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Design and Manufacturing of a Composite Lattice Structure Reinforced by Continuous Carbon Fibers

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Abstract: New techniques have been developed to make materials with a periodic three-dimensional lattice structure. The high stiffness per unit weight and multifunction of such lattice structures make them attractive for use in aeronautic and astronautic structures. In this paper, epoxy-soaked continuous carbon fibres were first introduced to make lattice composite structures, which maximize the specific load carrying capacity. A micromechanical analysis of several designs, each corresponding to a different manufacturing route, was carried out, in order to find the optimized lattice structure with maximum specific stiffness. An intertwining method was chosen and developed as the best route to make lattice composite materials reinforced by carbon fibers. A sandwich-weaved sample with a three-dimensional intertwined lattice structure core was found to be best. The manufacturing of such a composite lattice material was outlined. In addition to a high shear strength of the core and the integral manufacturing method, the lattice sandwich structure is expected to possess better mechanical capability.

Key words: composite lattice structure; carbon fibre; micromechanics; weaving

Introduction

Three dimensional lattice materials are made up of interconnected lattices of straight struts. These materials are optimized as light-weight structural materials, and can be synthesized from polymers or light-weight metals[1-3]. As structural materials, three-dimensional lattice systems possess advantages such as low density, high strength, and specific stiffness. Furthermore, three-dimensional (3-D) lattice materials can be designed as multifunctional materials[4]. For example, load supporting three-dimensional metal truss structures might also simultaneously provide mechanical impact absorption, thermal management, noise attenuation, and filtration[5].

Truss structures are conventionally employed in civil and mechanical engineering, with applications in bridges, cranes, and roofs. Recent developments in manufacturing techniques, however, have allowed for the manufacture of lattice materials at lengths ranging from millimetres to tens of centimetres[5]. For example, the injection moulding of polymeric structures and subsequent assembly into complex lattice materials is a cheap way to produce materials where the struts have aspect ratios less than 5[6,7]. Such polymeric materials can then be used as sacrificial patterns for investment casting of metallic lattice materials. Rapid prototyping techniques can be used to fabricate materials with lattice parameters on the order of 0.5 mm[8].

Sandwich beams can be constructed using a tetrahedral core, with the attachment of either solid face-sheets or triangulated face-sheets. The sandwich panels

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are investment cast from polystyrene pre-forms[7]. First, the tetrahedral cores and the triangulated face-sheets are injection-moulded as separate pieces in polystyrene, the cores are moulded with locating pins at the nodes, and the triangulated face-sheets are moulded with mating holes. Second, the polystyrene cores are solvent bonded to either triangulated or solid sheets, and third, an investment casting process is used to produce the sandwich beams from the polystyrene sacrificial patterns.

Strength knockdown due to casting defects has been observed in investment casting. Another manufacturing route utilizes a comparatively simple wrought metal based approach. The truss cores are made by deformation shaping of hexagonal perforated metal sheets. They are then bonded between thin facesheets using a transient liquid phase approach[9]. Lamina of plain square woven nichrome cloth have also been used for the construction of a three-dimensional truss system[10]. The lamina were first submersed into a polymer-based Nicrobraz cement. The solidus and liquidus of this braze alloy are lower than that for the nichrome. The coated laminae were then stacked peak to peak using pins to align all the square openings, and small compressive pressure was applied to the periphery of the lay-up. The stacked assemblies were then heated to melt the braze alloy powders which were drawn by capillary action to points of wire contact.

Another method, suggested by Brittain et al.[5], uses rapid prototyping and microcontact printing μCP in combination with microelectrochemistry to form the initial two-dimensional (2-D) grid patterns that make up the trusses. A 3-D truss can also be assembled from sets of grids, two of which are planar and one nonplanar. The planar sheet can be folded along an indicated axes with a machined brass die to form a nonplanar core.

