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Construction Process Control of Large Extra Caissons*

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Abstract: The complexity of geotechnical engineering and variability in construction circumstances of large extra caissons make the problem of maintaining appropriate sink attitude quite difficult, especially in keeping sink uniformity and achieving the expected final sink depth. A new construction control method is presented using H_∞ theory, considering uncertainties in the mechanics model and external noise in the construction site parameters. The design method of an H_∞ controller has consequently been obtained for large extra caissons. Control results using only constructor experiences are compared with simulation results using the H_∞ controller for a practical engineering situation, which indicates that the H_∞ controller is successful in maintaining sink uniformity, avoiding sink as well as in achieving the expected final sink depth.

Key words: large extra caissons; construction process; attitude control; H_∞ control

Introduction

The extra caisson is a structure with an open top and bottom, while the lateral surface is a closed-loop. This structure is applied to foundation engineering of bridges, pump stations, underground plants, underground storages, and foundations of buildings. Since extra caissons are often built in complex soil, in order to keep sink uniformity, to prevent sink sharp, and to achieve the expected final sink depth, the construction control of an extra caisson is very important. Although much experience has been obtained in construction control, there is still a lack of a mature theory and a credible numerical simulation stable for practical application. Abesener[1] adjusts excavation in terms of observed information; this is a rather passive approach without sink attitude analysis from the principle of a sink mechanics model. Hu and Nie[2] did not consider that uncertainties might occur in the process of construction, and consequently one of the extra caissons deflected severely. To solve this kind of control problem involving uncertainty of the model and statistical characters of parameter noise, a suitable control method should be developed to handle all these complications. In this paper, attitude control of a large extra caisson is studied using H_∞ control theory, taking into account uncertainties of the mechanics model and external noise around the construction site.

1 Equation of Sink State

Without external force, that is to say, when there is no excavation in the caisson, the extra caisson is subjected to its dead load, side friction, and reaction on the blade. It also suffers a floating force while undrained. The sink equation under these conditions is[3]

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$$m\ddot{h} = G - \left(U_1 fh - 2.5U_1 f\right) - K_m K_{mp} \bullet \left[\left(cN_c + \frac{1}{2}\gamma b N_\gamma\right)Ub + \gamma N_q Ubh\right] - N_w = -\left(U_1 f + K_{mp}\gamma N_q Ub\right)h + G + 2.5U_1 f - K_m K_{mp}\left(cN_c + \frac{1}{2}\gamma b N_\gamma\right)Ub - N_w \tag{1}$$

where U_1 is the external perimeter of the extra caisson; U_2 the internal perimeter of the caisson; U the mean perimeter of the extra caisson, $U=(U_1+U_2)/2$; f the side friction per unit area; $N_q = \dfrac{(1+\sin\varphi)e^{\pi\tan\varphi}}{1-\sin\varphi\sin(2\eta+\varphi)}$; $N_c = (N_q - 1)\cot\varphi$; $N_\gamma = (N_q - 1)\tan 1.4\varphi$; c cohesion for unit soil; φ the internal friction angle; γ gravity of the soil; b the thickness of the sidewall; G the dead load and additional load helping sinkage of the extra caisson (kN); N_w the weight of outlet water from the sidewall (kN); $N_w = 0$ (when sink drained); m the mass of the extra caisson; K_{mp} a factor influenced by the cross section and blade reaction; K_m a factor influencing reaction of the blade considering disturbance of soil; g gravitational acceleration; and h the sink depth of caisson.

Equation (1) can be simplified as

$$\ddot{h} = a_1 h + a_3 \tag{2}$$

where $a_1 = -\dfrac{K_m K_{mp}}{m}\gamma N_q Ub - \dfrac{1}{m}U_1 f$,

$$a_3 = g + \frac{2.5}{m}U_1 f - \frac{K_m K_{mp}}{m}\left(cN_c + \frac{1}{2}\gamma b N_\gamma\right)Ub - \frac{N_w}{m}.$$

