$ .

(6) Let  $\Delta t = \Delta t_0^*$ , determine the leakage location according to Eq. (2).

**Remark 1** As in Section 2.1, since the time delay searching interval is  $[-LF_s/(v2^N), LF_s/(v2^N)]$  at scale *N*, normally, the choice of *N* should make sure that the length of the series  $[-LF_s/(v2^N), LF_s/(v2^N)]$  is long enough for a TDE search.

## **3** Computational Complexity Analysis

Consider the leak location problem discussed in Section 2.1. With existing TDE approaches to estimate  $\Delta t$ , the searching interval length is

$$K = 2LF_{\rm s} / v \tag{7}$$

which will lead to  $K^2$  and  $3K \log_2 K$  times multiplications when the cross correlation function-based TDE and the FFT-based TDE are used to solve Eq. (2) respectively<sup>[2,7]</sup>.

Now analyze the computation efforts of the method proposed in this paper. When ignoring the influence of the wavelet filter length which is much shorter than the data length, the calculation of the approximation WT coefficients according to Mallat algorithm needs

$$\left(1 + \frac{1}{2} + \dots + \frac{1}{2^{N}}\right)K = 2\left(1 - \frac{1}{2^{N}}\right)K$$
(8)

multiplications.

According to Eqs. (4) and (5), at each scale j, the optimization of Eq. (4) will need  $2RK/2^{j}$  multiplications, so the whole procedure in Section 2.3 will need

$$2RK(1+1/2+\dots+1/2^{N}) + 2\left(1-\frac{1}{2^{N}}\right)K = 2\left(1-\frac{1}{2^{N}}\right)(2R+1)K$$
(9)

multiplications.

Therefore, the computational complexity of the current method is O(K), which is better than both the cross correlation-based and FFT-based TDE methods. And its advantage over the cross correlation-based TDE method is more obvious. For example, consider the example in Section 2.1, the current method needs only  $1.5 \times 10^6$  multiplications, while the method based on cross correlation function<sup>[2,11]</sup> needs at least  $10^8$  multiplications<sup>[6]</sup>.

**Remark 2** According to Section 2.3, since the calculating procedure will finally perform the TDE search at scale 0, which will ensure the sampling frequency same as that in normal TDE methods, the estimation accuracy will also be same as that in traditional TDE methods as long as the searching interval is correct.

### 4 Computational Accuracy

Real leakage data of a pipeline was used to evaluate the location accuracy of the current method. The pipeline was 6.4 km long, NPW's speed was 1.026 km/s, and the sampling rate was 20 Hz. A leak occurred at the distance of 2.94 km from the upstream end on 12th Oct., 2004. The time delay,  $\Delta t$ , corresponding to the leak location is -10 in sampling point according to Eq. (2).

The current WT-based TDE method also gave an estimated time delay of -10, which indicates that the method has satisfactory locating accuracy in this example.

## 5 Conclusions

A TDE method based on the wavelet transform was developed to locate leakage in pipelines. Compared with existing approaches, the current method can reduce the computational complexity while maintaining the same estimation accuracy.

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Yale University President Visits Tsinghua University

Yale University President Richard Levin and Vice President Linda Lorimer visited Tsinghua on September 5, 2009. Tsinghua University President Gu Binglin welcomed them and briefed them on the latest developments in education and research at Tsinghua University. They also discussed cooperation on the 10 000 Women Program and Yale's Summer Program.

(From http://news.tsinghua.edu.cn, 2009-09-07)