Other methods such as woven-fabric have been introduced for the manufacture of lattice sandwiches[11-15]. The shape of the core is always made as an X-core. Because the core and the panels are woven as an integrated structure, the mechanical behavior of such structures is improved. However, the compressive strength of the X-core fibres is neglected, and the pressure is borne by foams in the core.

1 Configuration of a Composite Lattice Structure

A sensible criterion for selection of materials to fabricate lattice structures is to make the structure ultralight and strong. The ideal candidate for this goal is a continuous carbon fibre with a resin substrate, with a density of only 1.55 to 1.60 g/cm^3 and a tensile strength far superior than the lightweight metals. Lattice structures made of such continuous carbon fibres will be lighter and stronger than the existing lattice structures made of aluminum alloys and polymers.

Lattice structures made of continuous carbon fibres can be generated by two manufacturing routes. The first route follows the laminae assembly approach of Sypeck et al.[10], originally developed for metallic lattice materials. Rods reinforced by continuous carbon fibres are first bent into a wavy configuration along the rod axis. Two groups of the bent rods are then assembled orthotropically to form a two-dimensional lamina. The folded laminae are then stacked peak to peak using pins to align all square openings to form a three-dimensional lattice structure, as shown in Fig. 1. The connection strength between the laminae is determined by the resin in the pin-joints.

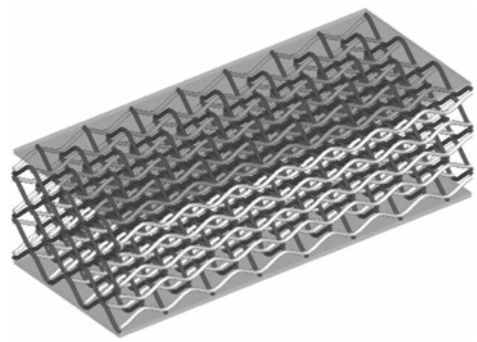

Fig. 1 Stacking-assembled lattice structure

The second manufacturing route adopts a method to generate such lattice materials. Two carbon fibre-reinforced epoxy plates with rows of holes are first designed and made. The core of the sandwich sample is made up of cuboid cells, and the rods of fibre bundles are placed at the diagonals of the cuboid cells. A fibre bundle is fixed on the bottom plate, and then intertwined from the bottom to the top hole. From the hole in the bottom plate and along the diagonal direction of the cuboid cell, a hole can be found on the top plate. Weaved downward from a nearby top hole, the fibre bundle travels along the diagonal direction of the cubic cell to another hole in the bottom plate. In this manner, the fiber is intertwined between the top and the bottom plates. During the process, all of the holes on the upper or bottom plate are aligned on the longitudinal direction of the lattice beam structure. Consequently, an inclined lattice lamina between the top and bottom plate along the longitudinal direction of the lattice beam is generated. The process is then repeated but with the fiber in an opposite inclination. Similar laminae with the same and opposite inclinations are generated by further consecutive steps. When the core is completed, the woven top and bottom plates are filled by layers of carbon fibre reinforced plies resulting in a defect free and smooth appearance. After completion, the lattice sandwich beam is made up of inclined lattice laminae and two plates, as illustrated in Fig. 2.

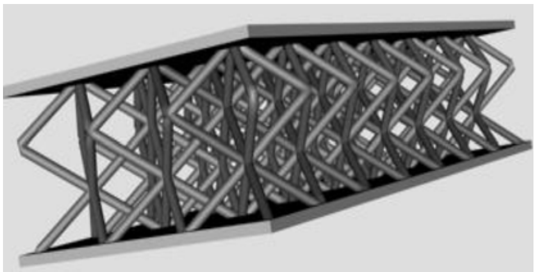

Fig. 2 Intertwined lattice sandwich

In both cases for the sandwich beam, the core is made up of fibres. The directions of the fibres differ, however, in the two manufacturing methods. The fibres for stacking-assembled laminae are placed parallel and perpendicular to the longitudinal direction of the beam. In contrast the fibres in an intertwined sandwich beam, on the other hand, are placed at the digonals of cuboid cells. In the next section, we analyze the mechanical behavior of these core structures using a micromechanics model, in order to select the optimal design of the lattice beams.