We introduce a control force $\boldsymbol{u}$. The equation of sink state is then given by

$$\begin{cases}\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_2\boldsymbol{u},\\ \boldsymbol{y} = \boldsymbol{C}_2\boldsymbol{x}\end{cases} \tag{3}$$

where $\boldsymbol{A} = \begin{bmatrix}0 & 1\\ a_1 & 0\end{bmatrix}$, $\boldsymbol{B}_2 = \begin{bmatrix}0\\ 1\end{bmatrix}$, $\boldsymbol{C}_2 = \begin{bmatrix}1 & 0\end{bmatrix}$,

$$\boldsymbol{x} = \begin{bmatrix}x_1\\ x_2\end{bmatrix},\ x_1 = h + \frac{a_3}{a_1},\ x_2 = \dot{x}_1.$$

Because there are many uncertainties in construction, the influence of model uncertainties (here indicated as parameter uncertainties) and external noise must be considered when examining control methods for Eq. (3), i.e., a suitable controller based on parameter uncertainties and external noise must be designed for large extra caissons.

2 Design Problem for H_∞ Controller

The most important thing for sink attitude control of an extra caisson is to let the extra caisson reach the designed elevation in the designated construction period t_f, that is, $\boldsymbol{x}(t_f) = \begin{bmatrix}h_{design} + a_3/a_1\\ 0\end{bmatrix}$. Furthermore, the controller must maintain uniform sinkage, i.e., make $y_r(t) = \dfrac{h_{design}}{t_f}t$. The control objective is, therefore, to make output near $y_r(t)$ during the construction period, but without using too much control energy.

Suppose the error vector is

$$\boldsymbol{e}(t) = \boldsymbol{y}_r(t) - \boldsymbol{y}(t) \tag{4}$$

and the performance index is given by

$$\boldsymbol{J}_{min} = \int_{t_0}^{t_f}[\boldsymbol{e}^T(t)\boldsymbol{Q}\boldsymbol{e}(t) + \boldsymbol{u}^T(t)R\boldsymbol{u}(t)]dt \tag{5}$$

where $\boldsymbol{Q}$ (positive definite) is the weighting matrix of the error vector and $R > 0$ is a weighting factor of the control vector.

For large extra caisson, the state equation is

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_2\boldsymbol{u},\quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{6}$$

To minimize Eq. (5), and to ensure a closed-loop system that is continuously stable, a state feedback controller is designed

$$\boldsymbol{u} = \boldsymbol{K}\boldsymbol{x} \tag{7}$$

The controller which makes $\boldsymbol{J}_{min}$ can be obtained through solving the relevant Riccati equations. So far the design problem does not consider the influence of external noise. That is to say, optimality of performance index can be achieved only when the controlled object is described accurately by Eq. (6). The optimality cannot be realized due to noise in real systems. To include this effect, we introduce noise $\boldsymbol{w}$. The controlled model is then given by

$$\dot{\boldsymbol{x}} = \left(\boldsymbol{A} + \Delta\boldsymbol{A}\right)\boldsymbol{x} + \boldsymbol{B}_1\boldsymbol{w} + \boldsymbol{B}_2\boldsymbol{u} \tag{8}$$

where $\boldsymbol{w}$ is unit pure noise signal.

We reduce the matrix using the singularity value analysis theorem,

$$\Delta\boldsymbol{A} = \boldsymbol{E}\boldsymbol{\Sigma}\boldsymbol{F} \tag{9}$$

where $\boldsymbol{\Sigma} \in \boldsymbol{\Omega}$ is arbitrary unknown matrix, $\Omega = \{\boldsymbol{\Sigma} \mid \boldsymbol{\Sigma}^{\mathrm{T}} \boldsymbol{\Sigma} \le \boldsymbol{I}\}$.

Considering a state feedback controlled system that makes the performance given by Eq. (5) a minimum, we defined an auxiliary signal,

$$z = \begin{bmatrix} \boldsymbol{Q}^{1/2} \\ 0 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ \sqrt{R} \end{bmatrix} \boldsymbol{u} \tag{10}$$

where $\boldsymbol{Q}^{1/2}$ is the square root of the matrix, and $\boldsymbol{Q} = \boldsymbol{Q}^{1/2}\boldsymbol{Q}^{1/2}$.