2 Optimal Design of the Lattice Beam

2.1 Stacking-assembled lattice structure

The stacking-assembled lattice structure is made up of rods reinforced by carbon fibers and nodes connected by resin. A representative volume element is shown in Fig. 3, with the x-axis aligned with the longitudinal direction of the beam. The solid lines denote the lower laminae while the dashed lines represent the upper laminae. Eight bars that are subjected to the axial tension or compression are joined together at a node, which is a resin adhesion point of the two lamina to resist the shear. The bars are mainly subjected to axial stress. The bending of the bars is neglected except for a buckling analysis under compression[16,17].

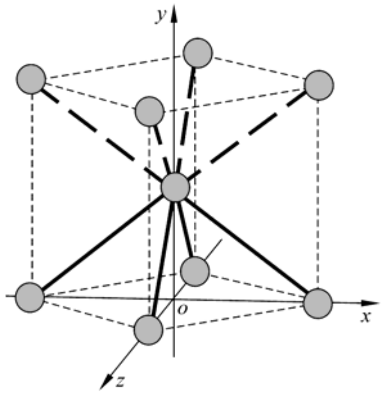

Fig. 3 Structure of the stacking-assembled lattice cell

Three assumptions are made in our micromechanics formulation: (1) the bending moment of each bar is neglected and the nodes are simplified as pin points; (2) the cross section of the beam remains planar after deformation; and (3) only fibres parallel to the longitudinal direction of the beam withstand the bending moment while the transverse fibres do not resist bending.

The basic cell structure is cuboid and the fibre bars are located on the diagonals of the cell. The relative density of the lattice structure, defined as the ratio of total fibre volume in a cell to the cell volume, can be calculated as follows:

$$\bar{\rho} \equiv \frac{\rho}{\rho_{\rm f}} = \frac{\pi}{2\cos^2\theta\sin\theta}\left(\frac{d}{w}\right)^2 \tag{1}$$

where d is the diameter of the fibre bar, w is the length of a bar between two nodes, and θ is the angle between the axial direction of the bar and the longitudinal direction of the beam, as shown in Fig. 4.

According to the assumption of a planar cross section, the distribution of the strain is linear along the height of the beam. Since the beam is made of lots of laminae, and the length w of a bar is small compared to the beam height, one may tentatively assume that the axial stresses transmitted by the bars in a cross-section are linear along the height of the beam. Each bar makes an angle with the longitudinal direction of the beam and only the horizontal components of the stresses balance the bending moment, as shown in Fig. 5.

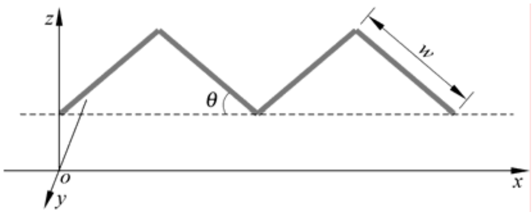

Fig. 4 Geometry of a single lamina in longitudinal direction

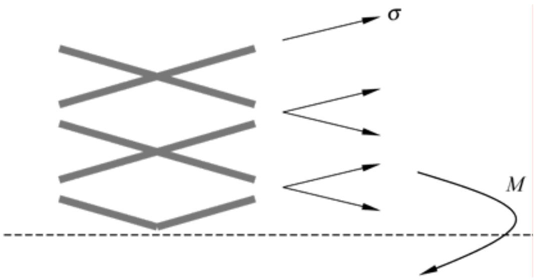

Fig. 5 Free-body diagram of cross-section section

Accordingly, the axial stress σ_i of the bar in the i-th lamina of the beam can be calculated as

$$\sigma_i = \frac{2y_i}{h}\sigma_h \tag{2}$$

where h denotes the height of the beam, y_i the vertical coordinate (measured from the neutral axis) of the i-th lamina, and σ_h the axial stress of the top fibre. The bending moment M supplied by the horizontal components of the stresses across the whole section can be computed through micromechanics as

$$M = \sigma_h \pi d^2 n \cos\theta \frac{h}{4}\sum_{i=1}^{m/2}\left(\frac{2y_i}{h}\right)^2 \tag{3}$$

where m denotes the number of cells across the height direction, and n denotes the number of cells across the width direction.

The effective moment of inertia $J(x)$ of a particular cross-section of the lattice beam can be identified as

$$J(x) = \frac{n}{2}\left(\pi d^2 \cos\theta\right)\sum_{i=1}^{m/2} y_i^2 \tag{4}$$

Averaging over the beam length, one arrives at the following expression for the effective moment of inertia of the beam:

$$J = \frac{m^3 n}{48}\pi d^2 w^2 \sin^2\theta\cos\theta \tag{5}$$

The tensile stress σ transmitted through the bar can be formally written as

$$\sigma = \frac{M}{J}y \tag{6}$$

The micromechanics analysis for the maximum shear stress, τ, transmitted through each node along the neutral axis, is computed based on the scheme depicted in Fig. 6. The beam was cut along two cross-sections and along the middle plane of the beam. The unbalanced tensile stresses across the adjacent cross-sections over the length of two cells are resisted by a shear stress τ at each node. Thus, the shear stress at the node is equal to the variation of the horizontal components of the axial stresses of the bars over the length of two cells integrated over the half-height of the beam:

$$\tau = Q\frac{\pi d^2}{2Js_\tau}\int_0^{w\cos\theta}\cos\theta\sum_{i=k}^{m/2} y_i \mathrm{d}x \tag{7}$$

where Q is the effective shearing force over the cross-section and s_τ is the shearing area of the node, determined by the adhesion area at each node between the upper and lower laminae. We denote the static area-moment $S(x)$ of a particular cross-section of the lattice beam as

$$S(x) = \frac{\pi d^2}{2s_\tau}\int_0^{w\cos\theta}\cos\theta\sum_{i=k}^{m/2} y_i \mathrm{d}x \tag{8}$$

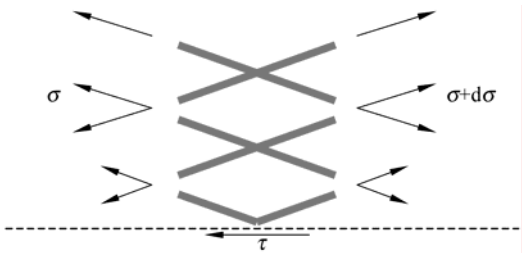

Fig. 6 Balance of shear stress

One may average the above expression over a unit cell in the longitudinal direction to get the averaged static area-moment S as

$$S = \frac{m^2}{16}\frac{\pi d^2}{s_\tau}w^2\sin\theta\cos^2\theta \tag{9}$$

The maximum shear stress of the resin node may be expressed formally as

$$\tau = \frac{QS}{J} \tag{10}$$

Substituting Eqs. (5) and (9) into Eq. (10), the final expression of the maximum shear stress is

$$\tau = \frac{m^2}{16}\frac{Q}{J}\frac{\pi d^2}{s_\tau}w^2\sin\theta\cos^2\theta \tag{11}$$

We conclude this subsection by considering the condition for compressive buckling of the rods. If the aspect ratio of the rod is large, the onset of buckling of the rod constitutes a failure mode. The restriction at either end of the bar is assumed to be a hinged joint. Following Euler's bifurcation analysis, the limiting compressive stress σ_c is

$$\sigma_c = \frac{\pi^2 E_f}{16}\left(\frac{d}{w}\right)^2 \tag{12}$$

where E_f is the axial Young modulus of the rod reinforced by carbon fibres. If the Euler buckling stress σ_c is less than the compressive strength σ_b, of the rod, the rod will fail by the loss of stability; if σ_c is larger than the compressive strength σ_b, the rod will fail by a loss of mechanical strength. The optimal design is to give the two failure modes equal importance. The aspect ratio in this critical case is[10]

$$\left(\frac{w}{d}\right)_c = \frac{\pi}{4}\sqrt{\frac{E_f}{\sigma_b}} \tag{13}$$