Then $\boldsymbol{J}$ in Eq. (5) is given by

$$\boldsymbol{J} = \int_0^{\infty} z^{\mathrm{T}}(t) z(t) \mathrm{d}t = \int_0^{\infty} \boldsymbol{h}^{\mathrm{T}}(t) \boldsymbol{h}(t) \mathrm{d}t \tag{11}$$

where $\boldsymbol{h}(t)$ is pulse response of closed-loop system constituted by Eqs. (7), (8), and (10).

In terms of Parseval identity[4], Eq. (11) can be given by

$$\boldsymbol{J} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \boldsymbol{T}^{\mathrm{T}}(\mathrm{j}\omega) \boldsymbol{T}(\mathrm{j}\omega) \mathrm{d}\omega = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathrm{tr}\{\boldsymbol{T}(\mathrm{j}\omega) \boldsymbol{T}^{\mathrm{T}}(\mathrm{j}\omega)\} \mathrm{d}\omega \tag{12}$$

where $\boldsymbol{T}(\bullet)$ is a closed-loop transfer function from w to z.

Equation (12) is the definition of the $\boldsymbol{H}_2$ norm of rational number $\boldsymbol{T}(s)$,

$$\|\boldsymbol{T}(s)\|_2 = \left\{ \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathrm{tr}\left[\boldsymbol{T}(\mathrm{j}\omega) \boldsymbol{T}^{\mathrm{T}}(\mathrm{j}\omega)\right] \mathrm{d}\omega \right\}^{1/2} \tag{13}$$

Therefore, the design mentioned above is able to solve the feedback controller K so as to make the closed-loop system stable and $\|\boldsymbol{T}(s)\|_2$ a minimum.

Interference suppression of the sink system is discussed next. Noise sets of a large extra caisson are defined[5] as

$$L_2 = \left\{ w(t) \middle| \int_0^{\infty} w^2(t) \mathrm{d}t < \infty \right\} \tag{14}$$

Consider the influence of noise $w \in L_2$ for controlled plant Eq. (8), and introduce the scalar parameter $\varepsilon > 0$ to represent the interference suppression level. For a dynamic feedback controller[6],

$$\boldsymbol{u} = \boldsymbol{K}\boldsymbol{x} \tag{15}$$

consider the design index as follows:

1) The closed-loop system is quadratic stable when $\omega = 0$.

2) The closed-loop system satisfies the interference suppression performance index when $\Delta \boldsymbol{A} = \boldsymbol{0}$ as initial state $\boldsymbol{x}(0) = \boldsymbol{0}$, to arbitrary $t_{\mathrm{f}} > 0$, there is

$$\int_0^{t_{\mathrm{f}}} \left[(y_{\mathrm{r}} - \boldsymbol{C}_2\boldsymbol{x})^{\mathrm{T}} \boldsymbol{Q} (y_{\mathrm{r}} - \boldsymbol{C}_2\boldsymbol{x}) + R\boldsymbol{u}^2(t) \right] \mathrm{d}t < \varepsilon \int_0^{t_{\mathrm{f}}} w^2(t) \mathrm{d}t, \quad \forall w \in L_2 \tag{16}$$

tenable to all noises w which satisfy Eq. (14), where $\varepsilon > 0$ is given constant.

Now discuss controller design problems that satisfy robust stable performance and interference suppression when uncertainties exist in the system.

While $\Delta \boldsymbol{A} = \boldsymbol{0}$, if $\boldsymbol{A} + \boldsymbol{B}_2\boldsymbol{K}$ is a stable matrix and satisfies closed-loop system.

$$\left\| \frac{1}{\sqrt{\varepsilon}} (\boldsymbol{C}_1 + \boldsymbol{D}_{12}\boldsymbol{K}) (s\boldsymbol{I} - \boldsymbol{A} - \boldsymbol{B}_2\boldsymbol{K})^{-1} \boldsymbol{B}_1 \right\|_{\infty} < 1.$$

To solve the state feedback controller satisfying Eq. (1) and Eq. (2), suppose

$$\hat{\boldsymbol{w}} = \begin{bmatrix} w \\ v_1 \end{bmatrix}, \quad \hat{\boldsymbol{z}} = \begin{bmatrix} z \\ v_2 \end{bmatrix} \tag{17}$$