An optimal design is to get a core cell with a small density but higher stiffness and strength. The variation of θ, w, and d will influence the density and the stiffness of the material. For maximum stiffness, the required objective function can be found from Eq. (5) as max($\sin^2\theta\cos\theta$), leading to a value for θ of 54.7°. For minimum density, the required objective function can be found from Eq. (1) as max($\sin\theta\cos^2\theta$), leading to a value for θ of 35.3°. A better objective is to maximize the stiffness while minimizing the relative density, in other words, to maximum the specific stiffness $J/\bar{\rho}$. The relevant optimizing formulation for this case is max($\sin^3\theta\cos^3\theta$). Consequently, the optimized value of θ is taken as 45°, where the ratio between the stiffness and the density of the lattice structure takes a maximum value.

2.2 Three-dimensional intertwined lattice structure

As shown in Fig. 7, the cell of the intertwined structure has a cubic lattice structure. Fibres intertwine along the diagonals of the cube. Each fibre strut represented by different lines in Fig. 7 is continuous at the nodes. The averaged moment of inertia can be evaluated as

$$J = \frac{m^3 n\pi d^2 w^2}{144\sqrt{3}} \tag{14}$$

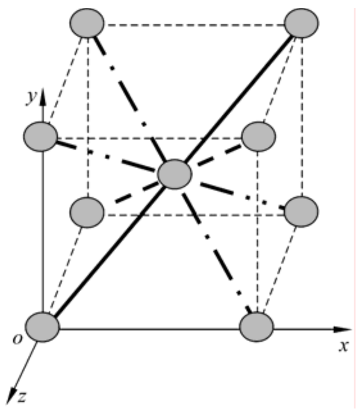

Fig. 7 Structure of intertwined lattice cell

The variation of the horizontal component of the axial force of the rods is balanced by the shear stress at the node as for the previous case of a stacking assembled lattice. In contrast, however, to that analysis, the shear stress in the intertwined structure is carried out by horizontal resistance of the intertwining rods. The horizontal force F_x carried at each contacting connection can be calculated as follows:

$$F_x = \frac{\pi d^2}{\sqrt{3}}\sigma \tag{15}$$

where σ is the axial force of the rod, as calculated from Eqs. (6) and (14). The shear stress can be calculated from the equilibrium analysis of the sectional element.

$$\tau s_\tau = Q\frac{\pi d^2}{2\sqrt{3}J}\int_0^{w/\sqrt{3}}\sum_{i=k}^{m/2} y_i \mathrm{d}x \tag{16}$$

$$\sigma = \frac{Q}{J}\int_0^{w/\sqrt{3}}\sum_{i=k}^{m/2} y_i \mathrm{d}x \tag{17}$$

The effective moment of area can, therefore, be defined as

$$S = \int_0^{w/\sqrt{3}}\sum_{i=k}^{m/2} y_i \mathrm{d}x \tag{18}$$

For a beam with m laminae, the shear stress is a maximum in the middle plane. The effective moment of area is

$$S = \frac{m^2 w^2}{24} \tag{19}$$

so the maximum shear stress is

$$\tau = 6\sqrt{3}\frac{Q}{mn\pi d^2} \tag{20}$$

The ratio of the axial stress caused by tension to that caused by shear is

$$\frac{\sigma_\tau}{\sigma_{h/2}}=\frac{mw}{2\sqrt{3}}\frac{Q}{M} \tag{21}$$

If the tension/compression failure mode occurs at the same time with the shear failure mode the following equation must be satisfied:

$$\frac{\sigma_\tau}{\sigma_{h/2}}=\frac{mw}{4\sqrt{3}}\frac{Q}{M}=1 \tag{22}$$

Under three point bending, the length of the beam is

$$L=\frac{mw}{2\sqrt{3}}=\frac{h}{2} \tag{23}$$

While the height of the structure is twice the span of the structure, it is not a beam. Therefore, the failure mode of the intertwined lattice beam is due either to tensile/compressive yielding or to buckling of the fibre bars.