The augmented controlled plant is then[7]

$$\begin{bmatrix} \hat{\boldsymbol{z}} \\ \boldsymbol{y} \end{bmatrix} = \boldsymbol{G}(s) \begin{bmatrix} \hat{\boldsymbol{w}} \\ \boldsymbol{u} \end{bmatrix} \tag{18}$$

$$\boldsymbol{G}(s) = \begin{bmatrix} \boldsymbol{A} & \begin{bmatrix} \frac{1}{\sqrt{\varepsilon}}\boldsymbol{B}_1 & \boldsymbol{E} \end{bmatrix} & \boldsymbol{B}_2 \\ \begin{bmatrix} \boldsymbol{C}_1 \\ \boldsymbol{F} \end{bmatrix} & \boldsymbol{0} & \begin{bmatrix} \boldsymbol{D}_{12} \\ \boldsymbol{0} \end{bmatrix} \\ \boldsymbol{C}_2 & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \tag{19}$$

Controllers that satisfy design indexes (1) and (2) can be used to obtain $\boldsymbol{K}(s)$ by solving H_{∞}, the standard design problem of augmented controlled plant $\boldsymbol{G}(s)$. The controller is then given by $\boldsymbol{u} = \boldsymbol{K}(s)\boldsymbol{x}$.

3 Practical Engineering Example

Consider as an example a large extra caisson of a certain pump station. The caisson is round with the external diameter of 52.4 m, the internal well is a stepped structure, the wall thickness is from 1.0 m to 1.92 m. The designed elevation of the blade is 14.1 m, the start sink elevation of the blade is 1.2 m, and the total height of the extra caisson is 15.9 m. The construction site is located at an ancient channel deposit area, so that the soil layers are distributed unevenly. Soil layers that the extra caisson cross are salty clay, mud salty clay, and mud clay in turn. We design the controller considering interference suppression with the aim of sink uniformity and obtain the final sink depth.

The state equation of the extra caisson is given by

$$\dot{\boldsymbol{x}}=\begin{bmatrix}0 & 1\\ -3.6558+\delta & 0\end{bmatrix}\boldsymbol{x}+\begin{bmatrix}0\\ 0.0001\end{bmatrix}\boldsymbol{w}+\begin{bmatrix}0\\ 1\end{bmatrix}\boldsymbol{u} \qquad (20)$$

where $\boldsymbol{x}=[x_1 \quad x_2]^{\mathrm{T}}$, $x_1=h+a_3/a_1=h-0.9211$, $x_2=\dot{h}$, and h is sink depth. The range of parameter uncertainties is taken as $|\delta|\le 3.6558\times 0.3=1.0967$; this denotes that there is a 30% uncertainty in a_1. The design state feedback controllers that make the closed-loop system must satisfy[8]:

1) For parameter uncertainties δ, the system is quadric stable;

2) For arbitrary given $t_{\mathrm{f}}>0$,

$$\int_0^{t_{\mathrm{f}}}\left[\left(y_r-\boldsymbol{C}_2\boldsymbol{x}\right)^{\mathrm{T}}Q\left(y_r-\boldsymbol{C}_2\boldsymbol{x}\right)+R\boldsymbol{u}^2(t)\right]\mathrm{d}t<\int_0^{t_{\mathrm{f}}}w^2(t)\,\mathrm{d}t \qquad (21)$$

Consider any δ satisfying $|\delta|\le 1.0967$. Suppose that the weighting function is given as:

$$\boldsymbol{Q}=\begin{bmatrix}7^2 & 0\\ 0 & 0\end{bmatrix},\quad R=1,\quad \boldsymbol{E}=\begin{bmatrix}0\\ 1\end{bmatrix},$$

$$\boldsymbol{F}=[1.0967 \quad 0],\quad \boldsymbol{\Sigma}=\frac{\delta}{1.0967}.$$

Define the evaluation signal $\boldsymbol{z}=\boldsymbol{C}_1\boldsymbol{x}+\boldsymbol{D}_{12}\boldsymbol{u}$ such that

$$\boldsymbol{C}_1=\begin{bmatrix}7 & 0\\ 0 & 0\\ 0 & 0\\ 1.0967 & 0\end{bmatrix},\quad \boldsymbol{D}_{12}=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix},\quad \sqrt{\varepsilon}=1.$$

The state feedback controller ($\boldsymbol{u}=\boldsymbol{K}\boldsymbol{x}$) that satisfies the mentioned conditions can be obtained through solving H_∞ as in a standard design problem of $\boldsymbol{G}(s)$ for Eq. (19). By substituting the data mentioned above for Eq. (19), the positive solution satisfying the Riccati equation is given by

$$\boldsymbol{X}=\begin{bmatrix}17.3472 & 3.2070\\ 3.2070 & 2.4346\end{bmatrix}.$$

The expected controller is thus given by

$$\boldsymbol{K}=[-3.2070 \quad -2.4346].$$

Since the start mark of measurement is 343 h, the excavation time is also taken from 343 h so as to allow a comparison in the relationship between excavation time and sink depth using the H_∞ controller and no controller. To maintain sink uniformity and arrive at the final sink depth, the designed excavation is compared with a practical excavation without a controller as shown in Fig. 1. A significant difference is seen between the two cases in Fig. 1.

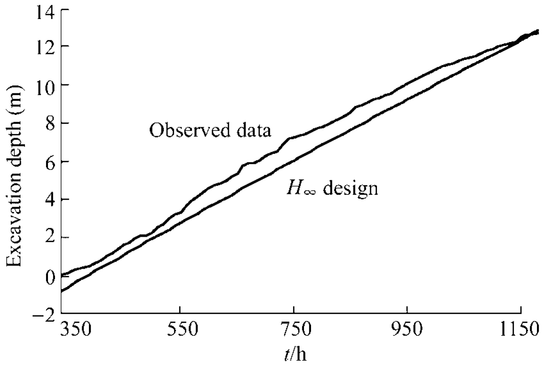

Fig. 1 Time history of excavation depth with H_∞ design and observed data without controller

Figures 2 and 3 give the time history of sink depth under H_∞ controller together with observed results without a controller. It can be seen that the sink curve under H_∞ controller is nearly a straight line, denoting that sink of the extra caisson is uniform, and that the controller achieved the expected results. Figures 2 and 3 also present the H_∞ controller suppression interference affecting sink depth.

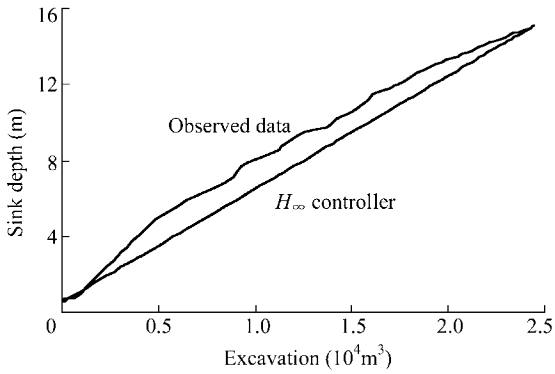

Fig. 2 Sink depth varying with excavation with H_∞ controller and observed data without controller

To find out the effect on the sink depth of parameter uncertainties, we make a perturbation analysis on parameter a_1 (Ref. [9]). Assume the nominal value in the range of 30%, that is, $a_1=-3.0591$, -3.5591, -4.0591, -4.5591 with a nominal value of -3.6558. The time history of sink depth and the relationship between the excavation and sink depth with the H_∞ feedback controller for the different a_1 are shown in Figs. 4 and 5.

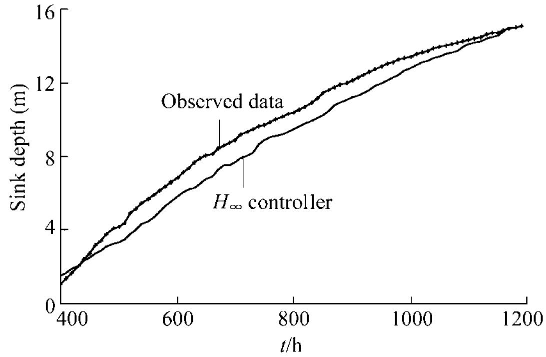

Fig. 3 Time history of sink depth with H_∞ controller and observed data without controller