2.3 Comparison

Due to the different inclination angles of the fibres, the relative densities of the stacking-assembled lattice structure and the three-dimensional intertwined lattice structure differ slightly. Consequently, the relative density of the stacked orthotropic laminae lattice structure is about 9% larger than that of the three-dimensional intertwined lattice structure.

If the dimensions of the two structures and the length of the fibre bars are the same, the stiffness of the two structures can be compared. Under these circumstances, the relationships between the number of cells in height and width are

$$m_1=\frac{\sqrt{6}}{3}m_2,\quad n_1=\frac{\sqrt{6}}{3}n_2 \tag{24}$$

Thus from Eqs. (5) and (14), the ratio of the stiffness of the two structures is 0.817. This means that the stiffness of the three-dimensional intertwined lattice structure is 22.4% greater than that of the stacked lattice structure.

The largest difference, however, of these two structures is the shear strength of the pin-joint nodes. For the stacking-assembled lattice structure, only the resin in the nodes resists the shear stress, even though the shear strength of the resin is very small. For the three-dimensional intertwined lattice structure, four fibres cross the nodes continuously so that the shear stress is balanced by the axial stress of the carbon fibers. Due to the high axial strength of the fibres, the shear strength of the nodes is much greater than that for nodes in the stacking-assembled lattice structure.

According to this analysis, under three-point bending for the same loads with the same dimension and length of fibre bars, the maximum shear stress of the stacking-assembled lattice structure is 1.732 times bigger than the shear stress of the three-dimensional intertwined lattice structure. In contrast, the shear strength of the three-dimensional intertwined lattice structure is about 30 to 40 times greater than that of the stacking-assembled lattice.

Thus, the shear strength of the stacking-assembled lattice structure is much smaller than that of the three-dimensional intertwined lattice structure. The lack of shear strength of the laminae interface in the stacking-assembled lattice structure restricts its application.

3 Manufacturing of Three Dimensional Intertwined Lattice Structure

To make the lattice structure, plates with rows of pinholes and fibers with a designed radius were first prepared. The plates were angle-ply composite laminates. Each ply was made up of unidirectional fibres. The pinholes were introduced after laying the plies. The preformed plates have in-plane dimensions larger than the final sandwich plates, but a thickness smaller than the designed thickness. The initial thickness differs to the final thickness by about one diameter of the fibre-bundled rods. The fiber-bundled rods were made of unidirectional fibres soaked in resin to form carbon/resin fibre rods. The cross section of each rod is almost circular, and the fibres were straight to ensure the strength and stiffness of the rods.

The plates were positioned by nuts on four threaded steel columns, as shown in Fig. 8. The initial spacing between the two plates was deliberately set smaller than the final value. A flexible rod reinforced by continuous fibres was fixed in a hole on the bottom plate to start the weaving process. The fibre was intertwined diagonally from the bottom hole to a hole on the upper plate. The fiber was then intertwined from the upper hole along another diagonal direction of a cube to the next bottom hole. This process was repeated to generate a planar lattice network of fiber rods. The intertwined lattice plane was formed parallel to the longitudinal direction of the sandwich. The planar lattice network made an angle of 45° with the plates, as shown in Fig. 9. When one planar network was finished, the fiber was intertwined to another pinhole at the end of the beam to start the next planar lattice network along the beam. In this way, the lattice sandwich core was manufactured with continous fibers. The plates and the core were woven integrally.