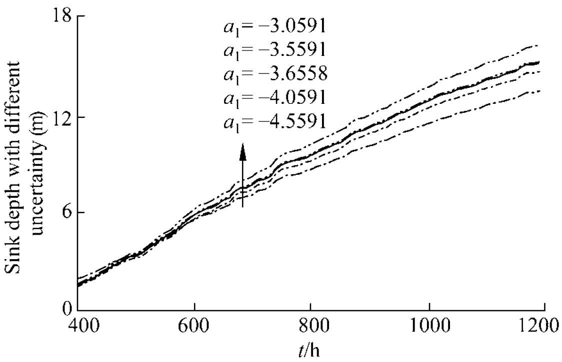

Fig. 4 Time history of sink depth with perturbation of parameter a_1 in system matrix

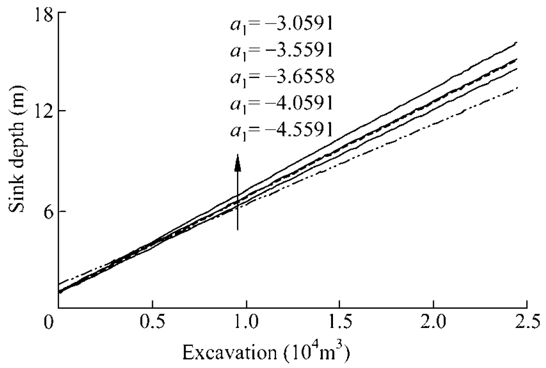

Fig. 5 Sink depth varying with excavation with perturbation of parameter a_1 in system matrix

It can be seen from Figs. 4 and 5 that although the interference explicitly affects the model of the extra caisson, the output is nevertheless quite similar for a certain input under the H_∞ controller. That is to say, the robustness of the controller to parameter uncertainties is good; and the influence of model uncertainties affecting sink depth is quite small under the H_∞ controller. The designed H_∞ controller therefore had a good application value in construction control of this large extra caisson.

4 Conclusions

To maintain sink uniformity and to obtain the final required sink depth in construction of large extra caissons, construction control must consider two important factors which affect the sink attitude severely. One is the geotechnical uncertainty; the other is interference associated with the construction circumstances. The important contribution of the control method presented in this paper is the ability to consider uncertainties and to suppress interference. The numerical results presented show that the H_∞ controller is successful in attitude control in construction of a large extra caisson, especially in maintaining sink uniformity and in arriving at the final sink depth[10].

References

[1] Abesener Joe. Automatic construction techniques for large extra caisson. *Construction of Civil Engineering*, 1999, **40**(2): 9-15.

[2] Yang Qihai. The method of righting and correcting inclination for large extra caisson of Yuanjiang bridge. *Construction of Bridge*, 2000, (1): 51-53. (in Chinese)

[3] Hu shaowei, Nie Jianguo. Dynamic behaviour of oilfield 'A' form derricks. *Tsinghua Science and Technology*, 2001, **6**(1): 57-62.

[4] Meyerhof G G. Bearing capacity and settlement of pile foundation. *Journal of the Geotechnical Engineering Division, Proc. ASCE*, 1965, **12**(1): 88-96.

[5] Hu Shaowei, Nie Jianguo, Wang Wei. Dynamic assessment of the carrying capacity of oilfield derricks. *Tsinghua Science and Technology*, 2001, **6**(1): 63-66.

[6] Sun Gengsheng, Zheng Datong. Soft Soil Foundations and Substructure Works. Beijing: China Building Press, 1984. (in Chinese)

[7] Lin Guoxiong, Fang Qinhan. Study on design and key technology of Wuhu Yangtse Bridge. *Construction of Bridge*, 1998, (4): 1-8. (in Chinese)

[8] Williams M L. Stress singularities resulting from various boundary conditions in angular corners of plates in extension. *Journal of Applied Mechanics*, 1952, **5**(4): 526-528.

[9] Xu Zilun. Elastic Mechanics. Beijing: Tsinghua University Press, 1994. (in Chinese)

[10] William H, Flannery Brain P. Numerical Recipes — The Art of Scientific Computing. Cambridge, London, New York: Cambridge University Press, 1986.