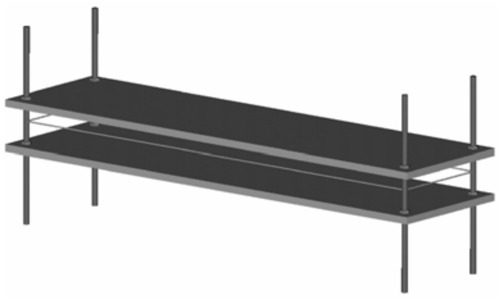

Fig. 8 Top and bottom plates positioned by nuts on four threaded columns

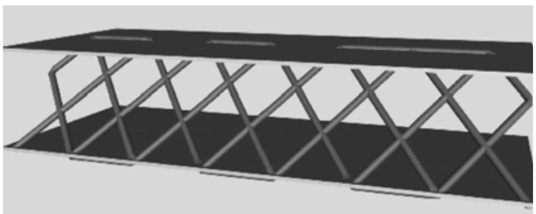

Fig. 9 Construction of an inclined lattice lamina

After the weaving process, the top and bottom plates were pushed outward by rotating the nuts on the threaded columns. When the height between the two plates reached a designated value, the fibre bars were tightened and the possibility of buckling of the bars decreased. The stitched plate surfaces were smoothed by over-laying new plies on the plates until the thickness of the plates reached the designated value.

Finally, the preformed sandwich beam was placed into an autoclave to set the structure. During heating, the resin was melted and redistributed, joining the fiber rods together at the nodes. After resolidification of the resin, the structure was formed. The last step was to remove the threaded columns and then the plates were cut out to the required in-plane dimensions. The fabricated lattice sandwich beam is shown in Fig. 10.

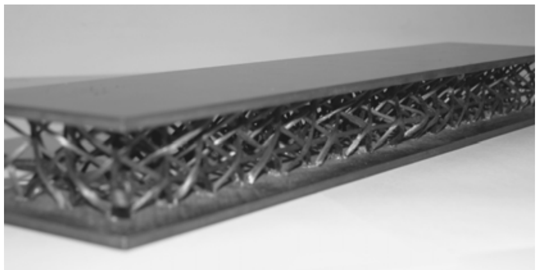

Fig. 10 Sample of the three-dimensional intertwined lattice sandwich

4 Tests and Conclusions

The optimization analysis indicates that the three-dimensional intertwined lattice structure has a superior specific stiffness compared to the stacking-assembled lattice structure. Furthermore, the intertwined fibre-rods at the nodes offer larger shear strength than that offered by the stacking-assembled lattice structure.

A sample of the three-dimensional intertwined lattice sandwich has been manufactured. Examination of the sandwich sample shows that the plates and the core were fabricated integrally, preventing the delamination that often occurs in conventional sandwich samples. Together with a high shear strength, the lattice sandwich structure is expected to possess better mechanical capability.

The sample was compressed to determine the stiffness of the lattice core. The resulting loading curve is shown in Fig. 11. Note that the sample was not loaded until collapse. The stiffness of the lattice structure is about 46 MPa with a relative density of the lattice just 0.017. For the same relative density, the stiffness of carbon foams is just about 37 MPa as well as 1.3 MPa for honeycombs. So the specific stiffness of the lattice structure is much higher than that for other cellular

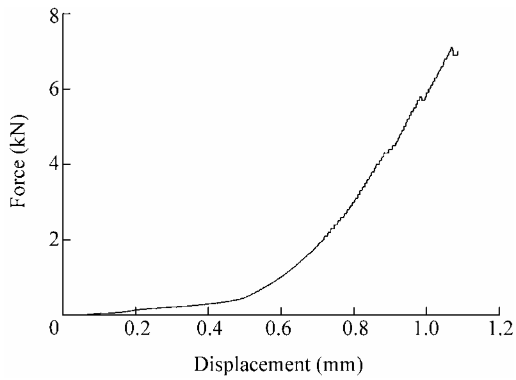

Fig. 11 Compression curve of the lattice structure

structures, giving the lattice structure a good mechanical efficiency.

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